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Near-rings with IFP

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G. Mason showed the following theorem in [2].

Theorem. [proposition 1 [2]]. *If a zero symmetric near-ring N is unital then the left regularity, the right regularity and the left strongly regularity are equivalent each other.*

In this paper we generalize his theorem to nonzero symmetric near-ring partially.

A *near-ring* N is a system $(N, +, \cdot)$ such that $(N, +)$ is a group, $(N, \cdot)$ a semigroup and the right distributive law holds. N is *regular* if for all x in N, there exists a in N with $x=xax$, and N is *right (left) strongly regular* if for all x in N, there exists a in N with $x=x^2a(x=ax^2)$ [2]. N is called a *right (left) regular* if N is regular and right (left) strongly regular [2]. Undefined terminology refer to [1].

Definition 1 [2]. A near-ring N is with *IFP* if for some a, b in N, $ab=0$ implies $axb=0$ for each x in N.

Theorem 2. *Let N be a right strongly regular near-ring with IFP. Then N is right regular.*

Proof. Assume that $x=x^2a$. It implies that $x^2=x^2ax$. Since N is with IFP, $(x-x^2a)ax=0$, so $xax=x^2a^2x$. Let $b=xa^2$, then $xbx=x^2a^2x=xax=x$ and $x^2b=x^3a^2=x^2a=x$. Thus N is right regular.

Remark If N is a left (or right) strongly regular near-ring, then it is reduced [2]. If N is a zero symmetric reduced near-ring, then it is with IFP [2].

Lemma 3. [2]. *Let N be a left regular with IFP. Then N is right regular.*

Theorem 4. *Let N be an unital near-ring with IFP. Then the left regularity is equivalent to right regularity.*

Proof. Assume that $x=x^2a=xax$ for some a in N. It implies that $0=x-xax=(1-xa)x$ where 1 is the identity in N. Since N is with IFP, $ax-xa^2x=0$. Thus $ax=xa^2x$. Since $x^2a^2x-x=x^2a^2x-xax=(x^2a-x)ax=0$, $x=x^2a^2x$. It follows that $x^2=(xa^2x)(xa^2x)=xa^2x=ax$. Thus $x=xa^2x=ax$. Thus $x=xax=x^3$. Hence N is a left regular near-ring.

By lemma, the converse is true.

Lemma 5. *Let N be an unital near-ring with IFP. Then the regularity is equivalent the right regularity.*

Proof. Assume that $x=xax$ for some a in N. It follows that $(1-xa)x=0$. Since N is with IFP, $(1-xa)ax=0$ so $ax=xa^2x$. Put $b=a^2x$ then $x^2b=x^2(a^2x)=xax=x$. Hence N is right regular.

The converse is true, in general.

Theorem 6. *Let N be an unital near-ring with IFP. Then the followings are equivalent.*

1) *N is regular.*

2) *N is right regular.*

3) *N is right strongly regular.*

4) *N is left regular.*

Remark. If N is zero symmetric, these are equivalent to left strongly regular [2].

Reference

1. G. Pilz, *Near-rings*, North-Holland, Amsterdam, 1977.
2. G. Mason, Strongly Regular Near-rings, *Proc. of the Edinburgh Math. Soc.* (1980), 23, 27-35.