論 文

大 韓 造 船 學 會 誌
第21卷 第3號 1984年 9月
Journal of the Society of
Naval Architects of Korea
Vol. 21, No. 3, September 1984

Extension of Guilloton's Method for the Calculation of Wave-making Resistance and Velocities at the Vicinity of a Ship Hull (1st Report)

by

D. K. Lee*

Abstract

Guilloton's wedge method is extended to evaluate velocity components on and around a ship hull. A ship is divided into a number of layers each of which is approximated by the superposition of so many wedges.

These wedges start from the stations evenly placed along the length of the ship. The Michell potential is used to obtain the field generating properties of a wedge. The derivatives of this potential represent then the velocity components induced by the wedge. Superposition of velocities induced at a fixed field point by all the wedges placed at the appropriate positions to approximate the hull will result in the velocity associated with the ship at a particular speed.

Nomenclature

ϕ : velocity potential
U : free stream velocity (i.e. speed of ship)
g : acceleration due to the gravity
(x, y, z): field point
(x_w, y_w, z_w): wedge position
Δ^2 : second difference of offsets
K : g/U^2, wave number
m : spacing between stations
n : spacing between waterlines
v : $ui+wj+wk$, disturbance velocity around the ship
v_w : $u_w i+v_w j+w_w k$, velocity induced by a wedge
ξ, η, ζ: wedge coordinate system, parallel to x, y, z-direction respectively
ζ_w: wave elevation

1. Introduction

It is well known that Guilloton's wedge method can predict wave-making resistance of a ship with moderate block coefficient to a degree of surprisingly good accuracy (1, 2). This relatively simple procedure is in fact the only practical means of estimating the resistance component when no high speed electronic computer is available. Although the possibility of theoretical prosperity might be limited in this line of approach to the ship wave-making problem, compared to that of highly versatile theoretical concept of singularities, the method still has inspired some investigators (3, 4) to analyse the problem under the light of the method.

Its accuracy and simplicity of application have motivated the present attempt to calculate the velocity components of ideal fluid at the vicinity of a ship hull within its frame. There are other methods (5, 6) which can be used for this purpose but it has been hoped that the wedge method could produce as good results on the velocity components as it does on the resistance component.

The prediction of velocity components around a

Manuscripts received on April 2, 1984; Revised Manuscripts received on July 13, 1984.
* Member, Dept. of Naval Architecture and Shipbuilding, Ulsan Institute of Technology

ship hull is one of the deeply interested problems to naval architects. It offers the important information for ship appendage design and propeller performance analysis as well as its design. However, in reality, the velocity around a ship hull is considerably different from the one which might be obtainable with the assumption of ideal fluid, due to the existence of boundary layer.

Hence the present investigation as it stands cannot be applicable to a region where the effect of viscosity is profound. Still, even for the calculation of three dimensional boundary layer, the velocity distribution of ideal fluid is prerequisite information from which the calculation can proceed.

The accuracy of Guilloton's wedge method comes up after the so-called space transformation is performed. This part of the method is, however, of quite different nature from the other part i.e. the evaluation of velocity components. Therefore it has been felt better to separate the present investigation into two parts: the first, this report, is concerned with providing the means of computing velocity components while the other will deal with the way of determining the hull shape to which the calculated result apply.

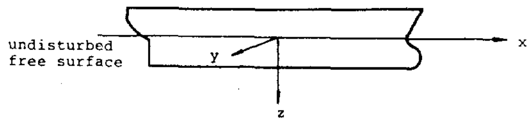

Fig. 1 Coordinate System

3. Michell Potential in a Series Form

Guilloton took Michell potential (7) for a ship and argued that the function representing hull slope could be approximated by some form of double series as shown below.

As the way of constructing this series, Guilloton summed the slopes of semi-infinite wedges which were so shaped and placed that superposition of their offsets could closely approximate those of the ship hull at question. More specifically, he broke the underwater part of a hull into six horizontal layers each of which was represented by linear combination of a number of evenly placed wedges (8). The hull contour on a waterline can be approximated to the accuracy of quadratic curve.

Taking the double summation symbols out of the integrals in eq. (1), it can easily be seen that the velocity potential around a ship may be obtained by

$$\begin{aligned}\phi=&\frac{2U}{\pi^2}\int_0^\infty\int_0^\infty\int_0^\infty\int_{-\infty}^\infty\sum_p\sum_q f_{pq}(\alpha,\beta)\frac{\cos(\tau z-\epsilon)\cos(\tau\beta-\epsilon)}{\sqrt{\sigma^2+\tau^2}}\cos[\sigma(\alpha-x)]e^{-y\sqrt{\sigma^2+\tau^2}}d\alpha d\beta d\sigma d\tau\\&-\frac{2U^3}{\pi g}\int_{g/U^2}^\infty\int_0^\infty\int_{-\infty}^\infty\sum_q\sum_p f_{pq}(\alpha,\beta)\frac{\sigma e^{-\sigma^2U^2(z+\beta)/g}}{\sqrt{\sigma^2U^4/g^2-1}}\sin\,[\sigma(x-\alpha)+\sigma y\sqrt{\sigma^2U^4/g^2-1}]d\alpha d\beta d\sigma\\&+\frac{2U^3}{\pi g}\int_0^{g/U^2}\int_0^\infty\int_{-\infty}^\infty\sum_q\sum_p f_{pq}(\alpha,\beta)\frac{\sigma e^{-\sigma^2U^2(z+\beta)/g}}{\sqrt{1-\sigma^2U^4/g^2}}\cos[\sigma(\alpha-x)]e^{-\sigma y\sqrt{1-\sigma^2U^4/g^2}}d\alpha d\beta d\sigma\end{aligned}\qquad(1)$$

the superposition of velocity potentials arised from each member wedge.

2. Coordinate System and Assumptions

The x-axis is directed forward and the z-axis is chosen to be vertically downward. The y-axis is toward starboard side to make the system right hand one, the x-y plane being coincident with the undisturbed free surface. The origin of the system moving with the ship may conveniently be placed amidship although this is not an absolute requirement. This system is shown in Fig. 1.

The assumptions are the same as those in Michell's paper (7), that is, ideal fluid, small free surface undulation, thin ship and etc.

4. Wedge Representation of a ship Hull

The underwater part of a hull is divided into a small number of layers of equal thickness. Following Guilloton (8), six layers are considered in Fig. 2. It demonstrates how a vertical section can be made up by combining wedge sections.

The section at $(q+1)$th station is assumed to be shown in this figure. The volume OAK represents that part already filled up by the wedge system placed

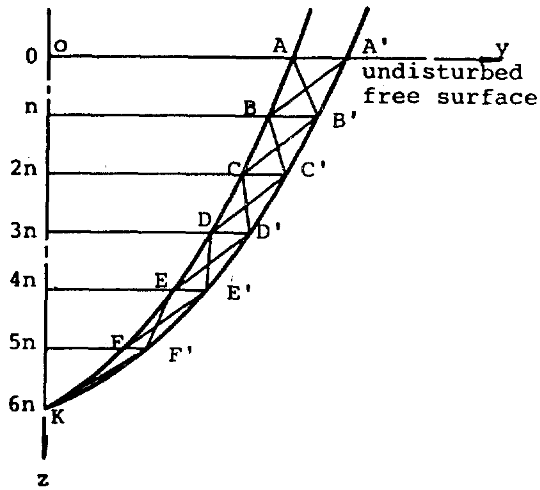

Fig. 2 Wedge Representation of a Section

up to $(q-1)$th station. The volume AKA' is the part that should be completed by the wedges to be located at q-th station. It is quite easy to see that five full wedges AB'C, BC'D, CD'E, and etc., with their vertices at $z=n$, $2n, \ldots$, $5n$ and one half wedge AA'B fulfil this function very well.

5. Wedge Equations

The wedges to be placed at the bow and stern should have noses of nonzero small angle. These wedges are called the sharp wedges. On the other hand, to produce a smooth hull form, the wedges that will be positioned between the both ends must have noses whose offsets increase gradually. These wedges produced by adding parabolic part at the nose of the sharp wedge are referred to as the rounded wedges. The both types of wedge are shown in Fig. 3.

The strength of a wedge is defined by the tangent of the angle (θ in Fig. 3) between the wedge base

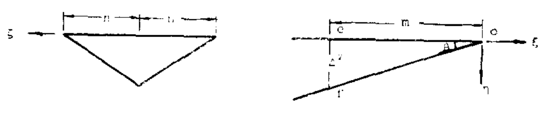

Fig. 3a Sharp Wedge

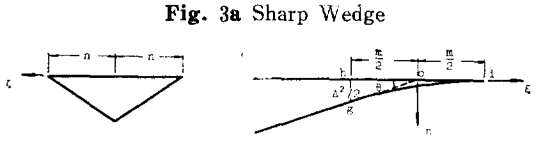

Fig. 3b Rounded Wedge

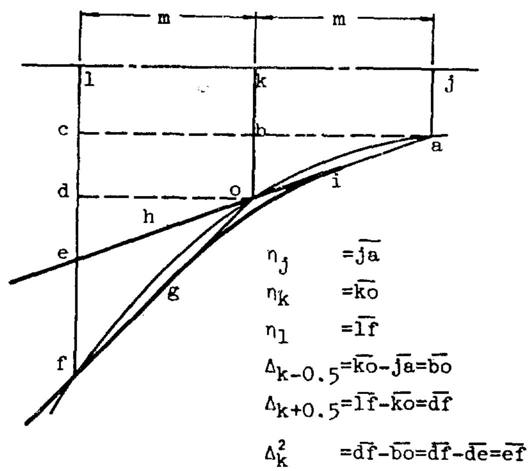

Fig. 4 Wedge Construction

and the straight part of the vertex line. Table 1 shows how the strengths of wedges to be distributed over the entire ship length can be determined from the hull offsets table. Fig. 4 reveals the meaning of the wedge strength as how the hull offsets are filled by successive location of wedges.

A wedge section in vertical plane can again be decomposed into four element wedges of right-angled triangular shape (Fig. 5).

In fact, right-angled triangular section is the fundamental wedge form and was used as the wedge in Guilloton's early works (9). Wedge of this form is referred to as an elementary wedge. The offset of an elementary wedge of which the baseline is located at $\zeta=a$ can be given by

* rounded elementary wedge

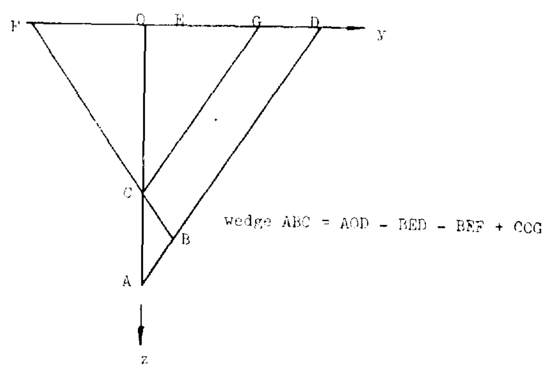

Fig. 5 Decomposition of a Wedge Section into Four Elementary Wedges

Table 1 Calculation of Strength of the Wedges

St. No.	Offsets	1st Diff. $\varDelta$	2nd Diff. $\varDelta^2$	Wedge Strength
0	η_0	$\varDelta_{-0.5}=\eta_0$	$\varDelta_0^2=\varDelta_{0.5}-\varDelta_{-0.5}$	$\varDelta_0^2/m$
1	η_1	$\varDelta_{0.5}=\eta_1-\eta_0$	$\varDelta_1^2=\varDelta_{1.5}-\varDelta_{0.5}$	$\varDelta_1^2/m$
2	η_2	$\varDelta_{1.5}=\eta_2-\eta_1$	$\varDelta_2^2=\varDelta_{2.5}-\varDelta_{1.5}$	$\varDelta_2^2/m$
3	η_3	$\varDelta_{2.5}=\eta_3-\eta_2$	$\varDelta_3^2=\varDelta_{3.5}-\varDelta_{2.5}$	$\varDelta_3^2/m$
4	η_4	$\varDelta_{3.5}=\eta_4-\eta_3$	·	·
⋮	⋮	⋮	⋮	

at $\zeta=\underline{a}$ can be given by

*rounded elementary wedge

$$\eta_{EW,R}=\frac{\varDelta^2}{mn}\left\{\frac{(\xi-m/2)}{2m}[H(\xi+m/2)-H(\xi-m/2)]-\xi H(-\xi-m/2)\right\}[(a-\zeta)H(a-\zeta)] \tag{2}$$

* sharp elementary wedge

$$\eta_{EW,S}=-\frac{\varDelta^2}{mn}\xi H(-\xi)(a-\zeta)H(a-\zeta) \tag{3}$$

In the above equations, H is Heaviside unit step function. A wedge whose vertex line is located at $z=pn$ can be constructed by combining the above expressions as suggested in Fig. 5. To be specific, the result is

$$\eta=(\eta_{EW})_{a=(p+1)n}-2(\eta_{EW})_{a=pn}+(\eta_{EW})_{a=(p-1)n} \tag{4}$$

This expression will serve as the wedge equation.

6. Velocity Potential Induced by an Elementary Wedge

Differentiating eq. (2) and eq. (3) with respect to ξ and substituting the results into the Michell potential, we obtain, after some algebraic manipulation, the following expressions for the velocity potential induced by an elementary wedge: (5) and (6).

7. Velocity Induced by Wedges

Expressions for the velocity components induced by elementary wedges are obtained by differentiating eq. (5) and eq. (6) with respect to ξ, η and ζ. These expressions can directly be used to calculate velocity components induced by half wedges located at the top layer (wedge $AA'B$ in Fig. 2). Expressions for a full wedge are produced by combining these results as Fig. 5 and eq. (4) imply. For instance, the velocity induced by a wedge whose vertex is located at $z=pn$ will appear as follows:

$$v_W=(v_{EW})_{a=(p+1)n}-2(v_{EW})_{a=pn}+(v_{EW})_{a=(p-1)n} \tag{7}$$

wherer

v_{EW}; velocity induced by elementary wedge

p ; integer representing waterlines(1, 2, ……)

* rounded wedge

$$\phi_{EW,R}(\xi,\eta,\zeta)=4\frac{\varDelta^2}{m}\frac{UK^2}{mn\pi^2}\int_0^\infty\int_0^\infty\frac{e^{-\eta\sqrt{\sigma^2+\tau}}}{\sqrt{\sigma^2+\tau^2}}\frac{\sin(m/2)}{\sigma^4+k^2\tau^2}\left(\cos\tau\zeta-\frac{\sigma^2}{K\tau}\sin\tau\zeta\right)\left(\frac{1}{\sigma^2}-\frac{\cos a\tau}{\sigma^2}-\frac{a}{K}+\frac{\sin a\tau}{K\tau}\right)\sin\sigma\xi\, d\sigma d\tau+4\frac{\varDelta^2}{m}\frac{UK^2}{mn\pi}\int_K^\infty\frac{e^{-\sigma^2\zeta/K}}{\sigma^5\sqrt{\sigma^2-K^2}}\left(\frac{\sigma^2 a}{K}+e^{-\sigma^2}a/K-1\right)\cos[\sigma(\xi+\frac{\eta}{K}\sqrt{\sigma^2-K^2})]\sin\frac{m\sigma}{2}d\sigma+4\frac{\varDelta^2}{m}\frac{UK^2}{mn\pi}\int_0^K\frac{e^{-\sigma\,\eta/K\sqrt{K2-\sigma2}}}{\sigma^5\sqrt{K^2-\sigma^2}}e^{-\sigma2\zeta/K}\left(\frac{\sigma^2 a}{K}+e^{-\sigma2a/K}-1\right)\sin(\sigma\xi)\sin\frac{m\sigma}{2}d\sigma \tag{5}$$

* sharp wedge

$$\Phi_{EW,S}(\xi,\eta,\zeta)=2\frac{\varDelta^2}{m}\frac{UK^2}{n\pi^2}\int_0^\infty\int_9^\infty\frac{\sigma e^{-\eta\sqrt{\sigma2+\tau2}}}{\sqrt{\sigma^2+\tau^2}(\sigma^4+K^2\tau^2)}(\cos\tau\zeta-\frac{\sigma^2}{K\tau}\sin\tau\zeta)\left(\frac{\sin a\tau}{K\tau}-\frac{a}{K}+\frac{1}{\sigma^2}-\frac{\cos a\tau}{\sigma^2}\right)\sin\sigma\xi\, d\sigma d\tau+2\frac{\varDelta^2}{m}\frac{UK^2}{n\pi}\int_K^\infty\frac{e^{-\sigma^2\zeta\sqrt{K}}}{\sigma^4\sqrt{\sigma^2-K^2}}\left(\frac{\sigma^2 a}{K}+e^{-\sigma a/K}-1\right)\cos[\sigma(\xi+\frac{\eta}{K}\sqrt{^2-K^2})]d\sigma+2\frac{\varDelta^2}{m}\frac{UK^2}{n\pi}\int_0^K\frac{e^{-\sigma^2\zeta/K}}{\sigma^4\sqrt{K^2-\sigma^2}}e^{-\sigma\eta/K\sqrt{K^2-\sigma^2}}\left(\frac{\sigma^2 a}{K}+e^{-\sigma}a/K-1\right)\sin(\sigma\xi)d\sigma \tag{6}$$

the result is

8. Velocity around a Hull and the Isobar Elevation

The disturbance velocity at an arbitrary field point can be calculated by superposing velocities induced by all the wedges as the following equation shows:

$$\boldsymbol{v}(x,y,z)=\sum_p\sum_q \boldsymbol{v}_w[(x-x_{p,q}),y,\ (z-z_{p,q})] \quad (8)$$

in which $(x_{p,q}, z_{p,q})$ indicates the location of each wedge which may usually be, following the convention in ship lines plan, the coordinates of p-th waterline and q-th station. The velocity observed from the employed coordinate system is then given by

$$\boldsymbol{v}(x,y,z)=U\boldsymbol{i}+\boldsymbol{v}(x,y,z) \quad (9)$$

The isobar elevation can then be estimated from the x-component of the disturbance velocity as stipulated in the linearised ship wave theory. If we denote the isobar elevation by ζ_w, it is given by

$$\zeta_w(x,y,z)=-\frac{U}{g}u(x,y,z) \quad (10)$$

The surface wave is particular case of this isobar elevation and is obtained by eq. (10) with $z=0$.

9. Programming

The offset at each junction of stations and waterlines is used as the input data to form the hull shape. Both stations and waterlines are assumed to be equally spaced. The wedge base must be rectangular which means that the usual curved bow and stern shape cannot be accommodated. Improvements regarding these points are required in the present programme before it is to be used for practical purposes.

Theoretically, the more stations are employed, the better accuracy is expected but practically, any reasonable number around, say, twenty may be quite adequate. Generally, for the slower speed, the more stations are required to maintain the accuracy. Increase of the number of stations is accompanied by the penalty of computing time.

In Guilloton's papers(8, 10), the underwater part of a hull is to be divided into six layers. In addition, it is required that the length of the ship is so adjusted that U^2/g is 2.5m. No such restriction and requirement are imposed in the present programming. Any number of waterlines between three and twentyone may be chosen. Again, in view of increase of computing time number less then ten would be sufficient.

10. Computed Results and Comparisons

Guilloton's H-function has been calculated by the present programme for a few cases and is shown in Table 2. The table for this function has been prepared by Guilloton(8) in such a way that extensive interpolation is required to obtain its values for any specific cases. The agreement is very good. It is felt that this agreement indicates the accuracy of Guilloton's table rather than that of the present calculation.

The wave profiles along the $x-z$ plane, by a single wedge are calculated for a few cases and shown in Fig. 6. The profiles appear excellent satisfying the radiation condition in the expected manner.

The Wigley model(11), Fig. 7, has been chosen as the test case of the present method. This model is particularly suitable for the wedge method because of its geometry.

The four places, bow, fore-shoulder, aft-shoulder and stern are the obvious positions where new wedges

Table 2 Comparison of the Values of the H-function

Case 1. Conditions;

wedge form ; full wedge wedge position ; $z=0.5$m

width of wedge ; $N=0.5$m level of field points ; $z=0.5$m

x	0.5	1.0	1.5	2.0	2.5	3.0
Guilloton's Table	239	349	388	382	346	292
Present Method	241	339	382	379	346	297

Case 2. Conditions;

wedge form ; full wedge wedge position ; z=0.1m
width of wedge ; N=0.1m level of field points ; z=0.2m

x	0.5	1.0	1.5	2.0	2.5	3.0
Guilloton's Table	294	414	465	460	391	311
Present method	306	433	485	460	393	315

Case 3. Conditions;

wedge form ; half wedge width of wedge ; N=0.1m
level of field points ; Z=0.0m

x	0.5	1.0	1.5	2.0	2.5	3.0
Guilloton's Table	671	435	290	221	176	140
Present method	651	430	292	219	174	139

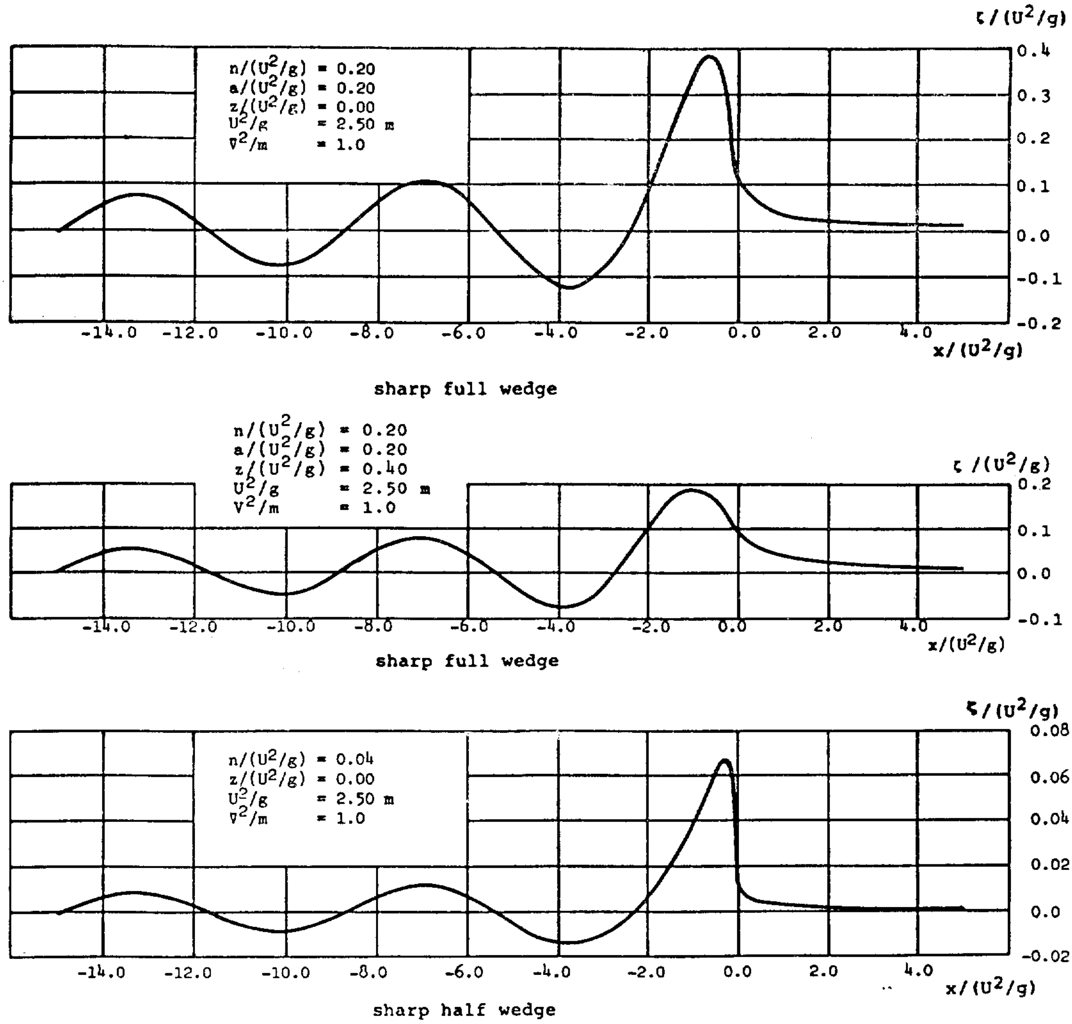

Fig. 6 Wave Profiles Created by a Wedge

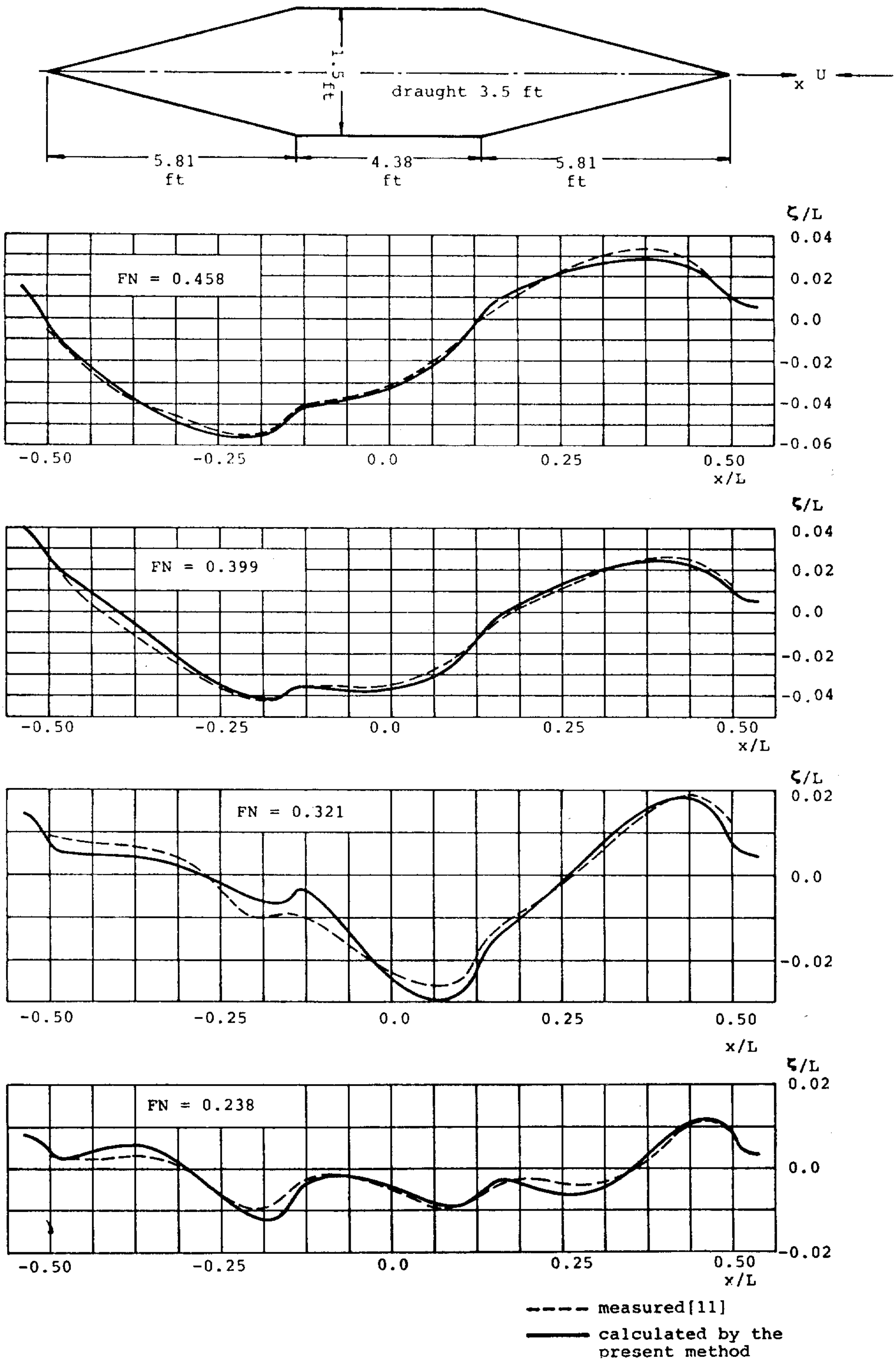

Fig. 7 Wigley Model and Its Wave Profiles

are to be put. All the wedges are sharp ones with their strengths

$\Delta^2/m=0.75/5.81=0.1291$

identically. Eight layers in vertical direction have been used in the calculation. The computed results for a few Froude numbers are shown in the form of wave profiles along the hull, projected to the center plane, together with the measured profiles taken from the reference [11]. The agreement is excellent. It can be seen that the accuracy of the present method is better with the higher Froude number.

11. Conclusions

It seems to be true that Guilloton's wedge method is superior to any other methods in connecting hull geometry to its wave-making characteristics. Havelock source method is excellent in wave-making property but reflection of hull shape to the wave created by the ship as a whole is almost impossible, if not tottally. There is no reason why the proved accuray of the method in predicting x-component of veloctiy cannot be maintained in predicting other components within the scope of ideal fluid, of course. Indeed, it has been felt that the degree of reliability may be quite safely anticipated in these regards.

It is well known (1) that the accuracy of the wedge method is achieved after the so-called space transformation (12) is introduced, this being not yet incorporated in the present method. Application to a practical hull form with curved bow and stern profile is not allowed at present. The second report regarding these aspects is to be made in short time.

Notwithstanding the imperfections and limitations of the present method at this stage, there seems to be abundant possibility of achieving fruitful results in this line of attacking the ship wave-making problems. There certainly are many aspects which deserve serious considerations from the ship hydrodynamicists.

This work has been supported by a research grant form the Asan Foundation and the author expresses his gratitude to the Foundation.

References

[1] Emerson, A.: The calculation of ship resistance: an application of Guilloton's Method, Trans. RINA. Vol. 109, 1967, p. 241.

[2] Gadd, G.E.: Wave resistance calculations by Guilloton's method, Trans. RINA, Vol. 115, 1973, p. 337.

[3] Noblesse, F.: A perturbation analysis of the wavemaking of a ship with an interpretation of Guilloton's method, JSR, Vol. 19, No. 3, Sept. 1975.

[4] Dagan, G.: A method of computing nonlinear wave resistance of thin ships by coordinate straining, JSR, Vol. 19, No. 3, Sept. 1975.

[5] Lee, D.K.: On the numerical evaluation of the wave pattern of a havelock source, Journal of KSNA, Vol. 16, No. 4, 1979.

[6] Lee, D.K.: The use of rankine source to evaluate velocities around a ship hull, Journal of KSNA, Vol. 18, No. 4, 1981.

[7] Michell, J.H.: The wave-resistance of a ship, Phil. Mag. London, Vol. 45, p. 106, 1898.

[8] Guilloton, R.: Potential theory of wave resistance of ships with tables for its calculation, Trans. SNAME, Vol. 59, p. 86, 1951.

[9] Guilloton, R: A new method of calculating wave profiles and wave resistance of ships, Trans. INA, Vol. 82, p. 69, 1940.

[10] Guilloton. R.: Compléments sur le potentiel linéarisé avec surface libre appliqué à l'étude des carènes, Bulletin ATMA, No. 55, p. 337, 1956.

[11] Wigley, C.: Ship wave resistance, NECI, 1931.

[12] Guilloton, R.: L'étude théorique du bateau en fluide parfait, Bulletin ATMA, No. 64, p. 537, 1964.