〈論 文〉

Multi-level linear Tracking to the Reservoir System Control

—貯水池시스템제어에 대한 多段階 線型追跡—

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Abstract

Linear tracking problem is analytically solved with the buadratic qerformance measure. This theory has the inherent drawbacks in application, because the tracking assumes no boundness of the control and state vectors.

The tracking was performed to the discrete system and interrupted subject to the violation. Multi-level tracking was intended based on the concept of the Bellman's Principle of Optimality in this paper. The tracking is iterated to get the desired trajectory which is not known in advance. An application was made to real operation of 6 rervoirs over 36 monthly periods for the Han river.

要 旨

선형 추적 문제는 2차 목적함수에서는 해석적으로 풀 수 있다. 그러나 이 이론은 제어 및 상태 변수를 無制約으로 가정하기 때문에 저수지운영에 대한 적응에 취약점을 갖고있다.

이 추적은 이산형 시스템에 대하여 수행하였으며 제약조건에 위배될 경우에 추적을 중단하였다. 이러한 多段階추적은 벨만의 最適性의 原理에 근거를 두고 시도되었다. 적용 실례로서 한강유역의 6개 기존 저수지에 36개월간의 자료로서 최적운영값을 계산 하였다.

1. Introduction

Optimizing the operation of a reservior system is a multi dimensional, hard-to-solve problem. This problem used to be handled by the operations research. However dynamic programming faces some significant difficulties due to the so called curse of dimensionality, and the boundness of the state and decision variables.[3,8] Linear programming has difficulty to describe the dynamic characteristics of the system. Nonlinear programming such as conjugate gradient projection method is suitable to the reservoir system control. However vast amount of efforts is required to get the matrix inverse for the projection.[6] Furthermore the design of the objective function itself is not easy in reflecting the multi-sites, multi-patterns of seasonally varied water demand.

This paper presents an application of the discrete linear tracking theory to the operation of the constrained reservior system.

The linear tracking theory has a great merit which can be analytically solved under the quadratic performance measure. The inherent difficulty in this theory is that the linear regulator/ tracking problem assumes the control and state vectors are not constrained by ny boundary, which is apparently different from the real circumstances. This problem was herein coped with the stage-wise system of the time horizon.

Another major problem is the fact that the desired trajectory is not known in advance. This was obtained by the recursive method.

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2. System Description

2.1 System Equation and Inequaliy Constraints

The dynamic equation of the reservior system can be expressed by Eq.(1) for the discrete axis. The system is assumed invertible, that is, the state and control vectors are one-to-one mapping, which is the most common pactice in the real world.

$$x(k+1)=\Phi x(k)+\Psi u(k)+y(k) \tag{1}$$

where,

$x(k)$: $n\times 1$ state vector, storage level at stage k

$u(k)$: $m\times 1$ control vector, relelase level at stage $k(m=n)$

$y(k)$: $n\times 1$ vector representing inflows(i), diversion(d), losses(l), etc.

$[y(k)=i(k)-d(k)-l(k)\cdots\cdots]$

Φ : $n\times n$ state transition matrix

Ψ : $n\times m$ control transition matrix

The inequality constraints to the state and control vectors are as eqs. (2) & (3).

$$x_{min}\leqq x(k)\leqq x(k)_{max} \tag{2}$$

where, x_{min} is the time-invariant inactive storage level, and

$x(k)_{max}$ is the maximum conservation pool level.

$$u_{min}\leqq u(k)\leqq u_{max} \tag{3}$$

2.2 Desired Trajectory/History

The objective of the operation is to keep the control as possible to the desired patterns which are related to the fluctuation of the monthly-varied water demand at every demand point.

Trott and Yeh[10] suggested a way to get the desired control history which is proportional to the single water demand ratio per month. This concept was generalized as follow;

$$u(k)=Wa(j) \tag{4}$$

where, $a(j)$; m-tuple vector, the ratio of water demand in month j to the yearly demand

$[\sum_{j=1}^{12} a(j)=I]$

j ; the running index to indicate the calender month of the stages

W ; $m\times m$ diagonal matrix, the maximum firm water supplies from m-reservoirs per annum

The corresponding state trajectory to the desired control history is obtained by forward-solving variables in terms of the control variable such as,

$$\hat{x}(k)=x(o)+\sum_{j=1}^{k}\Psi\hat{u}(j-1)+\sum_{j=1}^{k}y(j-1) \tag{5}$$

The desired trajectory and history should be known before tracking. However these are not generally acquainted in quantitative term for reservoir operation. So the initial trajectory was assumed and updated by repeating tracking until no more improving was achieved.

Annual firm water supply from reservoir i is defined as the minimum water amount rated to meet the water demand pattern such as,

$$W_i=\min[u_i(k)\beta_i(j)] \tag{6}$$

where, $\beta(j)=1/\alpha_i(j)$

3. Discrete linear tracking theory

The performance measure for the discrete system can be expressed as,

$$J=\frac{1}{2}[\hat{x}(N)-x(N)]^T V[x(N)-\hat{x}(N)]$$

$$+\frac{1}{2}\sum_{k=0}^{N-1}\{[x(k)-\hat{x}(k)]^T Q[x(k)-\hat{x}(k)]+u^T(k)Ru(k)\}$$

$$\triangleq\frac{1}{2}||x(N)-\hat{x}(N)||^2_V+\frac{1}{2}\sum_{k=0}^{N-1}\{||x(k)-\hat{x}(k)||^2_Q+||u(k)||^2_R\} \tag{7}$$

where, V; $n\times n$ real symmetric weighing matrix, PSD.

Q; $n\times n$ real symmetric weighing matrix, PSD.

R; $m\times m$ real symmetric weighing matrix, PD.

N; a stage integer greater than zero, scale of operation stages

V,Q and R matrices are assumed time-invariant in this work.

The Hamiltonian is defined as,

$$H[x(k),\ u(k),\ \lambda(k+1)]$$

$$=\frac{1}{2}||x(k)-\hat{x}(k)||^2_Q+\frac{1}{2}||u(k)||^2_R+\lambda^T(k+1)[\Phi x(k)+\Psi u(k)+y(k)] \tag{8}$$

The Lagrangian of the performance measure eq. (7) and the system equation (1), calculus of variation and the first order Taylors series expansions give the transversality condition as,

$$\frac{\partial}{\partial\lambda}\left[\frac{1}{2}||x(N)-x(N)||^2_V\right]\delta x(N)-\lambda^T(N)\delta x(N)=\theta \tag{9}$$

And the costate equation is,

$$\lambda^*(k)=\frac{\partial H}{\partial x}$$

$$=Qx^*(k)+\Phi^T\lambda^*(k+1)-Qx(k) \tag{10}$$

where, *denotes the optimal

Equation (10) is the discrete version of Euler Lagrange equation and λ (k) is solved backward provided the boundary condition. [$\lambda(N)$ is given.]

The boundary conditionis obtained by eq. (11) for arbitrary δx $(N)\neq\theta$,

$$\lambda^*(N)=Vx^*(N)-Vx(N). \tag{11}$$

4. Optimal control law

Under the assumption of unboundness of the control vector the optimal control is,

$$\frac{\partial H}{\partial u}=Ru^*(k)+\Psi^T\lambda^*(k+1)=\theta \tag{12}$$

therefore,

$$u^*(k)=-R^{-1}\Psi^T\lambda^*(k+1) \tag{13}$$

For simplicity, the optimal denotes hereafter without the superscript* when it is not ambiguous.

Substituting $u(k)$ in eq. (1) with eq. (13) yields,

$$x(k+1)=\Phi x(k)-\Psi R^{-1}\Psi^T\lambda(k+1)+y(k) \tag{14}$$

Eq. (10) and eq. (13) are the first order, linear and time-varying, nonhomogenous difference equations which are to be solved. Suppose its solution such as,

$$\lambda(k)=P(k)x(k)+s(k) \tag{15}$$

where, $P(k)$:$n\times n$ symmetric unknown matrix

$s(k)$:$n\times 1$ unknown vector

The Riccati equation, boundary condition, etc[1] are as follow;

$$P(k)=Q+\Phi^T[P^{-1}(k+1)+\Psi R^{-1}\Psi^T]^{-1}$$

and

$$s(k) = -\Phi^T\{[P^{-1}(k+1) + \Psi R^{-1}\Psi^T]^{-1}\Psi R^{-1}\Psi^T - I\}s(k+1) + \Phi^T[P^{-1}(k+1) + \Psi R^{-1}\Psi^T]^{-1}y(k) - Q\hat{x}(k) \tag{17}$$

and

$$\left.\begin{aligned} P(N) &= V \\ s(N) &= -Vx(N) \end{aligned}\right\} \tag{18}$$

The final results for th eoptimal control are as follow;

$$u(k) = -R^{-1}\Psi^T\Phi^{-T}[P(k) - Q]x(k) - R^{-1}\Psi^T\Phi^{-T}[s(k) + Q\hat{x}(k)] \triangleq F(k)x(k) + g(k) \tag{19}$$

where

$$F(k) = -R^{-1}\Psi^T\Phi^{-T}[P(k) - Q] \tag{20}$$

$$g(k) = -R^{-1}\Psi^T\Phi^{-T}[s(k) + Q\hat{x}(k)] \tag{21}$$

Kalman gains $F(k)$ and the command signal $g(k)$ are precomputed and stored. The optimal control $u(k)$ can be obtained by running forward equation (19) with the given intial condition $x(0)$. This is a closed-loop optimal discrete system.

5. Multi-level Tracking

The real environment might cause frequently violation the assumption that the state and control vectors satisfy their constraints in the optimal control law. Experience shows that both constraints, especially the state constraints play an important roles even for the monthly operation in the region like Korea where the flow range from low to high is significantly large.

According to the concept of the Rosen's gradient projection[9] Proj.$\left(-\frac{\partial J}{\partial u}\right)$, the control is changed along the projection of the negative gradient, which is is identical to the direction by the optimal control law onto the boundary of the admissible region. This can be shown in the Pontryagin's minimum principle so that, the unsaturated control is,

$$u^*(k) = -R^{-1}\Psi^T\lambda^*(k+1)$$

and, the saturated control is,

$$u^*(k) = \begin{cases} u^-(k), & \text{for } R^{-1}\Psi^T\lambda^*(k+1) > u^-(k) \\ u^+(k), & \text{for } R^{-}\Psi^T\lambda^*(k+1) < u^+(k) \end{cases}$$

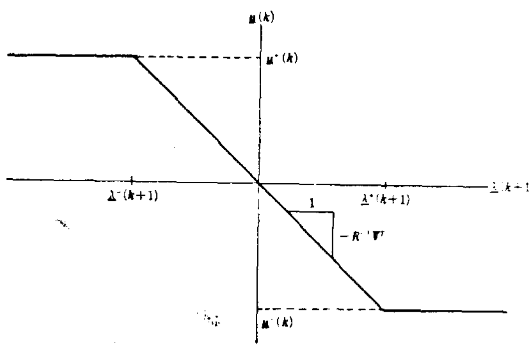

Fig.1 Behaviour of optimal control

When a violation happens at stage k on the processing the tracking, the vector moves to the boundary, and the feed-back correction is provided to keep the system equation upon necessity. When the vectors lie in the admissible region, the linear tracking starts again in accordance with the principle of optimalty, that is, whatever the initials are, the remaining must be an optimal with regard to the state resulting from the previous decision.

6. Application to the Han River in Korea

The Han river comprises the capital city Seoul and one of the most important source to feed the Republic of Korea. The existing reservoir system shows as Fig. 1.

The most critical years [Hall (2)] are selected as the operation period from January 1918 to December 1920, i.e., 36 stages.

The seasonal date and characteristics are tabulated in Table 1, 2.

Table 1. Reservoir charateristics

Item	Hwachon (1)	Soyang (2)	Chunchon (3)	Uiam (4)	Chongpyung (5)	Paldang (6)
CA(SQKM)	3,901	2,703	4,736	7,770	10,140	23,800
x_{min}(MCM)	360.4	1,000.0	89.0	41.0	102.9	226.0
x_{max}(MCM)	1,018.4	2,400.0 2,569.0*	150.0	80.0	185.0	244.0
u_{min}(MCM/M)**	80.4	70.0	90.0	110.0	130.0	200.0
u_{max}(MCM/M)**	20,000.0	17,000.0	23,000.0	29,000.0	33,000.0	51,000.0

*Maximum storage from Jun. through Sept.

**Assumed

Table 2. Water demand ratio by month

Month	Jan.	Feb.	Mar	Apr.	May	Jun.
Ratio	0.066	0.066	0.066	0.089	0.113	0.105
Month	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Ratio	0.105	0.105	0.082	0.071	0.066	0.066

*Assumed

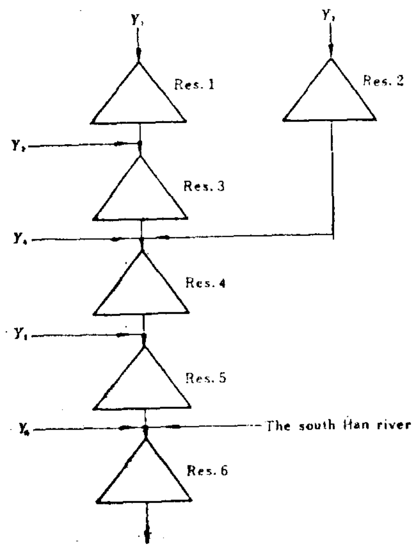

Fig.1 The existing reservoir system of the Han river, Korea

Table 3 Convergence to the optimal Firm Water Supply(MCM/Y)

Iter	Hwachon	Soyang	Chunchon	Uiam	Chongpyong	Paldang
1	500.00	500.00	500.00	1000.00	100.00	100.00
2	1212.12	1060.61	1363.64	1666.67	1969.70	3030.30
3	1212.12	1060.61	1363.64	1666.67	1969.70	3030.30

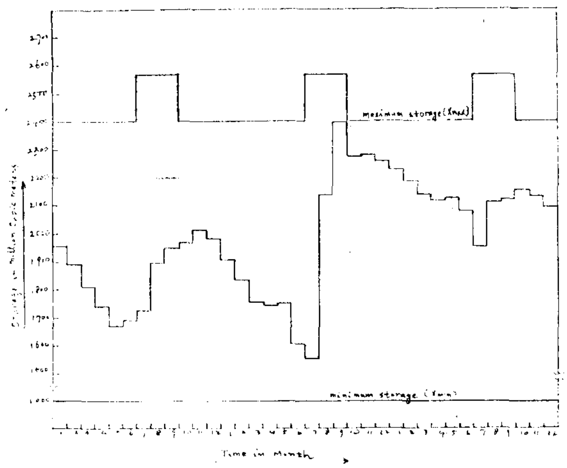

Fig.2 The optimal trajectory of the state for the Soyang Lake

7. Discussion and conclusion

1) The optimal control vector is generally determined by solving simultaneously the state and costate equations. It is therefore recognized that the optimal control history for the saturated part cannot generally be determined by calculating the optimal control history for the unsaturated part and by allowing it to saturated whenever the stipulated boundaries are violated [Kirk (4)].

In this work, the behavior of the costate vector subject to the violation was not fully identified but tracking was simplified by multi-leveling in accordance with the principle of optimality. Author is not in a positive place to insist that the results are optimal. However this paper was aimed at demonstrating the applicability of the algorithm on the practical point of view. Validity of the method would be verified by further study such as the Pontriagin'sminimum principle.

2) The boundary condition was obtained under the condition of fixed final time and final state free. In order to get strict evalutation of the potential, the fixed terminal condition would be desired.

3) The result was tested with three different initial conditions and the identical performance was obtained. As shown in the Table 3, the linear tracking presents rapid convergence. Almost the second iteration gives the saturated value.

4) In order to guarantee the convexity of the performance measure, the weighing matrices R, V, and Q should be symmetric nonnegative definite, that is, it makes the second variation to be positive $[\delta^2 J > \theta]$. Especially R must be positive definite to provide the existence of its inversion in deriving eq. (13).

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