# 시스템 파라미터가 불확실한 대규모 선형 이산시간 시스템의 비집중 안정화에 관한 연구

論 文
34～3～1

## Decentralized Stabilization of a Class of Large Scale Discrete-time Systems Subject to System Parameter Uncertainties

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요 약

본 논문에서는 시스템 파라미터가 불확실한 대규모 선형 이산 시간 시스템을 안정화하기 위하여 비집중 적응 방식이 제안되었다. 이 방식은 모르는 시스템 파라미터들의 영향을 상쇄하기 위한 적응 비선형 궤환과 상호간섭으로부터의 비안정적 영향을 제압하기 위한 기존의 선형 궤환을 결합한 형태이다. 전체 적응 시스템의 안정을 보장하는 충분조건이 유도되었고, 기존의 방식의 유용성을 보이기 위하여 컴퓨터 모사를 통한 수치 예가 제시되었다.

**Abstract**

This paper presents a decentralized adaptive scheme to stabilize a class of large-scale discrete-time linear systems subject to system parameter uncertainties. The scheme combines an adaptive nonlinear feedback control for compensating some effects by unknown system parameters and the exact model-based linear feedback control for overriding the unfavorable effects by interconnections. A condition of stability is derived, under which the overall adaptive system is assured to be globally stable. Also, a numerical example is provided to illustrate the feasibility of the scheme.

**Nomenclature**

| | |
|---|---|
| $\lvert r \rvert$ | : *Absolute value of a real number r* |
| $\lVert a \rVert$ | : *Euclidean norm of a finite dimensional vector a* |
| $A^T$ | : *Transpose of a matrix A* |
| $A^{-1}$ | : *Inverse of a square matrix A* |
| $\lambda_M(A)$ | : *Maximum eigenvalue of a square matrix A* |
| $\lVert A \rVert$ | : *Spectral norm of a matrix A defined as* $\lVert A \rVert = \lambda^{1/2}_M(A^T A)$ |
| $I_n$ | : *n-dimensional identity matrix* |
| $R^n$ | : *n-dimensional vector space* |

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## 1. Introduction

Recently, the real-time implementation of advanced complex control algorithms becomes feasible with the help of powerful microcomputers, and much effort has been made to solve adaptive control problems under discrete-time formulations[1]-[5]. But, the adaptive techniques in these works were for lower order systems, and hence are not directly applied to large scale systems either due to difficulties in simultaneous adjustment of a large number of parameters or due to difficulties in treating interconnections even under decentralized adaptive schemes[6]-[8].

Very few results are available for the problem of adaptive control of large scale systems. Hmamed and Radouane[7] proposed a new type of local adaptive controllers to stabilize a class of interconnected continuous systems. Also, in[8], for the same problem, an alternative simple approach was developed. However, these results were restricted to the class of continuous-time systems in which each subsystem has a single input and moreover is assumed to be given in a controllable form.

In this paper, we propose a decentralized adaptive scheme to stabilize a wider class of discrete-time systems in which each subsystem may have multi-input and have some relaxed assumptions on the system structure. It is noted that an extension of the results in the continuous case to a discrete version is not obvious. The adaptive scheme is a state-space version of[1] and a condition of stability is derived based on the result of[11]. Also, a numerical example is illustrated via computer simulation.

## 2. Problem Statement

Consider the large scale interconnected linear system described by

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{j=1,\ j\neq i}^{N} A_{ij} x_j(k),\ i = 1, 2, \ldots, N \qquad (1)$$

where $x_i \varepsilon R^{n_i}$ is the state of the i-th subsystem, $u_i \varepsilon R^{m_i}$ is its control input, and $A_i$, $B_i$ and $A_{ij}$ are constant matrices of appropriate dimensions. It is assumed here that

(A-1). The upper bounds on the dimensions $n_i$ and $m_i$ are known.

(A-2). The state $x_i$ is available for measurements only at the i-th subsystem.

(A-3). The elements of $A_i$ and $B_i$ are unknown, while the bounds on the elements of $A_{ij}$ are known.

(A-4). The interconnection matrix of any two subsystem i and j is factored as

$$A_{ij} = B_i \bar{A}_{ij},\ i\neq j. \qquad (1.a)$$

For convenience, an additional assumption will be made later.

Now, the problem is to determine a local control for each subsystem (decentralized control) which stabilizes the overall interconnected system (1). For this, we first present a method to design a local adaptive feedback control, and then show that the resultant closed-loop system is assured to be stable.

**Remark 1**

The assumption (A-3) is not unrealistic in the sense that a designer usually has information on the bounds of the interconnection elements, or one may evaluate then through some identification technique, while the system parameters of each subsystem itself may be assumed to be unknown due to inaccurate modelling of the complex dynamic system and/or due to its time-varying feature.

**Remark 2**

It is easy to see that the assumption (A-4) determine the class of systems in which each subsystem is interconnected by the following relation.

$$v_i = u_i + \sum_{j\neq i}^{N} \bar{A}_{ij} x_j.$$

where $v_i$ is the effective input which arises after aggregation of all the interactions affecting the i-th subsystem.

## 3. Design of Local Adaptive Controllers

The system (1) can be rewritten as

$$x_i(k+1) = \bar{A}_i x_i(k) + \bar{B}_i u_i(k) + (A_i - \bar{A}_i) x_i(k) + (B_i - \bar{B}_i) u_i(k) + \sum_{j\neq i}^{N} A_{ij} x_j(k),\ i = 1, 2, \cdots, N \qquad (2)$$

where $(\bar{A}_i, \bar{B}_i)$ is a predetermined controllable pair. In order to stabilize the system (2), the following local adaptive controllers are proposed:

$$u_i(k) = -(I_{mi} + G_i(k+1))^{-1} (K_i + F_i(k+1))\, x_i(k) \qquad i = 1, 2, \cdots, N \qquad (3)$$

where

$$K_i = (I_{mi} + \bar{B}_i^T P_i \bar{B}_i)^{-1} \bar{B}_i^T P_i \bar{A}_i .$$

The symmetric positive definite matrix $P_i$ is the solution of the following discrete algebraic Riccati equation:

$$P_i = \bar{A}_i^T P_i \bar{A}_i - \bar{A}_i^T P_i \bar{B}_i (I_{mi} + \bar{B}_i^T P_i \bar{B}_i) B_i^T P_i A_i + \beta_i I_{ni} \quad (5)$$

for a given positive constant $\beta_i$. Also, in (3), the terms $F_i(k+1)$ and $G_i(k+1)$ are the adjustable parameters whose adaptation laws are to be given later. Applying the local adaptive controls (3) to the system (2), the closed-loop system is then

$$x_i(k+1) = (\bar{A}_i - \bar{B}_i K_i) x_i(k) + (A_i - \bar{A}_i - \bar{B}_i F_i(k+1)) x_i(k) + (B_i - \bar{B}_i - \bar{B}_i G_i(k+1)) u_i(k) + \sum_{j\neq i}^{N} A_{ij} x_i(k), \quad i = 1, 2, \cdots, N \quad (6)$$

It is further assumed here that there exist matrices $F_i^*$ and $G_i^*$ satisfying the following relations:

$$\left. \begin{aligned} A_i - \bar{A}_i &= \bar{B}_i F_i^* \\ B_i - \bar{B}_i &= \bar{B}_i G_i^* \end{aligned} \right\} \quad (7)$$

These conditions actually imply that the column vectors of the matrices $(A_i - \bar{A}_i)$ and $(B_i - \bar{B}_i)$ should be linearly dependent on the column vectors of the matrix $B_i$ [9]. The typical cases where these conditions are satisfied are that [10]

1) the number of state variables in not greater than that of inputs,
2) the state equation is writen in partitioned phase variable canonical form.

It is remarked that the matching conditions (7) are necessary for deriving the generating schemes of $F_i(k+1)$ and $G_i(k+1)$.

Then, the closed-loop system (6) becomes

$$\begin{aligned} x_i(k+1) &= (\bar{A}_i - \bar{B}_i K_i) x_i(k) + \bar{B}_i [F_i^* - F_i(k+1)] x_i(k) + \bar{B}_i [G_i^* - G_i(k+1)] u_i(k) + \sum_{j\neq i}^{N} B_i \bar{A}_{ij} x_i(k), \quad i = 1, 2, \cdots, N \\ &= (A_i - B_i K_i) x_i(k) + \bar{B}_i \theta_i(k+1) \psi_i(k) + \sum_{j\neq i}^{N} \bar{B}_i L_{ij} x_j(k), \end{aligned}$$

where

$$\theta_i = [F_i^* - F_i(k+1), G_i^* - G_i(k+1)]$$

$$\psi_i(k) = \begin{bmatrix} x_i(k) \\ u_i(k) \end{bmatrix}$$

$$L_{ij} = (I_{mi} + G_i^*) \bar{A}_{ij} \quad (9)$$

The $i$-th decoupled subsystem of (8) is given by

$$x_i(k+1) = (\bar{A}_i - \bar{B}_i K_i) x_i(k) + \bar{B}_i \theta_i(k+1) \psi_i(k). \quad (10)$$

Defining

$$z_i(k+1) = x_i(k+1) - (\bar{A}_i - \bar{B}_i K_i) x_i(k) \quad (11)$$

the eqn. (10) is simplified as

$$z_i(k+1) = \bar{B}_i \theta_i(k+1) \psi_i(k) \quad (12)$$

To estimate $\theta_i(k+1)$ in (12), a modified version of the parameter adaptation law in [4, 5] is utilized as follows:

$$\theta_i(k+1) = \theta_i(k) + \Delta\theta_i(k) \quad (13)$$

$$\Delta\theta_i(k) = \delta_i(k) \bar{B}_i^T z_i(k) \psi_i(k-1) \quad (13.a)$$

$$\delta_i(k) = -\frac{\alpha_i}{1 + \psi_i^T(k-1) \psi_i(k-1)} \quad (13.b)$$

where $\delta_i(k)$ is a scalar sequence and $\alpha_i$ is a positive constant satisfying

$$(-2 \alpha_i I_{ni} + \alpha_i^2 \bar{B}_i \bar{B}_i^T) < 0 \quad (13.c)$$

It is noted that the generating schemes of $F_i(k+1)$ and $G_i(k+1)$ in (4) are easily derived from (13).

Regarding the above relations (12) and (13), the following properties hold:

**Lemna 1**

Consider the system (12) whose parameters are adaptively estimated by the law in (13). Let all the initial conditions of $z_i$, $\theta_i$ and $\psi_i$ be bounded. Then

(i) $\theta_i(k)$ is bounded for all $k \geq 0$.

$$\text{(ii)} \lim_{k\to\infty} \frac{\| z_i(k) \|^2}{1 + \| \psi_i(k-1) \|^2} = 0. \quad (14)$$

(Proof). (i) Choose a Lyapunov function candidate such that

$$V_i(k) = \text{tr}\{\theta_i^T(k) \theta_i(k)\} \quad (15)$$

Then, from eqns. (12), (13) and (15), the difference of $V_i(k)$ becomes

$$\Delta V_i(k) = V_i(k+1) - V_i(k)$$

$$= \text{tr}\,\{2\,\theta_i^T(k)\,\delta_i(k)\,B_i^T\,z_i(k)\,\psi_i^T(k-1) + \delta_i^2(k)\,z_i^T(k)\,z_i^T(k)\,B_i\,B_i^T\,z_i(k)\,\psi_i^T(k-1)\,\psi_i(k-1)\}$$

$$\leq \{1 + \psi_i^T(k-1)\,\psi_i(k-1)\}\left\{\frac{2\delta_i(k)\,z_i^T(k)\,z_i(k)}{1+\psi_i^T(k-1)\,\psi_i(k-1)} + \delta_i^2(k)\,z_i^T\,\bar{B}_i\,\bar{B}^T{}_i\,z_i(k)\right\}$$

$$= \frac{z_i^T(k)\,\{-2\alpha_i I_{ni} + \alpha_i^2\,\bar{B}_i\,\bar{B}_i^T\}\,z_i(k)}{1 + \psi_i^T(k-1)\,\psi_i(k-1)} \qquad (16)$$

Since $\alpha_i$ is chosen as in (13.b), $\Delta V_i(k) \leq 0$ for all $k \geq 0$. Hence, it follows that $\theta_i(k)$ is bounded for any bounded initial $\theta_i(0)$.

(ii) Since $V_i(k)$ is a monotonically non-increasing function which is bounded below, it converges to a limit as $k \to \infty$, i.e.,

$$\lim_{k\to\infty} V_i(k) = V_i(\infty) < \infty.$$

Therefore

$$\lim_{k\to\infty} \left|\sum_{l=0}^{k} \Delta V_i(l)\right| = |\,V_i(0) - V_i(\infty)\,| < \infty.$$

In other words, it follows from (16) that

$$\sum_{k=0}^{\infty} \frac{z_i^T(k)\,z_i(k)}{1 + \psi^T{}_i(k-1)\,\psi_i(k-1)} < \infty. \qquad (17)$$

For the inequality (17) to hold, it is obvious that

$$\frac{z_i^T(k)\,z_i(k)}{1 + \psi_i^T(k-1)\psi_i(k-1)} \to 0, \text{ as } k \to \infty.$$

Hence, the result (14) is established

Q.E.D.

## 4. A Condition of Stability

The closed-loop system (9) can be represented in an overall way as

$$x(k+1) = (\bar{A} + \bar{B}\,L - \bar{B}\,K)\,x(k) + z(k+1)$$
$$= A\,x(k) + z(k+1) \qquad (18)$$

where $x^T(k) = [x_1^T(k), \ldots, x_N^T(k)]$; $z^T(k) = [z_1^T(k), \ldots, z_N^T(k)]$; $\bar{A}$ = block diag $[\bar{A}_1, \ldots, \bar{A}_N]$; $\bar{B}$ = block diag $[\bar{B}_1, \ldots, \bar{B}_N]$; $K$ = block diag $[K_1, \ldots, K_N]$; $A = \bar{A} + \bar{B}\,L - \bar{B}\,K$, and the matrix $L$ is defined by

$$L = \begin{bmatrix} 0 & L_{12} & \cdots & L_{1N} \\ L_{21} & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ L_{N1} & \cdots & & 0 \end{bmatrix} \qquad (18.a)$$

Also, define the following terms for later use.

$u^T = [u_1^T, \ldots, u_N^T]$; $\psi^T = [\psi_1^T, \ldots, \psi_N^T]$;
$P$ = block diag $[P_1, \ldots, P_N]$

In order to establish the stability of the closed-loop system (18), the following four Lemmas are needed.

**Lemma 2**

The properties (ii) of Lemma 1 for $i = 1, 2, \ldots, N$ imply that

$$\lim_{k\to\infty} \frac{\|\,z(k)\,\|^2}{N + \|\,\Psi(k-1)\,\|^2} = 0. \qquad (19)$$

(Proof). The eqn. (14) in Lemma 1 implies that for a given $\varepsilon > 0$, there exists $K_{li}(\varepsilon)$ so that

$$\frac{\|\,z_i(k)\,\|^2}{1 + \|\,\psi_i\,(k-1)\,\|^2} \leqslant \varepsilon, \text{ for } k \geqslant k_{li}$$

or

$$\|\,z_i(k)\,\|^2 \leqslant \varepsilon\,(1 + \|\,\psi_i(k-1)\,\|^2), \text{ for } k \geqslant k_{li}. \qquad (20)$$

Let

$$k_l = \max_{1}\,[k_{l1}, k_{l2}, \ldots, k_{lN}]$$

Then, using (20), we sbtain

$$\frac{\|\,z(k)\,\|^2}{N + \|\,\psi(k-1)\,\|^2} = \frac{\sum_{i=1}^{N} \|\,z_i(k)\,\|^2}{N + \sum_{i=1}^{N} \|\,\psi_i(k-1)\,\|^2} \leqslant \varepsilon, \text{ for } k \geqslant k_l.$$

Since $\varepsilon < 0$ in arbitrary, the result (19) follows.

Q.E.D.

**Lemma 3**

There exists a bound $M > 0$ such that

$$\| u(k) \| \leq M \| x(k) \|. \tag{21}$$

(Proof). From (3) and the propeties (i) of Lemma 1 for $i = 1, 2, ..., N$, it easily follows that

$$\| u_i(k) \| \leq M_i \| x_i(k) \| \tag{22}$$

where

$$M_i = \| (I_{mi} + G_i(k+1))^{-1} \| \| (K_i + F_i(k+1)) \|.$$

Let

$$M = \max_i [M_1, M_2, ..., M_N].$$

Then, using (22), we obtain

$$\| u(k) \|^2 \leq \sum_{i=1}^{N} M_i^2 \| x_i(k) \|^2 \leq M^2 \| x(k) \|^2.$$

Hence, the result (21) follows.

Q.E.D

It is remarked here that when $G_i(k+1)$ is almost equal to $-I_{mi}$, $M_i$ in (22) may be nearly infinite. To avoid this degenerate case, the strategy in [1] for $\alpha_i$ satisfying (13.c) can be utilized.

**Lemma 4**

Consider the system (18). If the matrix $\bar{A}$ is asymptotically stable, the following relation holds:

$$\| \psi(k-1) \| \leq C_1 + C_2 \max_{0 \leq \tau \leq k} \| z(\tau) \| \tag{23}$$

Where $0 \leq C_1 < \infty$, $0 < C_2 < \infty$.

(Proof) The solution of (18) is given by

$$x(k) = A^k x(0) + \sum_{l=0}^{k-1} A^l z(k-l) \tag{24}$$

Taking the norm operation on both sides of (24), we get

$$\| x(k) \| \leq \| A^k \| \| x(0) \| + \sum_{l=1}^{k-1} \| A_l \| \max_{1 \leq \tau \leq k} \| z(\tau) \|$$

$$\leq \| A^k \| \| x(0) \| + \sum_{l=0}^{\infty} \| A_l \| \max_{0 \leq \tau \leq k+1} \| z(\tau) \|$$

$$\times \| A^k \| \| x(0) \| + \frac{M}{1-\mu} \max_{0 \leq \tau \leq k+1} \| z(\tau) \|$$

$$= C_3 + C_4 \max_{0 \leq \tau \leq k+1} \| z(\tau) \|. \tag{25}$$

It is noted that the first equality in (25) is induced from the fact that if the eigenvalues of A lie inside of the unit circle, one can find two positive constants $\bar{M}$, $\mu$ with $\mu < 1$ such that

$$\| A^k \| \leq \bar{M} \mu^k, \text{ for } k \geq 0.$$

Then, from (21) and (25), it follows that

$$\begin{aligned} \| \psi(k) \| &\leq \| x(k) \| + \| u(k) \| \\ &\leq (1 + M) \| x(k) \| \\ &\leq (1 + M) C_3 + (1 + M) C_4 \max_{0 \leq \tau \leq k+1} \| z(\tau) \| \\ &= C_1 + C_2 \max_{0 \leq \tau \leq k+1} \| z(\tau) \|. \end{aligned}$$

Replacing k by k-1, we obtain the relation (23).

Q.E.D

Let $L_{ij}^M$ be a matrix satisfying the relation

$$A_{ij}^M = \bar{B}_i L_{ij}^M \tag{26}$$

where $A_{ij}^M$ is a known matrix whose elements are composed of the bounds on the elements of $A_{ij}$. It is noted here that if $A_{ij}^M$ is arranged so that the column vectors of $A_{ij}^M$ be linearly dependent on those of $\bar{B}_i$, then such a $L_{ij}^M$ exists.

**Lemma 5**

Consider the homogeneous part of (18)

$$x(k+1) = (\bar{A} + \bar{B} L - \bar{B} K) x(k). \tag{27}$$

The system (26) is asymptotically stable, if $\beta$ can be chosen such that

$$\beta > \| L_M \|^2 (1 + \| \bar{B} \|^2 \| P \|) \tag{28}$$

where

$$\beta = \max_i \{\beta_1, \beta_2, ..., \beta_N\}.$$

and $L_M$ is a known matrix formed by replacing the component $L_{ij}$ in (18.a) by $L_{ij}^M$ in (26).

(Proof). According to Theorem 5 of [11], a sufficient condition for the system (27) to be stable is that

$$\beta > \| L \|^2 (1 + \| \bar{B} \|^2 \| P \|). \tag{29}$$

Since $\| L_M \|^2 \geq \| L \|^2$, we can obtain immediately (28) from (29).

Q.E.D.

Now, using the above four Lemmas, the following main result is obtained.

**Theorem 1 (A sufficient condition of stability)**

Consider the overall closed-loop system given in (18). If $\beta$ can be chosen such that

$$\beta > \| L_M \|^2 (1 + \| \bar{B} \|^2 \| P \| \tag{28}$$

then $z(k)$ and $x(k)$ converge to zero, as $k \to \infty$.
(Proof). If the condition (28) holds, we obtain the relation (23) by applying the result of Lemma 5 to Lemma 4. Then, with the property (19) in Lemma 2 and the relation (23), the following arguments can be developed along the line of the proof of Lemma 3.1 in[1].

If $\{\| z(k) \|\}$ is a bounded suquence, then by (23) $\{\| \psi(k\text{-}1) \|\}$ is a bounded sequence. Hence, it follows that from (19) that

$$z(k) \to 0, \text{ as } k \to \infty.$$

Now assume that $\{\| z(k) \|\}$ is unbounded. This implies that there exist a subsequence $\{\| z(k_j) \|\}$ such that $\{\| z(k_j) \|\}$ is monotonically increasing and

$$\lim_{k \to \infty} z(k_j) = \infty.$$

The relation (23) along the subsequence $\{\| z(k_j) \|\}$ leads to

$$\frac{\| z(k_j) \|^2}{N + \| \psi(k_j - 1) \|^2} \geq \frac{\| z(k_i) \|}{\sqrt{N} + \| \psi(k_i - 1) \|}$$

$$\geq \frac{\| z(k_i) \|}{\sqrt{N} + C_1 + C_2 \| z(k_i) \|}$$

Hence

$$\lim_{k_j \to \infty} \frac{\| z(k_j) \|^2}{N + \| \psi(k_j - 1) \|^2} \geq \frac{1}{C_2} > 0,$$

but this contradict (19) and hence the assumption that $\{\| z(k) \|\}$ is unbounded is false. Then, from the boundedness of $\{\| z(k) \|\}$, it follows that $z(k) \to 0$, as $k \to \infty$ and $\{\| \Psi(k) \|\}$ is bounded. Furthermore, by the definition of $\Psi(k)$, it is obvious that $x(k)$ is bounded for all $k \geq 0$.

The next step is to show that $x(k) \to 0$, as $k \to \infty$. The condition $z(k) \to 0$, as $k \to \infty$ implies that for a given $\bar{\varepsilon} > 0$, there exists an integer $k^*(\bar{\varepsilon})$ such that

$$\| z(k) \| \leq \bar{\varepsilon}, \text{ for } k \leq k^* \tag{30}$$

Also, the eqn. (24) can be rewritten with reference to this $k^*$ as follows:

$$x(k) = A^{k-k^*} x(k^*) + \sum_{i=0}^{k-k^*-1} A^i Z(k\text{-}1) \cdot \tag{31}$$

then, with eqns. (30) and (31), by the same way in the proof of Lemma 4, we get

$$\|x(k)\| \leq \| A^{k-k^*} \| \| x(k^*) \| + \sum_{i=0}^{\infty} \| A^i \| \max_{k^* \leq \tau \leq k} \| \| z(\tau) \|$$

$$\leq C_5 + C_6 \varepsilon$$

Since $C_5$ is zero for a large $k >> k^*$ and $\bar{\varepsilon} > 0$ is arbitrary, we conclude that

$$x(k) \to 0, \text{ as } k \to \infty .$$

Q.E.D.

**Remark 3**

From the parameter adaptation low (13) and the fact that $z(k)$ and $x(k) \to 0$, as $k \to \infty$, it follows that $\Delta\ \theta_i(k)$ go to zero, as $k \to \infty$. But this does not necessarily imply that

$$\theta_i(k) \to 0, \text{ as } k \to \infty$$

i.e., from (9),

$$F_i(k) \to F_i^* \text{ and } G_i(k) \to G_i^*, \text{ as } k \to \infty.$$

**Remark 4**

It is not possible to see immediately whether the condition (28) can be satisfied by some $\beta > 0$ because the solution $P_i$, $i = 1, \ldots, N$ of the Riccati eqn. (5) vary nonlinearly with $\beta_i$. To solve this problem, an algorithm in[11]to find a finite parameter $\beta^*$ such that the constraint (28) is verified for any $\beta > \beta^*$ can be utilized.

## 5. An Example

Consider the unstable discrete-time interconnected linear constant system given by

$$x_1(k+1) = \begin{bmatrix} -0.5 & 1.5 \\ 1 & 0 \end{bmatrix} x_1(k) + \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} u_1(k) + \begin{bmatrix} 0.4 & 0.2 \\ 0 & 0 \end{bmatrix} x_2(k)$$

$$x_2(k+1) = \begin{bmatrix} -0.4 & 1 \\ 0 & 0.5 \end{bmatrix} x_2(k) + \begin{bmatrix} 1 & 0.3 \\ 0 & 2 \end{bmatrix} u_2(k) + \begin{bmatrix} -0.2 & 0 \\ 0.1 & 0.4 \end{bmatrix} x_1(k)$$

with initial states

$$x_1^T(0) = [1, -1],\ x_2^T(0) = [-1, 1].$$

Here, bounds on the elements of interconnections are assumed to be known such that

$$A_{12}^M = \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0 \end{bmatrix},\ A_{21}^M = \begin{bmatrix} -0.3 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$$

As a preliminary step, the exact model for each subsystem $(A_i, B_i)$, and the design parameter $\beta_i$ are chosen such that

$$\bar{A}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\ \bar{A}_2 = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$$

$$\bar{B}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \bar{B}_2 = I_2$$

$$\beta_1 = 1,\ \beta_2 = 3.$$

Then, the solution $P_i$ of (5) is given by

$$P_i = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},\ P_2 = \begin{bmatrix} 3.2 & 0 \\ 0 & 3 \end{bmatrix}$$

Also, $L_{ij}^M$ in (26) is given by

$$L_{12}^M = [0.5\ 0.3],\ L_{21}^M = A_{21}^M.$$

Based on the above data, it can be easily shown that the condition of stability in (28) is satisfied.

Now, using the present adaptive scheme, computer simulations are carried out to stabilize the example system. The results with initial estimates

$$F_1(0) = [0, 0.5],\ G_1(0) = 1$$

$$F_2(0) = 0,\ G_2(0) = I_2$$

and with the scalar adaptation gain

$$\alpha_1 = \alpha_2 = 1$$

are presented in Fig. 1 - 2. It is noted that the notation $s^j_i$ in the figures denote the j-th component of the vector $s_i$.

As can be seen in the figures, the simulation results coincide with the expected ones given in section 4.

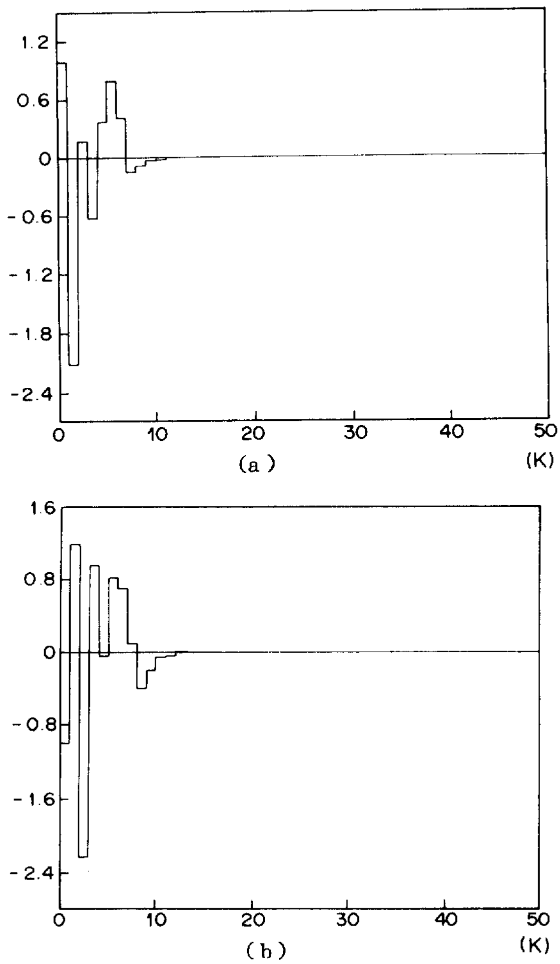
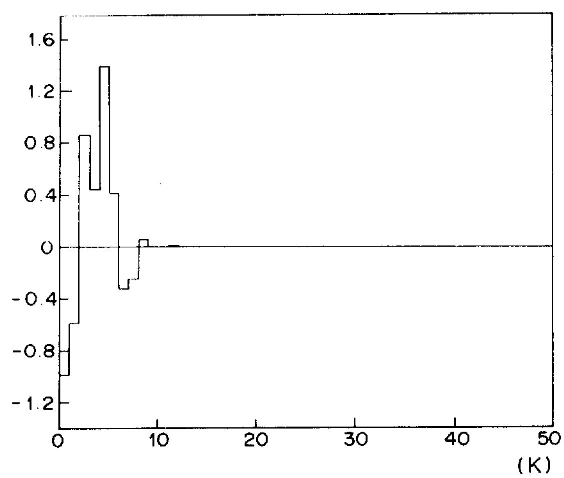



**Fig. 1.** The trajectories of (a) $x_1^1$ (k), (b) $x_1^2$ (k)




(a)

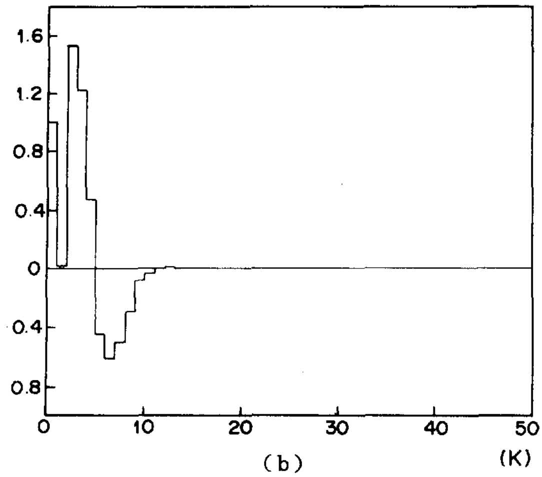



**Fig. 2.** The trajectories of (a) $x_2^1$ (k), (b) $x_2^2$ (k)

## 6. Conclusion

It has been shown that a class of large scale interconnected discrete-time linear systems could be stabilized via local state feedback. The proposed decentralized adaptive scheme has combined an adaptive scheme for compensating some effects by unknown system parameters and the exact model-based linear feedback control for overriding the unfavorable effects by interconnections.

A further topic of immediate interest is to develop some stabilization method which do not require the assumption of the existence of the matching conditions.

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