

# 조속기 입력의 변화율을 고려한 최적 주파수 제어

論 文  
33~3~2

## LFC Considering the Changing Rate of Governor Speed

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### 요 약

본 논문에서는 조속기 입력의 변화율을 포함하는 개선된 평가 함수를 이용한 최적 주파수 제어가 소개된다.

상태 변수 확장의 기법을 이용한 동적 제어기와 크라인만의 정리를 이용한 일정 이득 제어기를 유도한다.

실제적인 적용을 위해 축소화된 관측자를 사용하여, 측정 불가능한 상태 변수와 전력 수요를 얻어낸다.

본 논문에서 유도된 방법들은 전력 계통 주파수 제어 모델로 널리 사용되는 엘저드의 모델에 적용시켜 그 효용성을 입증한다.

### Abstract

The optimal Load-Frequency Control law is presented with the performance criterion which includes the changing rate of governor speed.

The authors propose two controllers. One is a dynamic controller using the method of state augmentation and the other is a constant gain controller with use of the trace function lemma by Kleinman.

For a more practical realization, a reduced-order Luenberger observer is applied in order to identify unmeasurable states and power demands.

The control schemes presented here are tested through the model developed by Elgerd, and the usefulness is demonstrated.

A conventional Load-Frequency Controller is designed to regulate these quantities by means of driving the ACE (Area Control Error) to zero.

In the systematic approach of this study, there are two important problems as follows.

- (i) mathematical model
- (ii) identification of unmeasurable states and unknown power demands

Elgerd and Fosha developed the model for LFC (Load-Frequency Control) problem which is based on the modern control theory.<sup>1)</sup>

### 1. Introduction

In a power system, the frequency and the tie-line power flow should be set to some scheduled values. However, power system disturbances caused by unknown power demands result in changes in the system frequency and tie-line power flow.

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This approach, however, is not feasible, since the control law requires unavailable informations.

To identify unmeasurable states and unknown power demands, Miniesy and Bohn suggested a method for the use of a Luenberger observer.<sup>2)</sup>

This has the advantage of the method using Kalman filter by Cavin et al. since Kalman filter cannot be applied in the absence of noise statistics.

Y.M. Park improved the study of LFC in a practical point of view with a exponential disturbance model, the optimal parameter determination of a observer system, the quadratic estimator for tie-line power flow, and a decentralized control scheme.<sup>3)</sup>

However, there remains the problem of LFC which follows rapidly changing load disturbances. The resultant governor action is to be inefficient, undertakes sudden changes, and so may causes damage to governor.

In order to treat this problem, Ross suggested the use of an EACC (Error Adaptive Control Computer) which is based on a probabilistic analysis, and Glover and Schweppe used the deadband logic whose basis is a magnitude bound analysis. These logics design the control law to reduce the number of LFC signals sent to the governor.

In this paper, alternatively, a systematic approach is introduced to treat such a problem.

The authors propose the modified performance criterion which includes the changing rate of control action, to obtain two controllers.

One is a dynamic controller using the method of state augmentation and the other is a constant gain controller with use of the trace function lemma by Kleinman.<sup>4)</sup>

## 2. Dynamic Controller

The typical LFC system dynamics can be represented as

$$\dot{x} = Ax + Bu + Dp, \tag{1}$$

where x, u and p are state, control input and disturbance vectors, respectively.

The terminal conditions to be satisfied at steady-state are [3]

$$x_f = EP_f, \quad u_f = SP_f, \tag{2}$$

where the lower subscript f means final time.

From above equations (1) and (2) one has

$$\begin{aligned} 0 &= AX_f + BU_f + DP_f \\ &= (AE + BS + D) P_f \end{aligned} \tag{3}$$

Therefore

$$O = AE + BS + D. \tag{4}$$

The change of variables and the inevitable assumption of  $P = P_f$  are used with equations (1), (2) and (4) to obtain

$$\begin{aligned} \dot{\bar{x}} &= A\bar{x} + B\bar{u} + (AE + BS + D) P_f + D\bar{p} \\ &= A\bar{x} + B\bar{u} + D\bar{p} \\ &= A\bar{x} + B\bar{u}, \end{aligned} \tag{5}$$

where  $\bar{x} \triangleq x - x_f$ ,  $\bar{u} \triangleq u - u_f$  and  $\bar{p} \triangleq P - P_f \dot{=} O$ .

A typical performance criterion is

$$J = \frac{1}{2} \int_0^\infty \{ \bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u} \} dt, \tag{6}$$

where Q and R are, respectively, a positive semi-definite and a positive definite weighting matrices with appropriate dimensions.

A proposed performance criterion is

$$J_m = \frac{1}{2} \int_0^\infty \{ \bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u} + \dot{\bar{u}}^T R_2 \dot{\bar{u}} \} dt, \tag{7}$$

where  $R_2$  is a positive definite weighting matrix.

Equations (5) and (6) are the typical form of an optimal linear quadratic problem, and equations (5) and (7) can be converted to such a form using the method of state augmentation; i.e.,

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{u}} \end{bmatrix} = \begin{bmatrix} A & B \\ O & O \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} + \begin{bmatrix} O \\ I \end{bmatrix} \dot{\bar{u}}, \tag{8}$$

$$J_m = \frac{1}{2} \int_0^\infty \{ \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix}^T \begin{bmatrix} Q & O \\ O & R \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} + \dot{\bar{u}}^T R_2 \dot{\bar{u}} \} dt. \tag{9}$$

The optimal dynamic controller, therefore, can be constructed by the following equations.<sup>5)</sup>

$$\dot{\bar{u}} = -F \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = -[F_1 \ F_2] \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = -F_1 \bar{x} - F_2 \bar{u},$$

$$\bar{u}(0) = O, \tag{10}$$

$$F = R_2^{-1} \begin{bmatrix} O \\ I \end{bmatrix}^T K, \tag{11}$$

$$O = \begin{bmatrix} A & B \\ O & O \end{bmatrix}^T K + K \begin{bmatrix} A & B \\ O & O \end{bmatrix} + Q + K \begin{bmatrix} O \\ I \end{bmatrix} R_2^{-1}$$

$$\begin{pmatrix} 0 \\ I \end{pmatrix}^T K, K > O. \tag{12}$$

Changing of variables to original ones results in

$$\begin{aligned} \dot{u} &= -F_1 x - F_2 u + (F_1 E + F_2 S) P_f \\ &\doteq -F_1 x - F_2 u + (F_1 E + F_2 S) P, \end{aligned} \tag{13}$$

where the approximation is reasonable because one cannot know the steady-state disturbance  $P_f$ .

This controller is similar to the conventional proportional-plus-integral controller that is also obtained from the method of state augmentation; i.e.,

$$u = -f_1 x - f_2 \int x d\tau - f_3 p - f_4 \int p d\tau, \tag{14}$$

and therefore

$$\dot{u} = -f_1 \dot{x} - f_2 x - f_3 \dot{p} - f_4 p. \tag{15}$$

However, in conventional one, augmented states are the integrals of some original states, and this scheme originates from the notion of the integrating action that is devised to eliminate the effect of constant disturbances.<sup>5)</sup>

### 3. Constant Gain Controller

An alternative method is proposed to the optimal control problem with equations (5) and (7); i.e., the simple structured controller without a integrator.

One can get the following results with some sophisticated manipulations introduced in appendix.

When one restricts the controller structure as

$$\bar{u} = -F \bar{x}, \tag{16}$$

the performance measure is, provided  $A-BF$  is stable,

$$\hat{J}_m \triangleq E(J_m) = \frac{1}{2} tr(K \Sigma_o), \tag{17}$$

where  $\Sigma_o$  is the variance matrix of initial states, i.e.,  $\Sigma_o = E[\bar{x}_o \bar{x}_o^T]$ ,  $E(\cdot)$  is the stochastic symbol for the mean value of a random variable,  $tr(\cdot)$  is the symbol for the trace of a square matrix, and  $K$  is the solution of the equation

$$\begin{aligned} O &= KA_o + A_o^T K + Q + F^T R F + A_o^T F^T R_2 F A_o, \\ K &\geq O, \end{aligned} \tag{18}$$

where  $A_o = A - BF$ ,

In order for  $F$  to be optimal, it is necessary that

$$O = (R F - B^T F^T R_2 F A_o) L + R_2 F A_o L A_o^T$$

$$-B^T K L, \tag{19}$$

where the matrix  $L$  is the solution of the equation

$$O = LA_o^T + A_o L + \Sigma_o, L > O \tag{20}$$

Consequently, the optimal solution is

$$\begin{aligned} u &= -F x + (FE + S) P_f \\ &\doteq -F x + (FE + S) P. \end{aligned} \tag{21}$$

In case of  $R_2 = 0$ , one can get the well-known results for an optimal linear quadratic problem from above equations (17), (18), (19) and (20).

### 4. Computer Simulations

The presented approach is applied to the two-area power system model by Elgerd.<sup>1)</sup>

The states, system matrices, and the variance matrix of the initial states are as follows.

$$x^T = (\Delta P_{tie}, \Delta f_1, \Delta P_{g1}, \Delta g_1, \Delta f_2, \Delta P_{g2}, \Delta g_2), \tag{22}$$

$$A = \begin{pmatrix} 0 & 0.54 & 0 & 0 & -0.54 & 0 & 0 \\ -6 & -0.05 & 0 & 6 & & & \\ 0 & -5.21 & -12.5 & 0 & & 0 & \\ 0 & 0 & 0.667 & -0.667 & & & \\ 6 & & & & -0.05 & 0 & 6 \\ 0 & & 0 & & -5.21 & -12.5 & 0 \\ 0 & & & & 0 & 0.667 & -0.667 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 12.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 12.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Sigma_o = E(\bar{x}_o \bar{x}_o^T) = \text{Diag} \left( 1, 1, \frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{2} \right),$$

where  $\Delta P_{tie}$  is the tie line power flow deviation,

$\Delta f_i$  is the frequency deviation in area  $i$ ,

$\Delta P_{gi}$  is the governor position deviation in area  $i$ ,

and  $\Delta g_i$  is the generator output deviation in area  $i$ .

A reduced-order Luenberger observer is used in

order to identify the unmeasurable states and disturbances, where the observer gain is selected optimally with ISE (Integral of the Square of the Error) criterion.<sup>2),3),6)</sup>

With the separation principle, one can design the controller and the observer separately.

For comparison, three types of simulations are examined.

Type A(—) is with the conventional optimal controller which minimizes  $J$ .

Type B(----) is with the optimal dynamic controller which minimizes  $J_m$ .

Type C(.....) is with the optimal constant gain controller which minimizes  $J_m$ .

The weighting matrices are as follows.

$$Q = \text{Diag} (1, 1, 0, 0, 1, 0, 0) ,$$

$$R = 3I , \text{ and } R_2 = \frac{1}{2} I ;$$

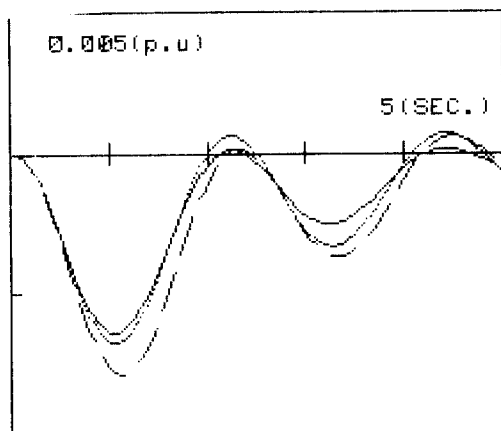


Fig. 1. Tie line power flow

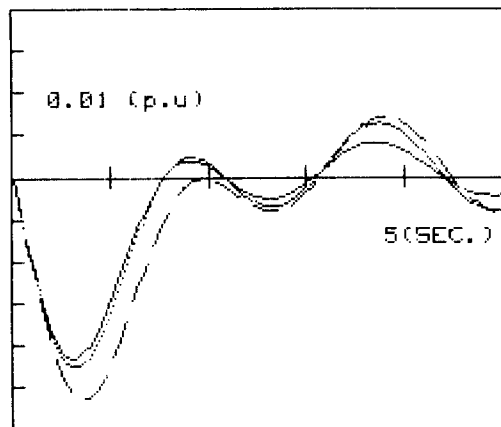


Fig. 2. Frequency of area 1

The figure 4 shows that the dynamic controller improves the steep governor action somuch.

And in order to achieve the similar smoothing effect, the constant gain controller should be calculated with somewhat large magnitude of the matrix  $R_2$ . This stems from the fact that the proportional control should be designed to mimic the dynamic control action, in addition to the existence of the rapidly changing disturbance term in the constant gain control law.

In the dynamic control law, this term is intergrated and so, smoothed enough to give desirable control. But the constant gain control scheme will also be desirable in a typical linear control problem.

And with dynamic controller, the deterioration of the state regulation is more apparent. an in figure 1,2,3.

The performance measures  $\hat{J}$  and  $\hat{J}_m$  are calculated for each type of simulations as,

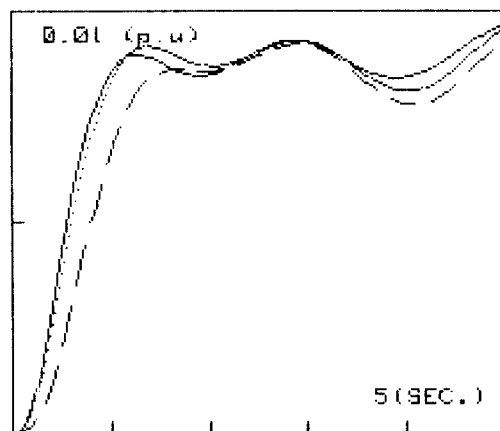


Fig. 3. Generator output of area 1

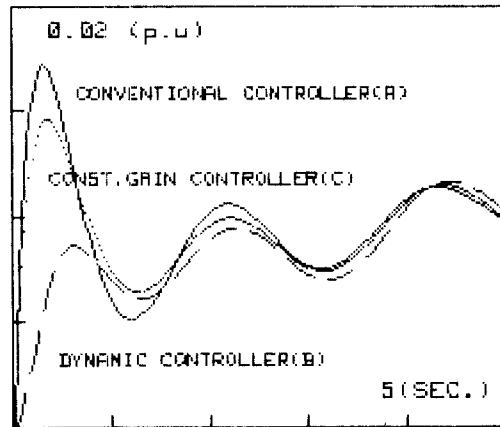


Fig. 4. Governor control input of area 1

$$\begin{aligned} \hat{J}(A) &= 10,367, & \hat{J}_m(A) &= 15,755, \\ \hat{J}(B) &= 12,567, & \hat{J}_m(B) &= 15,717, \\ \hat{J}(C) &= 11,211, & \hat{J}_m(C) &= 14,290, \end{aligned}$$

For optimization techniques, Fletcher-Powell-Davidon algorithm is used with Kleinman's iteration and Schur's theorem.

### 5. Conclusions

The optimal dynamic controller has a remarkable achievement in smoothing control actions, but also has disadvantages in implementing the controller and in the state regulation.

The optimal constant gain controller improves these aspects with somewhat less success in smoothing.

In any other control system that requires some smoothing scheme, these algorithms can be used.

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### Appendix

In order to derive the optimal constant gain control law, the following lemma by Kleinman should be examined.<sup>4)</sup>

(Lemma) Let  $f(x)$  be a trace function. Then if

one can write

$$f(x + \epsilon \Delta x) - f(x) = [M(x) \Delta x] \quad (23)$$

as  $\epsilon \rightarrow 0$ , where  $M(x)$  is an  $n \times m$  matrix and  $X$  is an  $m \times n$  matrix, then

$$\frac{\partial f(x)}{\partial x} = M^T(x), \quad (24)$$

One can write the performance criterion as

$$\begin{aligned} J_m &= \frac{1}{2} \int_0^\infty \{ \bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u} + \dot{\bar{u}}^T R_2 \dot{\bar{u}} \} dt \\ &= \frac{1}{2} \int_0^\infty \{ \bar{x}^T Q \bar{x} + \bar{x}^T F^T R F \bar{x} + \\ &\quad \dot{\bar{x}}^T F^T R_2 F \dot{\bar{x}} \} dt, \\ &= \frac{1}{2} \int_0^\infty \bar{x}^T [Q + F^T R F + (A - BF)^T F^T R_2 F \\ &\quad (A - BF)] \bar{x} dt. \end{aligned} \quad (25)$$

Since  $\bar{x}(t) = e^{(A-BF)t} \bar{x}_0$ , the expected value of  $J_m$  is as follows.<sup>7), 8)</sup>

$$\begin{aligned} \hat{J}_m &\triangleq E [ J_m ] \\ &= \frac{1}{2} E \{ \int_0^\infty \bar{x}_0^T e^{A_0^T t} [ Q + F^T R F + A_0^T F^T \\ &\quad R_2 F A_0 ] e^{A_0 t} \bar{x}_0 dt \} \\ &= \frac{1}{2} E \{ \text{tr} \int_0^\infty e^{A_0^T t} [ Q + F^T R F + A_0^T F^T \\ &\quad R_2 F A_0 ] e^{A_0 t} \bar{x}_0 \bar{x}_0^T dt \} \\ &= \frac{1}{2} \text{tr} \{ \int_0^\infty e^{A_0^T t} [ Q + F^T R F + A_0^T F^T \\ &\quad R_2 F A_0 ] e^{A_0 t} \cdot \Sigma_0 dt \}, \end{aligned} \quad (26)$$

Where  $A_0 = A - BF$ .

Therefore

$$J_m(F + \varepsilon \Delta F) = \frac{1}{2} \text{tr} \left\{ \int_0^\infty e^{A_0 - \varepsilon B \Delta F} \left[ (Q + (F + \varepsilon \Delta F)^T R (F + \varepsilon \Delta F) + (A_0 - \varepsilon B \Delta F)^T (F + \varepsilon \Delta F)^T R_2 (F + \varepsilon \Delta F) (A_0 - \varepsilon B \Delta F)) e^{A_0 - \varepsilon B \Delta F} \right] \Sigma_0 dt \right\} \quad (27)$$

The following relation is useful for further derivation<sup>8)</sup>

$$e^{A + \varepsilon B} = e^A + \varepsilon \int_0^1 e^{A(1-s)} B e^{As} ds + o(\varepsilon^2),$$

$$\therefore e^{(A_0 - \varepsilon B \Delta F)} = e^{A_0} - \varepsilon \int_0^1 e^{A_0(1-s)} B \Delta F e^{A_0 s} d\sigma + o(\varepsilon^2). \quad (28)$$

Using above equality, equation (27) can be approximated as

$$\hat{J}_m(F + \varepsilon \Delta F) = \frac{1}{2} \text{tr} \left\{ \int_0^\infty [ e^{A_0^T} (Q + F^T RF + A_0^T F^T R_2 FA_0) e^{A_0} + 2\varepsilon e^{A_0^T} (F^T R \Delta F - A_0^T F^T R_2 FB \Delta F + A_0^T F^T R_2 \Delta FA_0) e^{A_0} - 2\varepsilon e^{A_0^T} (Q + F^T RF + A_0^T F^T R_2 FA_0) \int_0^1 e^{A_0(1-\sigma)} B \Delta F e^{A_0 \sigma} d\sigma ] \Sigma_0 dt \right\} \quad (29)$$

$$\therefore \hat{J}_m(F + \varepsilon \Delta F) - \hat{J}_m(F) = \varepsilon \text{tr} \left\{ \int_0^\infty [ e^{A_0^T} \Sigma_0 (F^T R - A_0^T F^T R_2 FB) + A_0 e^{A_0} \Sigma_0 \right.$$

$$\left. e^{A_0^T} (A_0^T F^T R_2 - \int_0^1 e^{A_0 \sigma} \Sigma_0 e^{A_0^T} (Q + F^T RF + A_0^T F^T R_2 FA_0) \cdot e^{A_0(1-\sigma)} B d\sigma) \Delta F dt \right\}. \quad (30)$$

Hence, Kleinman's lemma gives the necessary condition as

$$O = \frac{\partial \hat{J}_m}{\partial F} = \int_0^\infty [ (RF - B^T F^T R_2 FA_0) e^{A_0} \Sigma_0 e^{A_0^T} + R_2 FA_0 e^{A_0} \Sigma_0 e^{A_0^T} A_0^T ] dt - \int_0^\infty B^T e^{A_0^T} [ Q + F^T RF + A_0^T F^T R_2 FA_0 ] e^{A_0} dt \int_0^\infty e^{A_0} \Sigma_0 e^{A_0^T} dt. \quad (31)$$

Equation (31) can be rewritten as

$$O = (RF - B^T F^T R_2 FA_0) L + R_2 FA_0 LA_0^T - B^T KL, \quad (32)$$

where the matrices L and K are

$$L = \int_0^\infty e^{A_0} \Sigma_0 e^{A_0^T} dt, \quad (33)$$

$$K = \int_0^\infty e^{A_0^T} [ Q + F^T RF + A_0^T F^T R_2 FA_0 ] e^{A_0} dt. \quad (34)$$

Equations (33) and (34) are identical with the following equations<sup>8)</sup>.

$$O = LA_0^T + A_0 L + \Sigma_0, \quad (35)$$

$$O = KA_0 + A_0^T K + Q + F^T RF + A_0^T F^T R_2 FA_0. \quad (36)$$

Now, the optimal gain matrix F can be calculated by equations (32), (35), and (36).