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Better Estimators of Multiple Poisson Parameters under Weighted Loss Function **

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Abstract

In this study, we consider the simultaneous estimation of the parameters of the distribution of p independent Poisson random variables using the weighted loss function. The relation between the estimation under the weighted loss function and the case when more than one observation is taken from some population is studied. We derive an estimator which dominates Tsui and Press's estimator when certain conditions hold. We also derive an estimator which dominates the maximum likelihood estimator(MLE) under the various loss function. The risk performances of proposed estimators are compared to that of MLE by computer simulation.

I. INTRODUCTION

Let $X_1,\ldots,X_p$ be p independent Poisson random variables, where X_i has parameter λ_i, i=1,...,p. Recently, considerable research has been devoted to the problem of finding better estimators of the λ_i than the MLE under the loss function,

$$L_k(\delta,\lambda) = \sum_{i=1}^{p} (\delta_i - \lambda_i)^2/(\lambda_i)^k, \quad k = 1, 2, \ldots.$$

In this paper, we consider the more general case where more than one observation may be taken from each population. It is possible that there are some situations in which more than one observation is taken from some of the populations. Suppose that $X_{i1},\ldots,X_{in_i}$, $n_i \geq 1$, $i=1,\ldots,p$, $p \geq 2$, are observed

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from the i^{th} Poisson distribution and that all the X_{ij}'s are independent. Consider the sufficient statistics

$$X_i = \sum_{j=1}^{n_i} X_{ij}, i = 1,....,p,$$

which are independent Poisson with means $n_1\lambda_1,..,n_p\lambda_p$, respectively. In many Poisson applications, this happens. For example, X_i is the total number of failures of component type i in n_i time periods with failure rate λ_i under the loss function L_k. Note that the MLE of λ is $(X_1/n_1,.....,X_p/n_p)$.

The risk of the MLE under the loss function L_k is

$$E\left[\sum_{i=1}^{p} (\lambda_i)^{-k} (\delta_i - \frac{X_i}{n_i})^2\right] = E\left[\sum_{i=1}^{p} n_i^{k-1} \cdot (X_i - n_i\lambda_i)^2/(n_i\lambda_i)^k\right].$$

If the n_i's are not equal, then the risk of the MLE becomes a weighted sum of component losses of

$$(X_i - n_i\lambda_i)^2/(n_i\lambda_i)^k , \ i=1,....,p.$$

Since X_i follows a Poisson distribution with parameter $n_i\lambda_i$, we consider the following problem. Suppose X_i follows Poisson (λ_i), i=1,...,p, p≥2. Find an estimators δ of λ such that δ dominates the MLE $X=(X_1,..., X_p)$ under the generalized loss function

$$L_k^c(\delta,\lambda) = \sum_{i=1}^{p} c_i(\delta_i - \lambda_i)^2/(\lambda_i)^k , \ c_i > 0. \ k \geq 1.$$

Then $\delta(X)$ dominates the MLE $(X_1/n_1,..,X_p/n_p)$ under the loss function

$$L_k(\delta,\lambda) = \sum_{i=1}^{p} (\delta_i - \lambda_i)^2/(\lambda_i)^k.$$

For the simultaneous estimation of p independent Poisson parameters, the generalized loss function L_k^c is considered by Tsui and Press(1982). Their estimators which dominates the MLE under the loss function L_k^c are defined componentwise as

$$\delta^{TP}(X) = X_i - k(p-1)\sqrt{c_{(1)}/c_i} \cdot X_i^{(k)} / (S^i + X_i^{(k)}), i = 1,....,p,$$

where (1) $c_{(1)} = \text{minimum}\{c_i\}$,

(2) $X_i^{(k)} = X_i(X_i - 1) ...... (X_i - k + 1)$,

$$(3)\quad S^i = \sum_{i=1}^{p} (X_j+k)^{(k)} - (X_i+k)^{(k)}.$$

The risk improvement, $R(X,\lambda) - R(\delta^{TP},\lambda)$, is at least

$$E\left[c_{(1)}k^2(p-1)^2 / \sum_{i=1}^{p} (x_i+k)^{(k)}\right].$$

II. DERIVED BETTER AND MINIMAX ESTIMATORS

The shrinkage terms of δ^{TP} always depend on the smallest weight c_1. If c_1 is relatively small compared to the other c_j, $j=2,\dots,p$, then the shrinkage terms corresponding to c_j's, $j=2,\dots,p$, are almost zero. Thus the estimators which correspond to $c_2,\dots,c_p$ are almost the same as the MLE. In this case, no significant improvement in risk would be expected by using the estimators δ^{TP}. If c_1 is relatively small, then the amount of loss corresponding to c_1 does not affect the total loss. It is natural to find an estimator which removes the influence of extreme weights.

In the next theorem, we derive an appropriate estimator δ^1 by modifying δ^{TP} to give good risk reduction even when there are extreme coefficients. It is shown that δ^1 dominates δ^{TP} under L_k^c for $p\geq 4$, when some conditions hold on c_i, $i=1,\dots,p$.

Theorem 1

Let $X_i \sim \text{Poisson}(\lambda_i)$, $i=1,\dots,p$, $p\geq 4$, where the X_i's are mutually independent.

Let $$D = \{ c_j : (p-1)(\sqrt{c_1} + \sqrt{c_j}) \leq 2 \sum_{k=j}^{p-1} \sqrt{c_k},\ 2\leq j\leq p-2 \} \qquad (1)$$

and let $j^* = \max\{ j : c_j \in D \}$.

Let
$$\begin{aligned}
c_i^* &= c_1 && \text{if } D = \phi,\ 1\leq i\leq p\\
&= c_1 && \text{if } D = \phi,\ i = 1\\
&= c_i && \text{if } D \neq \phi,\ 2\leq i\leq j^*\\
&= c_{j^*} && \text{if } D \neq \phi,\ j^* \leq i\leq p.
\end{aligned}$$

Define $\delta_i^1(X) = X_i - \sqrt{c_i^* / c_i}\cdot f_i(X)$, $i=1,\dots,p$,

where $f_i(X) = k(p-1)X_i^{(k)}/(S^i + X_i^{(k)})$.

Let $\delta^\circ(X)$ be the MLE.

Then δ^1 dominates δ^{TP} (and necessarily δ^0) under the loss function L_k^c.

Proof

Let $E[\Delta_{TP}^c] = R(\delta^{TP}, \lambda) - R(\delta^0, \lambda)$ under L_k^c and

let $E[\Delta_1^c] = R(\delta^1, \lambda) - R(\delta^0, \lambda)$ under L_k^c.

$$\Delta_{TP}^c = \sum_{i=1}^{p} c_1 k^2(p-1)^2 (X_i+k)^{(k)}/(S^2) - 2k(p-1) \sum_{i=1}^{p} (\sqrt{c_1 c_i}/(X_i+k-1)^{(k-1)}) \cdot A(X) \quad (2)$$

where $$A(X) = \left(\frac{(X_i+k)^{(k)}}{S} - \frac{(X_i+k-1)^{(k)}}{S^i + (X_i+k-1)^{(k)}}\right).$$

After some calculations, we have

$$\Delta_{TP}^c = \sum_{i=1}^{p} c_1 k^2(p-1)^2 (X_i+k)^{(k)}/(S^2)$$
$$-2k^2(p-1) \sum_{i=1}^{p} \sqrt{c_1 c_i}(S^i/(S \cdot (S^i + (X_i+k-1)^{(k)})).$$

Similarly,

$$\Delta_1^c = \sum_{i=1}^{p} c_i^* k^2(p-1)^2 (X_i+k)^{(k)}/(S^2)$$
$$-2k^2(p-1) \sum_{i=1}^{p} \sqrt{c_i c_i^*}\,(S^i/(S \cdot (S^i+(X_i+k-1)^{(k)}))).$$

Now we have

$$\Delta_{TP}^c - \Delta_1^c = k^2(p-1) \sum_{i=1}^{p} \{ (c_1 - c_i^*)(p-1)(X_i+k)^{(k)}/(S^2)$$
$$-2 \sum_{i=1}^{p} (\sqrt{c_1} - \sqrt{c_i^*}) \sqrt{c_i}\,(S^i/(S \cdot (S^i+(X_i+k-1)^{(k)}))) \} \quad (3)$$

$$\geq k^2(p-1) \sum_{i=1}^{p} \left[\frac{(c_1 - c_i^*)(p-1)(X_i+k)^{(k)} - 2(\sqrt{c_1} - \sqrt{c_i^*})\sqrt{c_i}S^i}{(S^2)}\right]. \quad (4)$$

Note that the second term of (3) is nonnegative and

$$(S^2) \geqslant (S) \cdot (S^i + (X_i + k - 1)^{(k)}).$$

We need to show that

$$\sum_{i=2}^{p} (c_1 - c_i^*)(p-1)(X_i + k)^{(k)} - 2\sqrt{c_i}\,(\sqrt{c_1} - \sqrt{c_i^*}) \cdot S^i \geqslant 0 \text{ for all } X. \quad (5)$$

Let $G(X, c)$ be the LHS of (5), then

$$G(X, c) = \sum_{i=1}^{p} (X_i + k)^{(k)} ((p-1)(c_1 - c_i^*) - 2 \sum_{j \neq i} (\sqrt{c_1} - \sqrt{c_j^*})\sqrt{c_j}).$$

$$\text{Let } H(c) = (p-1)(c_1 - c_p^*) - 2 \sum_{j \neq p} (\sqrt{c_1} - \sqrt{c_j^*})\sqrt{c_j})$$

$$= \min \{ (p-1)(c_1 - c_i^*) - 2p \sum_{j \neq i} (\sqrt{c_1} - \sqrt{c_j^*})\sqrt{c_j} \}. \quad (6)$$

We need to show that $H(c)$ is greater than or equal to 0.

$$H(c) = (p-1)(c_1 - c_p^*) - 2 \sum_{j \neq p} (\sqrt{c_1} - \sqrt{c_j^*})\sqrt{c_j}.$$

Let $c_p^* = c_k$, then

$$H(c) = (p-1)(c_1 - c_k) - 2 \sum_{j=k}^{p-1} (\sqrt{c_1} - \sqrt{c_k})\sqrt{c_j} - 2 \sum_{j=2}^{k-1} (\sqrt{c_1} - \sqrt{c_j})\sqrt{c_j}$$

$$= (\sqrt{c_1} - \sqrt{c_k})((p-1)(\sqrt{c_1} + \sqrt{c_k}) - \sum_{j=k}^{p-1} \sqrt{c_j}) - 2 \sum_{j=2}^{k-1} (\sqrt{c_1} - \sqrt{c_j})\sqrt{c_j}$$

$$\geqslant 0 \quad (\text{since } \sqrt{c_1} - \sqrt{c_k} \leqslant 0 \text{ and } (1)).$$

If $p = 2$ or 3 or $D = \phi$, then $\delta^1(X)$ is the same as the δ^{TP}.

The next example gives an application of Theorem 1 for multiple observations per Poisson population under the loss function L_1.

Example 2

Let $X_{ij} \sim \text{Poisson}(\lambda_i)$, $i=1,\ldots,p$, $j=1,\ldots,n_i$, and where the X_{ij}'sare mutually independent.

Let $X_i = \sum_{j=1}^{n_i} x_{ij}$ and let $n_1 \leqslant n_2 \leqslant \ldots \leqslant n_p$. By putting $c_i = n_i^{-1}$, let

$$D = \{ n_j : (p-1)(\sqrt{1/n_p} + \sqrt{1/n_j}) \leqslant 2 \sum_{k=2}^{j} \sqrt{1/n_k} \}, \text{ and}$$

$$j_* = \min\{ j : n_j \in D \}.$$

Define $$\delta_i(X) = X_i/n_i - \sqrt{n_i/n_i^*}(1/n_i)((p-1)X_i/(Z+p-1)) \qquad (7)$$

where (1) $i=1,\ldots,p$, and

(2)
$$\begin{aligned} n_i^* &= n_p & &\text{if } D = \phi & &1 \leqslant i \leqslant p \\ &= n_{j*} & &\text{if } D \neq \phi & &1 \leqslant i \leqslant j_*-1 \\ &= n_i & &\text{if } D \neq \phi & &j_* \leqslant i \leqslant p-2 \\ &= n_p & &\text{if } D \neq \phi, & &p-1 \leqslant i \leqslant p. \end{aligned}$$

Then $\delta(X)$ dominates the MLE $(X_1/n_1,\ldots,X_p/n_p)$ under the loss function L_1.

The previous estimators, δ^{TP} and δ^1, have the form $X_i - g_i(c)f_i(X)$, where the $f_i(X)$'s are decided under the loss function L_k. In the next, we construct a class of estimators which dominate δ^0 under the loss function L_k^c using a different form of an estimator. What we want is;

(1) Estimators which eliminate extreme weights automatically.

(2) Estimators which are robust with respect to loss functions. (i.e, if an estimator $\delta(X)$ dominates δ^0 under L_k^c then $\delta(X)$ also dominates δ^0 under $L_{k'}^c(\neq k)$.

(3) Estimators which reduce to the previous estimators under the loss function L_k when $c_1 = \ldots = c_p$.

First consider the estimators which have the form

$$\delta_i(X) = X_i - w_i(c, X)\frac{h_i(X)}{D(X)}X_i,\ i=1,\ldots,p,$$

where $w_i(c, X) \geqslant 0$. The next theorem defines an estimator $\delta^2(X)$ which dominates δ^0 under the loss function L_k^c for $p \geqslant 2$.

Theorem 3

Suppose X_i's are as given in the Theorem 1. Define the estimator of λ as

$$\delta_i^2(X) = X_i - w_i(c,X)\frac{X_i^{(k)}}{S^i + X_i^{(k)}}, \tag{8}$$

where (1) $i = 1,\ldots,p$, and

(2) $w_i(c,X) = k((p-1) - (i - \frac{\sum_{j=1}^{i} c_j}{c_i}))$.

Then $\delta^2(X)$ dominates δ^0 under the loss function L_k^c.

Proof

$$\text{Let } D_k^c = R(\delta^2,\lambda) - R(\delta^0,\lambda) \text{ under } L_k^c.$$

We have

$$D_k^c = E[\sum_{i=1}^{p}\{c_i w_i^2 (X_i+k)^{(k)}/(S^2)$$

$$-2c_i w_i((X_i+k)/S - X_i/(S^i+(X_i+k-1)^{(k)}))\}]$$

$$\leqslant E[\sum_{i=1}^{p}(c_i w_i^2 (X_i+k)^{(k)} - 2kc_i w_i S^i)/(S^2)]$$

$$= E[(1/(S^2))\sum_{i=1}^{p}(X_i+k)^{(k)}(c_i w_i^2 - 2k\sum_{j\neq i} c_j w_j)].$$

We need to show that

$$c_i w_i^2 \leqslant 2k \sum_{j\neq i} c_j w_j \quad \text{for any } i. \tag{9}$$

After some calculations, (9) is

$$(c_i(p-i) + \sum_{k=1}^{i-1} c_k)^2 \leqslant 2c_i \sum_{j\neq i}((p-j)c_j + \sum_{n=1}^{j-1} c_n). \tag{10}$$

Let A(c) denote the LHS of (10) and B(c) denote the RHS of (10), respectively.

Then

$$B(c) = 2c_i\{ \sum_{1\leqslant j<i} ((p-j)c_j + \sum_{n=1}^{j-1} c_n) + \sum_{i+1\leqslant j\leqslant p} ((p-j)c_j + \sum_{n=1}^{j-1} c_n\}$$

$$\geqslant 2\{ \sum_{1\leqslant j\leqslant i} ((p-j)\, c_jc_i + \sum_{n=1}^{j-1} c_nc_i) + ((p-i)c_i + \sum_{k=1}^{i-1} c_k)\,(c_i\,(p-i))\}. \quad (11)$$

Note that $\sum_{n=1}^{j-i} ((p-j)\, c_j + \sum_{n=1}^{j-1} c_n) \geqslant ((p-i)\, c_i + \sum_{k=1}^{i=1} c_k)$ for $j > i$,

and also note that

$$\sum_{1\leqslant j<i} ((p-j)\, c_jc_i + \sum_{n=1}^{j-1} c_nc_i)$$

$$= \sum_{1\leqslant j<i} \{ (p-i)\, c_jc_i + \sum_{n=1}^{j-1} c_nc_i + (i-j)\, c_ic_j \}$$

$$= (p-i)\, c_i \sum_{j=1}^{i-1} c_j + c_i \sum_{1<j<i} (\sum_{n=1}^{j-1} c_n + (i-j)\, c_j)$$

$$\geqslant ((p-i)c_i + \sum_{j=1}^{i-1} c_j)\cdot(c_1+c_2+..+c_{i-1}). \quad (12)$$

Substituting (12) into (11), we have

$$B(C) \geqslant 2\{ ((p-i)c_i + \sum_{k=1}^{i-1} c_k)(\sum_{k=1}^{i-1} c_k) + ((p-i)\, c_i + \sum_{k=1}^{i-1} c_k)\,(c_i(p-i))\} \quad (13)$$

$$= 2A(c).$$

So the inequality (9) is proved.

$\delta^2(X)$ has the following properties:

(1) If C_i is relatively large compared to $\sum_{j=1}^{i-1} c_j$, then the shrinkage term $w_i(c, X)$ involves $k(p-i)$ instead of $k(p\text{-}1)$. This guards against the effect of extreme coefficients.

(2) If $c_1=...=c_p$, then the shrinkage terms of δ^2 under L_k^c are the same as the ones of the estimator under L_k.

The estimators, δ^1 and δ^2, have different shrinkage terms depending on the loss functions L_k^c

Focusing on the problem of robustness with respect to loss functions, the estimators of the form

$$\delta_i(X) = X_i - w(c, X)X_i, \ i=1,\dots,p, \tag{14}$$

where $w(c,X) \geqslant 0$, are considered under the loss function L_1^c.

The next theorem gives a condition on w(c,X) for the estimator δ^3 to dominate δ^0 under L_1^c.

Theorem 4

Suppose the X_i are as given in Theorem 3.
Define the estimator of λ as

$$\delta^3(X) = (1-w(c, Z))X,$$

where (1) w(c,Z) is a real valued function, $Z = \sum_{i=1}^{p} X_i$,

(2) $w(c,Z)/w(c,Z\text{-}1) \geqslant (Z\text{-}1)/Z$ for $Z \geqslant 1$ and

$$(3)\ 0 \leqslant w \leqslant \left(\frac{\sum_{i=1}^{p} c_i}{c_p} - 1 \right) \Big/ \left(Z + \frac{\sum_{i=1}^{p} c_i}{c_p} - 1 \right). \tag{15}$$

Then $\delta^3(X)$ dominates δ^0 under the loss function L_1^c.

Proof

Let $D_1^c = R(\delta, \lambda) - R(\delta^0, \lambda)$ under L_1^c.

Then $$D_1^c = E_\lambda \left[\sum_{i=1}^{p} c_i \lambda_i^{-1} (w^2 X_i^2 - 2wX_i (X_i - \lambda_i)) \right]. \tag{16}$$

By using the reparameterization method,

$$D_1^c = E[\Lambda^{-1} \sum_{i=1}^{p} c_i \theta_i^{-1} \{((w^2-2w)(Z^2-Z) + 2w\Lambda Z)\theta_i^2 + (w^2-2w)Z\theta_i\}]$$

$$= E[\Lambda^{-1} \{((w^2-2w)(Z^2-Z) + 2w\Lambda Z) \sum_{i=1}^{p} c_i\theta_i + (w^2-2w)Z \sum_{i=1}^{p} c_i\}]$$

$$= E[\Lambda^{-1} (\sum_{i=1}^{p} c_i)\{((w^2-2w)(Z^2-Z) + 2w\Lambda Z)(\frac{\sum_{i=1}^{p} c_i\theta_i)}{\sum_{i=1}^{p} c_i}) + (w^2-2w)Z\}].$$

Using condition (2), we have

$$E[w\cdot Z^2-w\Lambda Z] \geqslant 0. \tag{17}$$

Using this result (17), we have

$$D_1^c \leqslant E[\Lambda^{-1} (\sum_{i=1}^{p} c_i)\{(w^2(Z^2-Z)+2wZ)(\frac{\sum_{i=1}^{p} c_i\theta_i}{\sum_{i=1}^{p} c_i}) + (w^2-2w)Z\}]$$

$$\leqslant E[\Lambda^{-1} (\sum_{i=1}^{p} c_i)\{(w^2(Z^2-Z) + 2wZ)(c_p / \sum_{i=1}^{p} c_i) + (w^2-2w)Z\}].$$

Since $c_p = \max\{c_i\}$ and $\sum_{i=1}^{p} \theta_i = 1$, we have

$$D_1^c \leqslant E[\Lambda^{-1} (\sum_{i=1}^{p} c_i Z)\{w^2((Z-1)(c_p/ \sum_{i=1}^{p} c_i)+1) - 2w(1-c_p/ \sum_{i=1}^{p} c_i)\}]$$

$\leqslant 0.$ (by condition (11))

The next theorem shows that the dominating estimator $\delta^3(X)$ under the loss function L_1^c also dominates under the loss function $L_{k(\geqslant 1)}^c$. This gives the robustness of the estimator $\delta^3(X)$ under the variety of loss functions.

Theorem 5

$\delta^3(X)$ dominates the MLE(δ^0) under the loss functions $L_{k(\geqslant 2)}^c$.

Proof

Let $D_k^c = R(\delta^3, \lambda) - R(\delta^\circ, \lambda)$ under the loss function L_k^c.
From (17),

$$D_k^c = E_\Lambda [\Lambda^{-k} (\sum_{i=1}^p c_i\theta_i^{1-k}) \{ ((w^2-2w)(Z^2-Z) + 2w\Lambda Z) (\frac{\sum_{i=1}^p c_i\theta_i^{2-k}}{\sum_{i=1}^p c_i\theta_i^{1-k}})$$

$$+ (w^2-2w)Z \}]$$

$$\leqslant E[\Lambda^{-k} (\sum_{i=1}^p c_i\theta_i^{1-k}) \{ (w^2(Z^2-Z)+2wZ) (\frac{\sum_{i=1}^p c_i\theta_i^{2-k}}{\sum_{i=1}^p c_i\theta_i^{1-k}}) + (w^2-2w)Z \}].$$

Using a method similar to that of Theorem 1, we have the inequality

$$(\frac{\sum_{i=1}^p c_i\theta_i}{\sum_{i=1}^p c_i}) \geqslant (\frac{\sum_{i=1}^p c_i\theta_i^{2-k}}{\sum_{i=1}^p c_i\theta_i^{1-k}}) \quad \text{for } k\geqslant 1. \tag{18}$$

It follows that

$$D_k^c \leqslant E[\Lambda^{-k} (\sum_{i=1}^p c_i\theta_i^{1-k}) \{ (w^2(Z^2-Z)+2wZ) (\frac{\sum_{i=1}^p c_i\theta_i}{\sum_{i=1}^p c_i}) + (w^2-2w)Z \}]$$

$$\leqslant 0 \qquad (\text{by } (17)). \tag{19}$$

Remarks:

(1) If c_p is much greater than the other c_i's, i=1,....,p-1, then $\delta^3(X)$ is the same as the MLE. Intuitively, we would not expect an improvement in risk over the MLE. Because we know that the MLE is admissible for p=1.

(2) Proper Bayes minimax estimators derived by Clevenson and Zidek(1975) under the loss function L_1 are defined as

$$\delta^{CZ} = (1-(ß+p-1) / (Z+ß+p-1))X, \qquad \text{where} \quad 1<ß\leqslant p-1.$$

If $ß+p-1\leqslant 2 (\sum_{i=1}^p c_i/c_p-1)$, then δ^3 contains the proper Bayes minimax estimator δ^{CZ}.

III. SIMULATION RESULTS UNDER L_1^c.

In this section, the results of the computer simulation are described. We compare the risk performances of estimators $S^{TP}, \delta^1, \delta^2$, and δ^3 under the loss function L_1^c, where

$$W(c, Z) = \left(\sum_{i=1}^{p} c_i/c_p - 1 \right) / \left(Z + \sum_{i=1}^{p} c_i/c_p - 1 \right)$$

is used for $\delta^3(X)$. The procedures of computation are describes below:

(1) The number of independent Poisson random variable were chosen(p=5).

(2) The parameters λ_i were generated uniformly from the intervals (0,3) and (0,6).

(3) Observations from each p independent Poisson distributions were generated 500 times.

(4) The average loss under the loss function L_1^c of each estimators was calculated.

(5) The percentage reduction of average loss(PRAL) over MLE was calculated.

(6) The procedure was repeated 3 times and the average PRAL was calculated and tabled.

(7) The measure of difference among c_i's is defined as

$$\text{Diff.} = \left(\prod_{i=1}^{p} c_i \right)^{1/p} / \left(\sum_{i=1}^{p} c_i/p \right).$$

In most cases the percentage of risk savings is seen to be an increasing function of p, the number of independent Poisson parameters. Also, we see that the risk improvement percentage over the MLE decreases as the range of λ_i's increases. δ^1 has considerable appeal since its improvement in average risk is largest among dominating estimators and δ^1 dominates δ^{TP} under the loss function L_k^c by Theorem 1.

Table 1. Considered Cases on C_i, i=1,...,p

Cases	Number of Parameters	c_2/c_1	c_3/c_1	c_4/c_1	c_5/c_1	Diff.
case 1	5	1	1.5	2	2	0.95
case 2	5	2	3	4	5	0.87
case 3	5	3	6	0	11	0.74
case 4	5	4	6	29	35	0.50

Table 2. PRAL over the MLE under L_1^c for p=5 where $\lambda\epsilon(0,3)$ †

Cases	δ^{TP}	δ^4	δ^5	δ^y
Case 1 (Diff=0.95	35 35 34 (35)	35 35 34 (35)	35 35 34 (35)	34 34 34 (34)
Case 2 (Diff=0.87	29 23 28 (27)	36 29 35 (33)	33 26 32 (30)	31 25 29 (28)
Case 3 (Diff=0.74)	16 22 23 (20)	24 33 34 (30)	23 30 31 (28)	21 29 29 (26)
Case 4 (Diff=0.50)	13 10 14 (12)	26 31 28 (25)	21 17 22 (20)	19 16 23 (17)

The first PRAL in each cell is based on λ=(0.4,0.6,1.1,1.2,2.6).
the second is based on λ=(0.5,1.4,2.1,2.5,2.9) and the third is based on λ=(0.1,0.5,1.2,1.4,2.9).
The parenthetical values are the averages for the three sample PRALs in the cell.

Table 3. PRAL over the MLE under L_1^c for p=5 where $\lambda\epsilon(0,6)$

Cases	δ^{TP}	δ^4	δ^5	δ^6
Case1 (Diff=0.95)	29 20 16 (22)	29 20 16 (22)	29 20 16 (22)	28 19 15 (21)
Case 2 (Diff=0.87)	12 16 23 (17)	16 19 27 (21)	14 17 25 (19)	13 15 24 (17)
Case 3 (Diff=0.75)	10 12 17 (13)	14 16 24 (18)	13 15 22 (17)	12 14 22 (16)
Case 4 (Diff=0.50)	11 7 6 (8)	21 14 12 (16)	16 11 10 (12)	17 10 9 (12)

The first PRAL in each cell is based on λ=(0.2,2.1,2.4,4.6,5.3).
The second is based on λ=(0.1,1.4,3.8,4.5,4.8) and the third is based on λ=(0.1,0.2,1.7,3.0,3.8).
The parenthetical values are the averages for the three sample PRALs in the cell.

REFERENCES

Albert, J. (1979). Robust Bayes Estimation. Mimeograph Series. No. 79-9, Dept of Stat, Purdue University.

Albert, J. (1981). Simultaneous Estimation of Poisson Means. Journal of Multivariate Analysis, 11, 400-417.

Albert, J. (1981). Pseudo-Bayes Estimation of Multinomial Proportions. Commun. in Statist. Theory and Methods. A 10 (16), 1587-1611.

Berger, J. (1977). Pseudo-Bayes Estimation of Multinomial Proportions. Commun. in Statist. Theory and Methods. A 10 (16), 1587-1611.

Ghosh, M. (1983). Estimation of Multiple Poisson Means: Bayes and Empirical Bayes. Statistics and Decisions. 1, 183-195.

Tsui, K. (1978). Simultaneous Estimation of Several Poisson Parameters under Squared Error Loss. Technical Report No. 37, Dept. of Stat., Univ of california, Riverside.

Tsui, K. (1981). Difference Inequalities and Inadmissblity of the Usual Estimator in Multiparameter Estimation of Discrete Exponential Families. Technical Report No. 653. Dept. of Stat., Univ. of Wisconsin.

Tsui, K. and Press, S. (1982). Simultaneous Estimation of Several Poisson Parameters under K-normalized Squared Error Loss. Ann. of Statist. 10.93-100.

Berger, J. (1977). Admissible minimax estimation of normal mean with arbitrary quadratic loss. Ann. of Statist. 4. 223-226.

Berger, J. (1978). Generalized Bayes estimation in multivariate problems. Ann. of Statist. 6. 783-801.

Berger, J. (1979). Multivariate Estimation with Nonsymmetric Loss Functions. In optimizing Methods in Statistics, J.S. Rustagi(Ed). Academic Press. New York.

Berger, J. (1980). *Statistical Decision Theory.* Springer-Verlag, New YOrk.

Bock, J. (1975). Minimax estimators of the Mean of Multivariate Normal Distribution. Ann. of Statist. 3. 209-218.

Brown, L (1975). Estimation with Incompletely Specified Loss Functions (the case of several location parameters). J. Amer. Statist. Assoc. 70.471-427.

Clevenson, M. and Zidek, J. (1975). Simultaneous Estimation of the Means of Independent Poisson Laws. J. Amer. Statist. Assoc. 70,698-705.

Feinberg, S. and Holland, P. (1973). Simultaneous Estimation of Multinomial Cell Probabilities. J. Amer. Statist. Assoc. 68. 683-691.