

## The Application of Rule of Mixtures to Fiber-Reinforced Composites(3)<sup>1</sup>

— Determination of Constant "a" and "b" for the Modified Rule of Mixtures  
Applied to Fiber-Reinforced, Sulfur-Based Composites —

Byung G. Lee<sup>2</sup>

## 목재 섬유 복합材에 混合理論의 적용에 관한 研究(3)<sup>1</sup>

— 硫黃 化合物을 사용한 木材 纖維 複合材에  
수정된 混合理論의 常數 決定 —

李 丙 根<sup>2</sup>

### 要 約

섬유의 방향성이 무질서한 composites에 적용되는 Smith와 Cox의 理論을 포함한 Paul과 Jones의 混合理論式은 硫黃 化合物을 사용하여 제조한 목재섬유 복합材에도 1-次的인 linear regression constant가 주어질 때는 사용할 수가 있음을 보여준다.

$E_c = \frac{1}{3} a E_f V_f + b E_m V_m$ 으로 표시된 이 linear regression form에 math. rom pack을 사용한 Hewlett Packard 75C(HP 75C) computer의 계산 결과는 목재 섬유 복합材에 사용된 matrix의 종류, 섬유판의 밀도와 목재 및 목질 섬유의 종류에 관계없이  $a = 3.27 \sim 3.54$ 와  $b = -2.47 \sim -2.80$ 의 일정한 범위의 값을 보여주므로, 지금까지 無질서한 방향성을 지닌 長섬유로 된 복합材에만 적용되어 왔던 Paul과 Jones의 混合理論과 이것과 같은 방향을 지닌 短 섬유로 된 木材나 木質 섬유 composites에도 적용될 수 있음을 증명하고 있다.

### Summary

It is shown that Paul and Jones' Rule of Mixtures modified by Smith and Cox's theory can be used for the fiber-reinforced, sulfur-based composites, when the constant for the linear regression equation is given.

The computation results, programmed by Hewlett Packard 75C (HP 75C) using math rom pack for the linear regression form, expressed as  $E_c = \frac{1}{3} a E_f V_f + b E_m V_m$ , turn out to be  $a = 3.27 - 3.54$   $b = -2.47 \sim -2.80$ .

This results indicate that the factors such as density of fiber mat and the amount of matrix used have nothing for affecting the numerical value of the constants a and b of the linear regression form.

Conclusively this results also show that the Paul and Jones' Rule of Mixtures which has been used for the composites made by randomly-oriented long fiber can also be used for the composites made by short fiber with the same fiber orientation such as wood and lignocellulosic fibers.

*Key words: fiber-reinforced composites, Smith and Cox's theory, Paul and Jones' rule, fiber orientation.*

<sup>1</sup> 接授 4月 14日 Received April 14, 1984.

<sup>2</sup> 江原大學校 林科大學 College of Forestry, Kangwon National University, Chuncheon 200, Korea.

## 1. Introduction

Bryant and Lee identified in their early papers (1983)<sup>1,2)</sup> that the modified rule of mixtures applicable to the composites having random fiber orientation and long fiber in it can be prepared on the basic equation of Paul and Jones' Rule of Mixtures using Smith and Cox's theory.

Bryant and Lee also applied this new modified rule of mixtures to the experimental results of fiber-reinforced, sulfur-based composites (1983).<sup>1,2)</sup> They found that the new modified rule of mixtures can only be applicable in a certain condition of fiber mat and matrix used in the composites, and also it is very difficult to use this modified rule of mixtures as the generalized equation for the composites (1981).<sup>3,4)</sup> In this context, Bryant and Lee proposed their hypothesis as the first attempt in the papers (1983)<sup>1,2)</sup> that the modified rule of mixture will have a constant of the first degree on the equation of the modified rule of mixtures expressed as  $E_c = \frac{1}{2}aE_fV_f + bE_mV_m$ . It is assumed that the constants of the first degree of the equation can be obtained by applying the experimental results to the equation, using linear regression form programmed.

## 2. Experimental procedure

A series of fiber mats were prepared from kraft pulpmill screening rejects provided by the Crown Zellerback pulpmill at Wallula, Oregon in U.S.A.. Rice straw pulp was made by soaking chopped straw in a 1% NaOH solution overnight and beating it for about 2 minutes in a laboratory model Hollander. After beating, the pulp was thoroughly washed to remove pith and fines.

Fiber mats were made in a 12-inch-square sheet mold, using enough fiber to produce pressed mats of about 1/8-inch thickness at the required density. In order to control density, mats were first air dried and then pressed between screens in a steam-heated laboratory hot press using 1/8-inch-thick

metal stops at the edges of the mats.

Pressed fiber mats were cut into 2-inch by 5 1/4-inch specimens. These were immersed in molten sulfur modified by tall oil and polyester, which characterizes synthetic resinous material at 135°-140°C, until they were thoroughly saturated. The time required for thorough saturation varied from 10 to 30 minutes.

Instron Tester was used for mechanical strength properties of the composites, which are modulus of elasticity and modulus of rupture.

In order to apply the experimental results to Bryant and Lee's hypothesis, primarily linear regression form is examined using Hewlett Packard 75C (HP 75C) computer with mathematic rom pack.

## 3. Results and discussion

In order to calculate the constants a and b in the Bryant and Lee's hypothesis, the experimental data used are followed in the Tables.

Table 1. Mechanical strength properties and volume fraction of constituents of the composites.

Fiber source : rice straw

Matrix : the compound(sulfur + tall oil)

Mechanical Strength unit: psi

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$31.4 \times 10^4$	$233 \times 10^4$	$26 \times 10^4$	0.7853	0.1001
$32.1 \times 10^4$	"	"	0.7841	0.1012
$33.3 \times 10^4$	"	"	0.7830	0.1023
$34.6 \times 10^4$	"	"	0.7820	0.1032
$35.1 \times 10^4$	"	"	0.7809	0.1041
$36.7 \times 10^4$	"	"	0.7794	0.1055
$37.7 \times 10^4$	"	"	0.7783	0.1062
$39.0 \times 10^4$	"	"	0.7774	0.1074
$41.1 \times 10^4$	"	"	0.7764	0.1081
$41.8 \times 10^4$	"	"	0.7753	0.1092
$42.1 \times 10^4$	"	"	0.7742	0.1103
$43.0 \times 10^4$	"	"	0.7732	0.1114
$44.0 \times 10^4$	"	"	0.7721	0.1124
$45.1 \times 10^4$	"	"	0.7710	0.1133
$45.9 \times 10^4$	"	"	0.7701	0.1145
$47.3 \times 10^4$	"	"	0.7690	0.1157
$48.3 \times 10^4$	"	"	0.7671	0.1166
$49.6 \times 10^4$	"	"	0.7660	0.1186
$50.5 \times 10^4$	"	"	0.7651	0.1197
$51.1 \times 10^4$	"	"	0.7643	0.1209

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$	$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$52.2 \times 10^4$	$233 \times 10^4$	$26 \times 10^4$	0.7632	0.1219	$84.2 \times 10^4$	$233 \times 10^4$	$26 \times 10^4$	0.7355	0.1582
$53.4 \times 10^4$	"	"	0.7621	0.1230	$85.0 \times 10^4$	"	"	0.7356	0.1594
$54.1 \times 10^4$	"	"	0.7610	0.1242	$86.1 \times 10^4$	"	"	0.7350	0.1604
$55.3 \times 10^4$	"	"	0.7600	0.1254	$87.3 \times 10^4$	"	"	0.7345	0.1613
$56.2 \times 10^4$	"	"	0.7591	0.1268	$88.8 \times 10^4$	"	"	0.7340	0.1628
$57.1 \times 10^4$	"	"	0.7582	0.1277	$89.1 \times 10^4$	"	"	0.7330	0.1642
$57.8 \times 10^4$	"	"	0.7573	0.1289	$90.3 \times 10^4$	"	"	0.7322	0.1657
$58.1 \times 10^4$	"	"	0.7560	0.1300	$93.1 \times 10^4$	"	"	0.7315	0.1670
$58.8 \times 10^4$	"	"	0.7550	0.1310	$93.4 \times 10^4$	"	"	0.7303	0.1685
$59.1 \times 10^4$	"	"	0.7541	0.1321	$94.1 \times 10^4$	"	"	0.7289	0.1699
$61.1 \times 10^4$	"	"	0.7530	0.1331	$95.4 \times 10^4$	"	"	0.7277	0.1713
$62.8 \times 10^4$	"	"	0.7522	0.1339	$96.7 \times 10^4$	"	"	0.7264	0.1725
$63.4 \times 10^4$	"	"	0.7510	0.1348	$97.7 \times 10^4$	"	"	0.7253	0.1740
$65.1 \times 10^4$	"	"	0.7500	0.1355	$98.4 \times 10^4$	"	"	0.7244	0.1756
$66.6 \times 10^4$	"	"	0.7491	0.1367	$99.6 \times 10^4$	"	"	0.7230	0.1773
$67.8 \times 10^4$	"	"	0.7480	0.1379	$101.0 \times 10^4$	"	"	0.7221	0.1783
$68.3 \times 10^4$	"	"	0.7470	0.1390	$102.4 \times 10^4$	"	"	0.7210	0.1800
$69.6 \times 10^4$	"	"	0.7462	0.1403	$103.7 \times 10^4$	"	"	0.7201	0.1814
$70.7 \times 10^4$	"	"	0.7450	0.1412	$104.4 \times 10^4$	"	"	0.7198	0.1826
$72.2 \times 10^4$	"	"	0.7440	0.1424	$105.6 \times 10^4$	"	"	0.7189	0.1838
$73.3 \times 10^4$	"	"	0.7430	0.1435	$106.1 \times 10^4$	"	"	0.7181	0.1850
$74.1 \times 10^4$	"	"	0.7421	0.1447	$108.0 \times 10^4$	"	"	0.7174	0.1862
$75.2 \times 10^4$	"	"	0.7383	0.1495	$109.2 \times 10^4$	"	"	0.7166	0.1870
$77.3 \times 10^4$	"	"	0.7370	0.1510	$110.1 \times 10^4$	"	"	0.7154	0.1880
$78.1 \times 10^4$	"	"	0.7361	0.1524	$112.0 \times 10^4$	"	"	0.7147	0.1890
$79.2 \times 10^4$	"	"	0.7320	0.1581	$113.1 \times 10^4$	"	"	0.7136	0.1907
$80.1 \times 10^4$	"	"	0.7310	0.1602	$114.4 \times 10^4$	"	"	0.7127	0.1916
$81.1 \times 10^4$	"	"	0.7300	0.1621	$115.2 \times 10^4$	"	"	0.7120	0.1925
$82.6 \times 10^4$	"	"	0.7290	0.1643	$116.1 \times 10^4$	"	"	0.7117	0.1937
$83.4 \times 10^4$	"	"	0.7282	0.1659	$117.0 \times 10^4$	"	"	0.7111	0.1951
$84.0 \times 10^4$	"	"	0.7271	0.1681	$118.1 \times 10^4$	"	"	0.7104	0.1958
					$119.2 \times 10^4$	"	"	0.7097	0.1972
					$120.0 \times 10^4$	"	"	0.7084	0.1984
					$121.1 \times 10^4$	"	"	0.7073	0.1992
					$122.0 \times 10^4$	"	"	0.7066	0.2001
					$123.1 \times 10^4$	"	"	0.7059	0.2011
					$124.4 \times 10^4$	"	"	0.7051	0.2025
					$126.1 \times 10^4$	"	"	0.7042	0.2038
					$126.6 \times 10^4$	"	"	0.7031	0.2047
					$128.1 \times 10^4$	"	"	0.7024	0.2062
					$129.3 \times 10^4$	"	"	0.7016	0.2070
					$131.0 \times 10^4$	"	"	0.7011	0.2075
					$132.0 \times 10^4$	"	"	0.7001	0.2081
					$133.1 \times 10^4$	"	"	0.6987	0.2104
					$134.2 \times 10^4$	"	"	0.6970	0.2126
					$135.1 \times 10^4$	"	"	0.6964	0.2135
					$136.3 \times 10^4$	"	"	0.6946	0.2150
					$137.0 \times 10^4$	"	"	0.6940	0.2161
					$138.1 \times 10^4$	"	"	0.6932	0.2174
					$139.0 \times 10^4$	"	"	0.6911	0.2182
					$141.1 \times 10^4$	"	"	0.6906	0.2191
					$142.6 \times 10^4$	"	"	0.6901	0.2204
					$143.3 \times 10^4$	"	"	0.6894	0.2211
					$144.6 \times 10^4$	"	"	0.6889	0.2227

Table 2. Mechanical strength properties and volume fraction of constituents of the composites.

Fiber Source : pulp rejects

Matrix : the compound (sulfur + tall oil)

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$68.1 \times 10^4$	$233 \times 10^4$	$26 \times 10^4$	0.7533	0.1330
$69.4 \times 10^4$	"	"	0.7544	0.1344
$70.0 \times 10^4$	"	"	0.7527	0.1364
$71.1 \times 10^4$	"	"	0.7510	0.1384
$72.2 \times 10^4$	"	"	0.7510	0.1403
$73.3 \times 10^4$	"	"	0.7500	0.1428
$73.9 \times 10^4$	"	"	0.7481	0.1457
$75.2 \times 10^4$	"	"	0.7465	0.1473
$76.3 \times 10^4$	"	"	0.7439	0.1498
$77.1 \times 10^4$	"	"	0.7428	0.1509
$78.0 \times 10^4$	"	"	0.7418	0.1520
$79.1 \times 10^4$	"	"	0.7407	0.1530
$80.2 \times 10^4$	"	"	0.7400	0.1540
$81.1 \times 10^4$	"	"	0.7393	0.1552
$82.1 \times 10^4$	"	"	0.7381	0.1561
$83.4 \times 10^4$	"	"	0.7370	0.1574

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$145.4 \times 10^4$	$233 \times 10^4$	$26 \times 10^4$	0.6884	0.2232
$146.0 \times 10^4$	"	"	0.6880	0.2245
$147.1 \times 10^4$	"	"	0.6889	0.2258
$149.4 \times 10^4$	"	"	0.6867	0.2266
$149.9 \times 10^4$	"	"	0.6849	0.2279
$151.6 \times 10^4$	"	"	0.6841	0.2289

**Table 3.** Mechanical strength properties and volume fraction of constituents of the composites

Fiber source : rice straw

Matrix : the synthetic resin (sulfur + polyester)

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$28.1 \times 10^4$	$233 \times 10^4$	$18 \times 10^4$	0.7867	0.0993
$29.1 \times 10^4$	"	"	0.7859	0.0998
$29.8 \times 10^4$	"	"	0.7850	0.1004
$31.2 \times 10^4$	"	"	0.7849	0.1014
$32.5 \times 10^4$	"	"	0.7831	0.1022
$33.3 \times 10^4$	"	"	0.7822	0.1034
$34.2 \times 10^4$	"	"	0.7810	0.1042
$35.3 \times 10^4$	"	"	0.7795	0.1055
$35.9 \times 10^4$	"	"	0.7782	0.1063
$37.0 \times 10^4$	"	"	0.7773	0.1075
$38.2 \times 10^4$	"	"	0.7764	0.1083
$39.1 \times 10^4$	"	"	0.7752	0.1092
$40.0 \times 10^4$	"	"	0.7743	0.1103
$41.4 \times 10^4$	"	"	0.7730	0.1110
$42.6 \times 10^4$	"	"	0.7724	0.1123
$43.3 \times 10^4$	"	"	0.7721	0.1140
$44.7 \times 10^4$	"	"	0.7710	0.1141
$44.9 \times 10^4$	"	"	0.7704	0.1149
$46.2 \times 10^4$	"	"	0.7692	0.1157
$47.3 \times 10^4$	"	"	0.7671	0.1178
$48.1 \times 10^4$	"	"	0.7660	0.1184
$49.2 \times 10^4$	"	"	0.7651	0.1197
$50.1 \times 10^4$	"	"	0.7645	0.1209
$51.4 \times 10^4$	"	"	0.7633	0.1218
$52.2 \times 10^4$	"	"	0.7626	0.1230
$53.3 \times 10^4$	"	"	0.7610	0.1242
$54.0 \times 10^4$	"	"	0.7601	0.1255
$55.6 \times 10^4$	"	"	0.7588	0.1266
$57.1 \times 10^4$	"	"	0.7577	0.1290
$57.9 \times 10^4$	"	"	0.7562	0.1301
$58.4 \times 10^4$	"	"	0.7554	0.1312
$60.6 \times 10^4$	"	"	0.7546	0.1322
$61.2 \times 10^4$	"	"	0.7534	0.1334
$62.3 \times 10^4$	"	"	0.7526	0.1338
$63.4 \times 10^4$	"	"	0.7510	0.1344
$64.2 \times 10^4$	"	"	0.7502	0.1357
$66.1 \times 10^4$	"	"	0.7494	0.1368
$68.2 \times 10^4$	"	"	0.7481	0.1378
$70.1 \times 10^4$	"	"	0.7470	0.1387
$71.1 \times 10^4$	"	"	0.7462	0.1407

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$71.8 \times 10^4$	$233 \times 10^4$	$18 \times 10^4$	0.7451	0.1416
$72.6 \times 10^4$	"	"	0.7440	0.1424
$73.1 \times 10^4$	"	"	0.7426	0.1437
$75.2 \times 10^4$	"	"	0.7422	0.1449
$75.8 \times 10^4$	"	"	0.7392	0.1491
$76.1 \times 10^4$	"	"	0.7372	0.1512
$78.3 \times 10^4$	"	"	0.7362	0.1527
$78.8 \times 10^4$	"	"	0.7321	0.1584
$79.1 \times 10^4$	"	"	0.7316	0.1606
$79.7 \times 10^4$	"	"	0.7304	0.1627
$81.1 \times 10^4$	"	"	0.7291	0.1647
$81.4 \times 10^4$	"	"	0.7284	0.1657
$81.9 \times 10^4$	"	"	0.7277	0.1680

**Table 4.** Mechanical strength properties and volume fraction of constituents of the composites

Fiber Source : pulp rejects

Matrix : synthetic resin (sulfur + polyester)

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
$65.0 \times 10^4$	$233 \times 10^4$	$18 \times 10^4$	0.7553	0.1330
$66.6 \times 10^4$	"	"	0.7544	0.1344
$67.8 \times 10^4$	"	"	0.7527	0.1364
$68.2 \times 10^4$	"	"	9.7513	0.1385
$69.6 \times 10^4$	"	"	0.7510	0.1403
$70.2 \times 10^4$	"	"	0.7500	0.1428
$71.1 \times 10^4$	"	"	0.7481	0.1460
$72.0 \times 10^4$	"	"	0.7467	0.1475
$73.1 \times 10^4$	"	"	0.7449	0.1487
$75.2 \times 10^4$	"	"	0.7439	0.1498
$76.4 \times 10^4$	"	"	0.7428	0.1509
$76.7 \times 10^4$	"	"	0.7418	0.1520
$77.1 \times 10^4$	"	"	0.7407	0.1530
$77.8 \times 10^4$	"	"	0.7400	0.1541
$78.1 \times 10^4$	"	"	0.7391	0.1552
$78.9 \times 10^4$	"	"	0.7380	0.1560
$80.1 \times 10^4$	"	"	0.7370	0.1573
$81.1 \times 10^4$	"	"	0.7359	0.1581
$82.3 \times 10^4$	"	"	0.7356	0.1594
$83.3 \times 10^4$	"	"	0.7350	0.1604
$84.6 \times 10^4$	"	"	0.7345	0.1613
$85.7 \times 10^4$	"	"	0.7340	0.1628
$86.2 \times 10^4$	"	"	0.7330	0.1642
$87.1 \times 10^4$	"	"	0.7322	0.1657
$88.2 \times 10^4$	"	"	0.7315	0.1670
$89.1 \times 10^4$	"	"	0.7303	0.1685
$90.6 \times 10^4$	"	"	0.7289	0.1699
$91.4 \times 10^4$	"	"	0.7277	0.1713
$92.6 \times 10^4$	"	"	0.7266	0.1729
$93.1 \times 10^4$	"	"	0.7254	0.1741
$94.2 \times 10^4$	"	"	0.7244	0.1756
$94.6 \times 10^4$	"	"	0.7230	0.1773
$95.5 \times 10^4$	"	"	0.7220	0.1784

$E_c$	$E_f$	$E_m$	$V_m$	$V_f$
96.7x10 <sup>4</sup>	233x10 <sup>4</sup>	18x10 <sup>4</sup>	0.7209	0.1801
97.4x10 <sup>4</sup>	"	"	0.7220	0.1784
98.1x10 <sup>4</sup>	"	"	0.7209	0.1801
99.6x10 <sup>4</sup>	"	"	0.7201	0.1814
100.6x10 <sup>4</sup>	"	"	0.7198	0.1826
101.2x10 <sup>4</sup>	"	"	0.7189	0.1839
103.1x10 <sup>4</sup>	"	"	0.7180	0.1850
103.7x10 <sup>4</sup>	"	"	0.7172	0.1861
104.4x10 <sup>4</sup>	"	"	0.7163	0.1871
105.2x10 <sup>4</sup>	"	"	0.7154	0.1880
106.6x10 <sup>4</sup>	"	"	0.7143	0.1892
107.1x10 <sup>4</sup>	"	"	0.7134	0.1903
180.4x10 <sup>4</sup>	"	"	0.7125	0.1913
109.6x10 <sup>4</sup>	"	"	0.7120	0.1925
110.1x10 <sup>4</sup>	"	"	0.7117	0.1938
111.2x10 <sup>4</sup>	"	"	0.7110	0.1949
114.2x10 <sup>4</sup>	"	"	0.7101	0.1959
114.6x10 <sup>4</sup>	"	"	0.7091	0.1970
115.0x10 <sup>4</sup>	"	"	0.7082	0.1981
116.1x10 <sup>4</sup>	"	"	0.7073	0.1992
117.2x10 <sup>4</sup>	"	"	0.7064	0.2003
118.6x10 <sup>4</sup>	"	"	0.7055	0.2015
119.6x10 <sup>4</sup>	"	"	0.7050	0.2027
121.1x10 <sup>4</sup>	"	"	0.7042	0.2038
121.4x10 <sup>4</sup>	"	"	0.7033	0.2050
122.6x10 <sup>4</sup>	"	"	0.7024	0.2061
123.4x10 <sup>4</sup>	"	"	0.7016	0.2070
123.8x10 <sup>4</sup>	"	"	0.7010	0.2075
125.0x10 <sup>4</sup>	"	"	0.7001	0.2091
126.0x10 <sup>4</sup>	"	"	0.6993	0.2102
126.8x10 <sup>4</sup>	"	"	0.6977	0.2114
128.1x10 <sup>4</sup>	"	"	0.6970	0.2124
129.0x10 <sup>4</sup>	"	"	0.6962	0.2135
129.9x10 <sup>4</sup>	"	"	0.6954	0.2146
131.0x10 <sup>4</sup>	"	"	0.6940	0.2161
131.9x10 <sup>4</sup>	"	"	0.6932	0.2172
133.3x10 <sup>4</sup>	"	"	0.6924	0.2183
134.1x10 <sup>4</sup>	"	"	0.6916	0.2194
135.6x10 <sup>4</sup>	"	"	0.6908	0.2202
136.4x10 <sup>4</sup>	"	"	0.6901	0.2214
137.5x10 <sup>4</sup>	"	"	0.6893	0.2225
138.1x10 <sup>4</sup>	"	"	0.6887	0.2236
139.1x10 <sup>4</sup>	"	"	0.6880	0.2247
141.0x10 <sup>4</sup>	"	"	0.6868	0.2258
142.2x10 <sup>4</sup>	"	"	0.6861	0.2269
143.6x10 <sup>4</sup>	"	"	0.6853	0.2279
144.4x10 <sup>4</sup>	"	"	0.6847	0.2286
145.2x10 <sup>4</sup>	"	"	0.6840	0.2294

Hewlett Packard 75C (HP 75C) computer. Mathematic rom pack is also used in this programming for this linear regression form.

For the programming, the following processes are applied.

$E_c = \frac{1}{3}aE_fV_f + bE_mV_m$  is converted to the form,  $y = ax_1 + bx_2$  just like this formation,

$$y_1 = ax_{11} + bx_{21}$$

$$y_2 = ax_{12} + bx_{22}$$

$$y_3 = ax_{13} + bx_{23}$$

⋮

⋮

$$y_n = ax_{1n} + bx_{2n}$$

If  $X = \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ x_{13} & x_{23} \\ \vdots & \vdots \\ x_{1n} & x_{2n} \end{bmatrix}$  is, then  $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$  and  $C = \begin{bmatrix} a \\ b \end{bmatrix}$

This is also expressed as  $Y = XC$  and  $X^T Y = X^T X C$ , which can be converted to the linear regression form of  $(X^T X)^{-1} X^T Y = C$  as a matrix.

Table 5 shows the computative results of the programmed. This results indicate that the factors such as density of fiber mat and the amount of matrix used have nothing for affecting the numerical value of the constants a and b of the linear regression form.

Table 5. The constants a and b of the modified rule of mixtures derived from linear regression form

composites	constant		a	b
	sulfur + tall oil	sulfur + polyester		
rice straw	sulfur + tall oil	sulfur + polyester	3.52	-2.51
	sulfur + tall oil	sulfur + polyester	3.54	-2.47
pulp reject	sulfur + tall oil	sulfur + polyester	3.27	-2.80
	sulfur + tall oil	sulfur + polyester	3.29	-2.61

#### 4. Conclusions

It is found that the constants a and b applied to the modified rule of mixtures ( $E_c = \frac{1}{3}aE_fV_f + bE_mV_m$ ) are well exploited for the fiber-reinforce-

When  $E_c = \frac{1}{3}aE_fV_f + bE_mV_m$  is admitted to calculate the "Bryant and Lee's constant", a and b, it is natural to adopt linear regression form using

ed sulfur-based composites, no matter what density of fiber mat and amount of matrix used in the composites they have, even though it is natural consequence that different numerical value of constants a and b are given, when different kinds of matrix and fiber species are used.

The direction of further study for the wood-originated, fiber-reinforced composites is to seek as many numerical values of constants a and b as different kinds of fiber and matrix are used in order to establish matrix and fiber species effects on the modified rule of mixtures theorized by Bryant and Lee.

## References

1. Lee, B. G. 1983. The application of rule of mixtures to fiber-reinforced composites (1). *Mogjae-gonghak* (The Korean society of wood science & technology) Vol. 11(3): 3-13.
2. Lee, B. G. 1983. The application of rule of mixtures to fiber-reinforced composites (2). *Mogjae-gonghak* (The Korean society of wood science & technology) Vol. 11(5):33.
3. Jones, R. M. 1981. Mechanics of composite materials, *Metallurgical reviews*, 20, 91.
4. Bryant, B. S. and B. G. Lee. 1981. *Proceedings of Sulphur-81*:581.