Synthesis of a Four-Bar Linkage to Generate a Prescribed Coupler Curve

連桿軌跡을 利用한 4링크機構의 合成

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要 約

特殊한 機能을 遂行하기 위한 4링크 機構의 設計에서는 링크의 連桿軌跡이 重要한 設計條件이된다. 移秧機의 移植機構나 바인더의 放出암은 모두 4링크 機構를 利用하여 作業遂行에 必要한 連桿軌跡을 얻고 있는 것이다. 必要한 連桿軌跡을 얻기위한 4링크 機構의 合成은 圖解的, 解析的 方法을 通하여 많은 研究가 이루어져 왔으며 最近에는 콤퓨터를 利用한 機構合成에 對한 研究가 활발하게 이루어지고 있다. 本 研究에서는 連桿軌跡上의 點들을 利用하여 주어진 連桿軌跡을 얻기위한 4링크 機構의 合成에 對한 새로운 方法을 開發하고 이 方法을 콤퓨터 프로그래밍하여 주어진 連桿軌跡과 콤퓨터로 合成한 4링크 機構의 連桿軌跡을 比較검토 하였다.

1. Introduction

Four-bar linkages have been most useful and common mechanisms appearing in a variety of machines from small instruments to heavy mechanical equipment. Figure 1 shows a typical four-bar linkage mechanism. It consists of fixed frame 1, crank 2, connecting rod 3 which is also called coupler, and rocker 4.

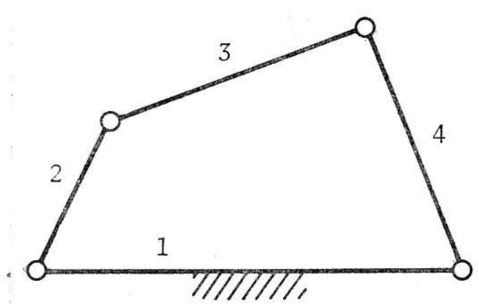

Fig. 1. Four-bar linkage mechanism

Since the four-bar mechanism has been studied extensively by many engineers, a considerable amount of work has been done on its kinematic theory. This may be because of the followings;

(1) Four-bar mechanisms have been widely used, and they are simple and basis of lower pair plane mechanisms.

(2) Complex mechanisms may be converted kinematically to a combination of four-bar mechanisms in some aspects of their motions. Therefore, thorough understanding of the kinematic theories on the four-bar mechanism may be inevitable.

Kinematic analysis can be performed using several methods with applications of vectors, complex variables and graphics. For a given mechanism, such motion characteristics as displacements, velocities, and accelerations are obtained in the analysis. Synthesis is the opposite of analysis. In other word, it is the creation of mechanism which will generate a desired set of

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input and output motion characteristics. For the mechanism synthesis, some graphical methods giving direct results have been used many years. However, only in recent years have analytical approaches been introduced.

It is often desired to have a mechanism trace the points along a previously specified path. The path generated by a point on the coupler is called a coupler curve and the tracing point is called a coupler point. The use of the coupler curves has numerous applications in machine design. Good examples can be found in many automatic machines such as those for wrapping, packing, weaving, vending, and so forth.

The objective of this study is to develop a method for synthesizing four-bar mechanisms that will generate such a coupler curve that the tracing point on the coupler passes through any previously specified path.

2. Review of Literature

A tracing point on the coupler of the four-bar mechanism generates a coupler curve. The equation of the coupler curve may be obtained using analytic geometry.

Robert, referring to Hartenberg et al. (1964), derived an equation of the coupler point curve as follows;

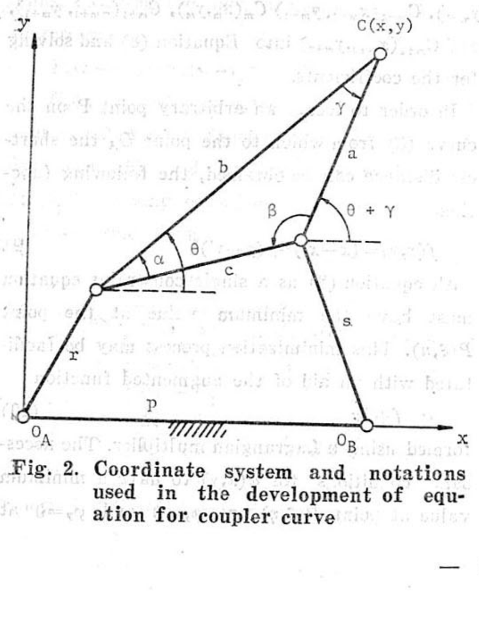

Fig. 2. Coordinate system and notations used in the development of equation for coupler curve

$$\sin\alpha((x-p)\sin\gamma-y\cos\gamma)(x^2+y^2+b^2-r^2)$$
$$+y\sin\beta((x-p)^2+y^2+a^2-s^2))^2$$
$$+(\sin\alpha((x-p)\cos\gamma-y\sin\gamma)(x^2+y^2+b^2-r^2)$$
$$-x\sin\beta((x-p)^2+y^2+a^2-s^2))^2$$
$$=4k^2\sin^2\alpha\sin^2\beta\sin^2\gamma(x(x-p)-y-py\cot\gamma)^2 \quad (1)$$

where k is a constant obtained by the sine law applied to triangle ABC,

$$k=\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma} \quad (2)$$

If designers are concerned only with the coupler curve traced by a coupler point, four-bar linkage may be synthesized by solving Equation (1), and two cognate linkages tracing an identical coupler curve may be found by an application of the Robert-Chebyshev theorem.

Hrones and Nelson (1951) published "The Hrones and Nelson atlas" in which 7,000 coupler curves of the crank and rocker mechanism are contained. This atlas can be used to select coupler curves having one and two cusps or having crossovers, for curves having segments which approximate circle arcs, and for curves having straight-line segments.

In the book of Shigley (1969), it was reported that Hartenberg et al. illustrated most of the classical straight-line generators. Tesar et al. futher investigated many approximate straight line mechanisms in great detail and developed a considerable amount of theory on the straight-line mechanisms.

Freudenstein and Sandor (1959) synthesized four-bar link mechanisms for generating a path through up to five arbitrary points corresponding to the prescribed crank rotations. His method is needed corresponding crank angles, to the five points, but with the five points, only desired curve may not be obtained.

Freudenstein (1965) also synthesized the linkage dimensions of a four-bar mechanism which would generate partial parabola and ellipse coupler curves.

In the book of Suh (1978), it was reported that if the nine points are given, a unique synthesis of four-bar mechanism is possible, and beyond six points, the synthesis is highly dependent on the choice of coordinates to be used.

3. Synthesis of Linkage

A. Determinations of Driving Link Length and Length between Moving Center of Crank and Coupler Point

In a crank and rocker mechanism, it occurs twice-stretched and folded that the driving link and the coupler link lie on the the same straight line during one complete revolution of the driving link. As shown in Figure 3, let R_l and R_s be the longest and shortest distances respectively from the axis of rotation of the crank passing through an arbitrary point O_A to a given coupler curve. Then the length of driving link, a_2, and the length between moving center A and coupler point C_j, a_5, can be determined from the following relations;

$$a_2 + a_5 = R_l \tag{3}$$

$$|a_2 - a_5| = R_s \tag{4}$$

If R_l is not equal to R_s, that is, the coupler curve is not a circle having its center at the point O_A, a_2 and a_6 are determined as follows;

$$a_2 = \frac{R_l - R_s}{2}, \quad a_5 = \frac{R_l + R_s}{2} \tag{5}$$

$$\text{or} \quad a_2 = \frac{R_l + R_s}{2}, \quad a_5 = \frac{R_l - R_s}{2} \tag{6}$$

By Robert-Chebychev theorem, a four-bar linkage synthesized using $a_2 = (R_l - R_s)/2$ and $a_5 = (R_l + R_s)/2$ generates another two cognate linkages which contain solutions of $a_2 = (R_l + R_s)/2$ and $a_5 = (R_l - R_s)/2$.

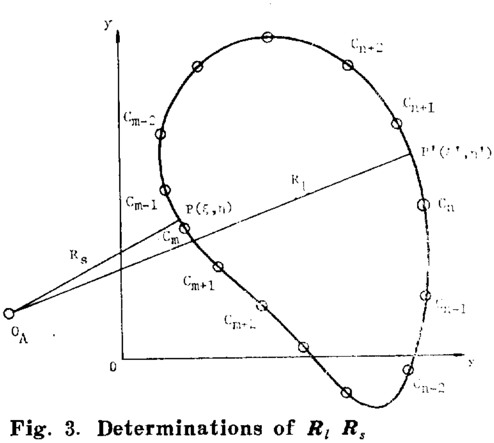

Fig. 3. Determinations of R_l R_s

The foregoing method for the determinations of R_l and R_s is a graphical approach. However, in order to use a digital computer, it is inevitable to develop an analytical method. If points C_j's are arbitrary points on the coupler curve, the length from O_A to C_j may be given by the folowing equation;

$$\delta_j = \sqrt{(x_j - x')^2 + (y_j - y')^2}$$

$$j = 1, 2, \cdots, n, \; n \geq 5 \tag{7}$$

where

n = total number of data point on the specified curve,

δ_j = length from O_A to C_j,

(x', y') = coordinates of O_A,

(x_j, y_j) = coordinates of C_j.

Assuming that δ_m is the distance from O_A to C_m on which gives the minimum of δ_j and locates the coupler curve which contains the successive adjacent coupler points, such as C_{m-2}, C_{m-1}, C_m, C_{m+1}, and C_{m+2}, can be expressed as a second order equations as follows;

$$g(x,y) = x^2 + b_1 y^2 + b_2 xy + b_3 x + b_4 y + b_6 = 0. \tag{8}$$

Then, the coefficients of Equation (8), b_1, b_2, b_3, b_4, and b_6 may be determined by substituting those five points on the coupler curve, $C_{m-2}(x_{m-2}, y_{m-2})$, $C_{m-1}(x_{m-1}, y_{m-1})$ $C_m(x_m, y_m)$, $C_{m+1}(x_{m+1}, y_{m+1})$, and $C_{m+2}(x_{m+2}, y_{m+2})$ into Equation (8) and solving for the coefficients.

In order to locate an arbitrary point P on the curve (8) from which to the point O_A the shortest distance can be obtained, the following function

$$f(x,y) = (x - x')^2 + (y - y')^2 \tag{9}$$

with equation (8) as a single constraint equation must have the minimum value at the point $P(\xi, n)$. This minimization process may be facilitated with an aid of the augmented function

$$\varphi = f + \lambda g \tag{10}$$

formed using a Lagrangian multiplier. The necessary conditions for $\varphi(x,y)$ to have a minimum value at point $P(\xi, \eta)$ are $\varphi_x = 0$ and $\varphi_y = 0$ at point $P(\xi,\eta)$. That is,

$$f_x+\lambda g_x=0 \text{ at } (\xi,\eta) \tag{11}$$

$$f_y+\lambda g_y=0 \text{ at } (\xi,\eta) \tag{12}$$

with the side condition

$$g(x,y)=0 \text{ at } (\xi,\eta) \tag{13}$$

equations (11), (12), and (13) give three equations in the three unknown quantities ξ,η, and λ. If the parameter λ is eliminated, Equations (11) and (12) become

$$f_x g_y-f_y g_x=0 \text{ at } (\xi,\eta) \tag{14}$$

Now, solving the second order simultaneous Equations (13) and (14) gives the point $P(\xi,\eta)$. Rewriting equations (13) and (14) in the following fashion facilitates the application of the numerical method.

$$G(x,y)=0 \tag{15}$$

$$F(x,y)=0 \tag{16}$$

Let (x_m,y_m) be an approximate solution of equations(15) and (16). Assuming that F and G are continuously differentiable functions and taking Taylor expansions of F and G about (x_m,y_m) yields

$$F(x,y)=F(x_m,y_m)+F_x(x_m,y_m)(x-x_m)+F_y(x_m,y_m)(y-y_m)+\cdots\cdots \tag{17}$$

$$G(x,y)=G(x_m,y_m)+G_x(x_m,y_m)(x-x_m)+G_y(x_m,y_m)(y-y_m)+\cdots\cdots \tag{18}$$

If (x_m,y_m) is sufficiently close to the solution (ξ,η), the higher order terms can be neglected. Thus, equating the linear terms of the expansion to zero gives

$$F_x(x-x_m)+F_y(y-y_m)=-F, \tag{19}$$

$$G_x(x-x_m)+G_y(y-y_m)=-G. \tag{20}$$

It is noted that all functions and derivatives in equations (19) and (20) are to be evaluated at (x_m,y_m). Solving equations (19) and (20) by Cramer's rule yields

$$x-x_m=\frac{\begin{vmatrix}-F & F_y\\ -G & G_y\end{vmatrix}}{\begin{vmatrix}F_x & F_y\\ G_x & G_y\end{vmatrix}}=\left[\frac{-FG_y+GF_y}{J(F,G)}\right](x_m,y_m) \tag{21}$$

$$y-y_m=\frac{\begin{vmatrix}F_x & -F\\ G_x & -G\end{vmatrix}}{\begin{vmatrix}F_x & F_y\\ G_x & G_y\end{vmatrix}}=\left[\frac{-GF_x+FG_y}{J(F,G)}\right](x_m,y_m) \tag{22}$$

where $J(F,G)=F_xG_y-G_xF_y$. $J(F,G)$ is the Jacobian of the functions F and G. For successive approximations, the recursion formulas may be formed as follows;

$$x_{i+1}=x_i-\left[\frac{FG_y-GF_y}{J(F,G)}\right]_i \tag{23}$$

$$y_{i+1}=y_i-\left[\frac{GF_x-FG_x}{J(F,G)}\right]_i \tag{24}$$

Consequently, The point $P(\xi,\eta)$ may be obtained by the iteration method using these recursion formulas. Then, R_s is given by

$$R_s=\sqrt{(\xi-x')^2+(\eta-y')^2} \tag{25}$$

Similarly, R_l can be obtained as follows. Determine C_n which gives the maximum value of δ_j and develop a second order equation using five successive adjacent coupler points, C_{n-2}, C_{n-1}, C_n, C_{n+1}, and C_{n+2}, as the same method as done for the determination of R_s. In order to determine the coordinates (ξ',η') of a point P' on the curve (8) from which to the point $O_A(x',y')$ the longest distance is obtained, function

$$f(x,y)=(x-x')^2+(y-y')^2 \tag{26}$$

with a single constraint Equation (8), must be extremized at the point $P'(x',y')$ to have the maximum value. In similar approach to the previous one, using the recursion formulas (19) and (20), and initial values of x, and y, the coordinates of point C_n, ξ' and η' can be determined by the iteration method. Then, R_l is given by

$$R_l=\sqrt{(\xi'-x')^2+(\eta'-y')^2} \tag{27}$$

B. Determination of Angular Position of Driving Link

Angular position of the driving link associated with a coupler point C_j, θ_{a_2,j_2} may be determined by the link lengths a_2 and a_5, and the coordinates of points C_j and O_A. As shown in Figure 4,

$$\vec{AC_j}=\vec{O_AC_j}-\vec{O_AA}=(x_j-x'-a_2\cos\theta_{a_2,j})i+(y_j-y'-a_2\sin\theta_{a_2,j})j, \tag{27}$$

and

$$a_5^2=\left|\vec{AC_j}\right|^2=(x_j-x'-a_2\cos\theta_{a_2,j})^2+(y_j-y'-a_2\sin\theta_{a_2,j})^2. \tag{28}$$

Rearranging Equation (28) yields

$$(y_j-y')\sin\theta_{a_2,j}+(x_j-x')\cos\theta_{a_2,j} = \frac{(x_j-x')^2+(y_j-y')^2+a_2^2-a_5^2}{2a_2} \quad (29)$$

Solving for $\theta_{a_2,j}$ gives

$$\theta a_{2,j}=\sin^{-1}\left[\frac{(x_j-x')^2+(y_j-y')^2+a_2^2-a_5^2}{2a_2\sqrt{(x_j-x')^2+(y_j-y')^2}}\right] -\tan^{-1}\left(\frac{x_j-x'}{y_j-y'}\right) \quad (30)$$

and

$$\theta_{a_2,j}=\pi-\sin^{-1}\left[\frac{(x_j-x')^2+(y_j-y')^2+a_2^2-a_5^2}{2a_2\sqrt{(x_j-x')^2+(y_j-y')^2}}\right] -\tan^{-1}\left(\frac{x_j-x'}{y_j-y'}\right) \quad (31)$$

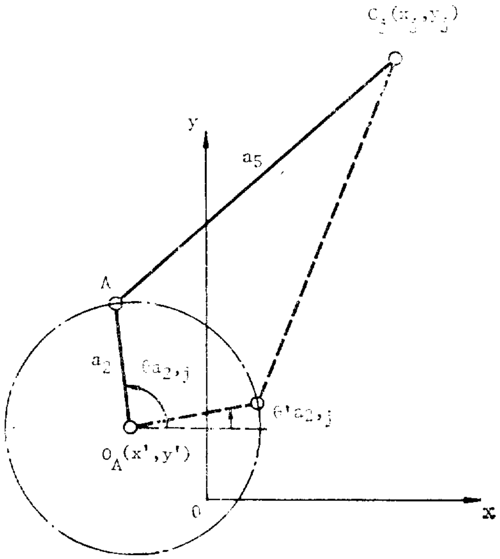

Fig. 4. Relations of driving and coupler links

These are two angular positions of driving link associated with a coupler point C_j. If n coupler points on the coupler curve are chosen, 2^n angular position groups are obtained, where the angular position group is a set of n angular positions of driving link corresponding to n coupler points.

From these 2^n angular position groups, the proper angular position group must be found. If the four-bar linkage is the crank-rocker mechanism and C_1, C_2, C_3, ⋯, and C_n are the successive n adjacent coupler points, corresponding angular positions of driving link must be located successively in either clockwise or counterclockwise during only one revolution of the driving link. In order to satisfy the above condition, the values of all $\sin(\theta_{a_2,i+1}-\theta_{a_2,i})$ must be either positive or negative. Thus, using equation

$$N=\sum_{i=1}^{n}\frac{\sin(\theta_{a_2,i+1}-\theta_{a_2,i})}{|\sin(\theta_{a_2,i+1}-\theta_{a_2,i})|} \quad (32)$$

where $\theta_{a_2,n+1}=\theta_{a_2,1}$

the proper angular position group is determined when $N=\pm n$.

C. Determinations of Fixed Link Length, Connecting Link Length, Follower Link Length, Angular Position of Fixed Link, and Angle on Coupler

From the condition that point B is oscillating about O_B, an equation relating a_4 and α may be derived. This equation will be be written with respect to the new coordinate system with the X-axis along the line paralleled to $0x$ at point O_A and the Y-axis along the perpendicular to $0x$ at O_A as shown in Figure 5.

Let (X_j, Y_j) be the coordinates of point C_j and

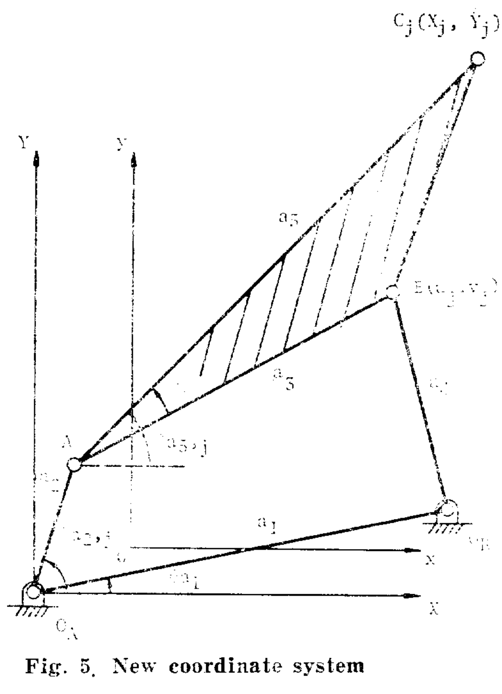

Fig. 5. New coordinate system

(u_j, v_j) be the coordinates of point B with respect to the new coordinate system X-Y, then

$$X_j = x' + x_j \tag{33}$$

$$Y_j = y' + y_j \tag{34}$$

$$u_j = a_2\cos\theta_{a_2,j} + a_3\cos(\theta_{a_3,j} - \alpha) \tag{35}$$

$$v_j = a_2\sin\theta_{a_2,j} + a_3\sin(\theta_{a_3,j} - \alpha) \tag{36}$$

where

$$\theta_{a_3,j} = \tan^{-1}\left(\frac{Y_j - a_2\sin\theta_{a_2,j}}{X_j - a_2\cos\theta_{a_2,j}}\right). \tag{37}$$

Since point B oscillates along the arc of a circle with its center at point O_B,

$$u_i^2 + v_i^2 + Au_i + Bv_i + C = 0, \quad i = 1, 2, 3, 4 \tag{38}$$

where A, B, and C are constants, and index i indicates the point to be precisely passed.

Since this system involves three unknowns, A, B, and C, and four equations, a unique solution can be obtained only when the coefficient matrix of the system is of rank 3. Thus, for a system (38), its characteristic determinant must vanish. That is,

$$\begin{vmatrix} u_1^2+v_1^2 & u_1 & v_1 & 1 \\ u_2^2+v_2^2 & u_2 & v_2 & 1 \\ u_3^2+v_3^2 & u_3 & v_3 & 1 \\ u_4^2+v_4^2 & u_4 & v_4 & 1 \end{vmatrix} = 0 \tag{39}$$

In order to simplify the Equation (39), it will be convenient to rewrite Equations (35) and (36) as follows;

$$u_j = q_j + a_3 r_j \tag{40}$$

$$v_j = p_j + a_3 s_j \tag{41}$$

where

$$p_j = a_2\cos\theta_{a_2,j} \tag{42}$$

$$q_j = a_2\sin\theta_{a_2,j} \tag{43}$$

$$r_j = \cos(\theta_{a_3,j} - \alpha) \tag{44}$$

$$s_j = \sin(\theta_{a_3,j} - \alpha) \tag{45}$$

Evaluation of Equation (39) will be summarized in Appendix Ⅱ. Since the r_j and s_j are function of α, u_j and v_j have two variables α and a_3. Rewriting Equation (39) is as follows;

$$A_1(\alpha)a_3^3 + A_2(\alpha)a_3^2 + A_3(\alpha)a_3 = 0. \tag{46}$$

The condition $a_3 > 0$ gives

$$a_3 = \frac{-A_2(\alpha) \pm \sqrt{A_2^2(\alpha) - 4A_1(\alpha)A_3(\alpha)}}{2A_1(\alpha)} \tag{47}$$

If three equations $(i = 1, 2, 3)$ are selected from Equation (38), the unknowns, A, B, and C, are determined as follows;

$$A = \frac{\begin{vmatrix} -(u_1^2+v_1^2) & v_1 & 1 \\ -(u_2^2+v_2^2) & v_2 & 1 \\ -(u_3^2+v_3^2) & v_3 & 1 \end{vmatrix}}{\begin{vmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{vmatrix}} \tag{48}$$

$$B = \frac{\begin{vmatrix} u_1 & -(u_1^2+v_1^2) & 1 \\ u_2 & -(u_2^2+v_2^2) & 1 \\ u_3 & -(u_3^2+v_3^2) & 1 \end{vmatrix}}{\begin{vmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{vmatrix}} \tag{49}$$

$$C = \frac{\begin{vmatrix} u_1 & v_1 & -(u_1^2+v_1^2) \\ u_2 & v_2 & -(u_2^2+v_2^2) \\ u_3 & v_3 & -(u_3^2+v_3^2) \end{vmatrix}}{\begin{vmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{vmatrix}} \tag{50}$$

If α is given by an arbitrary constant, a_3, A, B, and C are determined. If the conditions $a_3 > a_2$ is satisfied, a_1, a_4, and θ_{a_1} may be determined as follows;

$$a_1 = \frac{1}{2}\sqrt{A^2 + B^2} \tag{51}$$

$$a_4 = \frac{1}{2}\sqrt{A^2 + B^2 - 4C} \tag{52}$$

$$\theta_{c_1} = \tan^{-1}\left(\frac{-B}{-A}\right) \tag{53}$$

So far, all dimensions of a four-bar linkage for an arbitrary angle ∠CAB have been determined. The coupler point of this four-bar linkage must pass precisely through the prescribed four points.

It is now necessary to minimize the sumation of distances between the coupler curve and other n-4 points. But it is difficult to find directly distances between the coupler curve and other n-4 points. So, from the condition that point B derived from above dimensions must be on the arc of a circle with the center O_B and radius a_4, following equation was used

$$d_j = \left| a_4 - \sqrt{\left(u_j - \frac{A}{2}\right)^2 + \left(v_j - \frac{B}{2}\right)^2} \right| \tag{54}$$

where d_j denotes the distance from a circle with

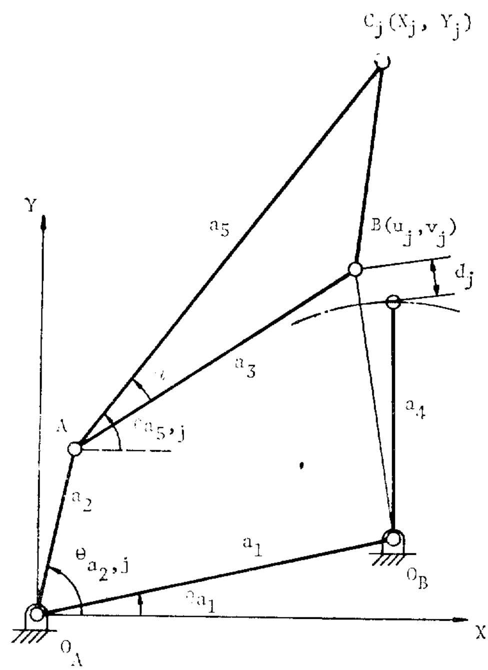

Fig. 6. Determination of d_j

the radius a_4 and the center O_B, obtained from Equations (51), (52), and (53), to point B_j derived from C_j in Figure 6.

When the angle ∠CAB changes from 0° to 360° with a constant step size, each angle can produce corresponding four-bar linkages and d_j. If a four-bar linkage resulted from an angle ∠CAB makes $\sum d_j$ minimum, the coupler point of that four-bar linkage can pass precisely through four given points and most approximately through other n-4 points.

D. Determination of Linkage Type

A four-bar linkage is shown in Figure 7. From the Grashoff's law that expresses $a_2+a_3>a_1+a_2$ and $a_2+a_3<a_1+a_4$, angular position of the rocker measured from the the positive O_BX'-axis can be neither 0° nor 180° during one complete revolution of the crank. When the angular position of the crank is 0°, the angular position of the rocker is either between 0° and 180° or between 180° and 360°. Therefore the crank-rocker mechanism synthesized above has two possible types, upper-type and lower type. The upper-type is the mechanism whose angular position of the rocker lies between 0° and 180°, and the lower-type is the mechanism whose angular position of the rocker lies between 180° and 360°.

From the one arbitrary prescribed point, the angular position of the rocker is determined by the following equation;

$$\theta'_{a_4,j}=\tan^{-1}\left(\frac{v_j-a_1\sin\theta_{a_1}}{u_j-a\cos\theta_{a_1}}\right)-\theta_{a_1} \quad (55)$$

The type of four-bar linkage can be determined by this equation.

4. Results and Discussion

Assuming that it is desired to synthesize a

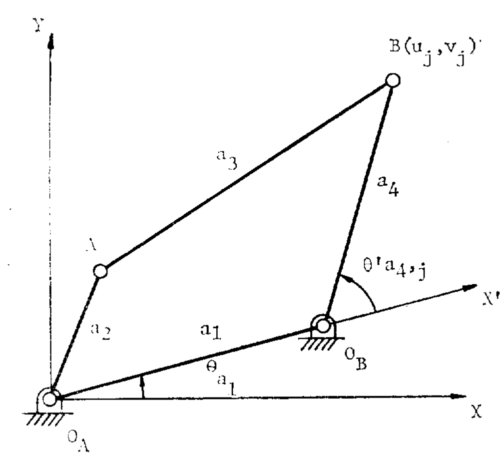

Fig. 7. The angular position of the rocker

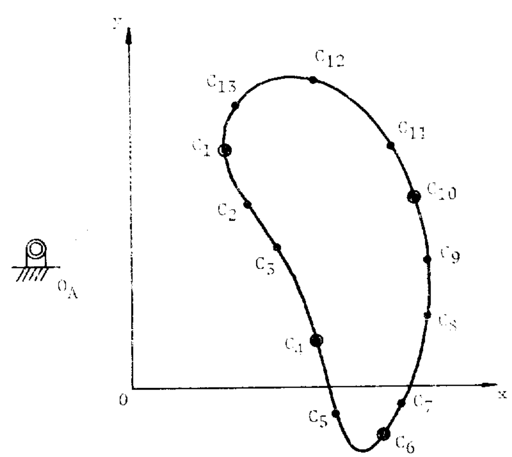

Fig. 8. A prescribed curve with four precision points

four-bar linkage to generate the coupler curve shown in Figure 8 passing through the precisely specified four points, (20, 50), (39, 10), (52, −10), and (59, 40), with the rotation axis of the crank at (−20, 30)

$C_{13}(22, 60)$,

where C_1, C_4, C_6 and C_{10} are points precisely given.

The synthesis results are listed in Table 1.

Table 1. Input data, output 4-bar linkage, and error

INPUT DATA	PRECISION POINT	OUTPUT 4-BAR LINKAGE	*error
OA (−20.0, 30.0)			
C_1 (20.0, 50.0)	C 1	a_1 =51.6441	EC_1 ; .0122
C_2 (25.0, 39.0)	C 4	a_2 =19.6254	EC_2 ; .0172
C_3 (31.0, 30.0)	C 6	a_3 =59.0861	EC_3 ; .2768
C_4 (39.0, 10.0)	C 10	a_4 =28.8355	EC_4 ; .0324
C_5 (43.0, − 6.0)		a_5 =63.7024	EC_5 ; .2137
C_6 (52.0, −10.0)		θ_{a_1} =237.985(DEG)	EC_6 ; .0051
C_7 (56.0, − 4.0)		α =149.000(DEG)	EC_7 ; .0548
C_8 (61.0, 15.0)		TYPE=LOWER-TYPE	EC_8 ; .3887
C_9 (61.0, 27.0)			EC_9 ; .5693
C_{10} (59.0, 40.0)			EC_{10} ; .0005
C_{11} (54.0, 51.0)			EC_{11} ; .1309
C_{12} (38.0, 65.0)			EC_{12} ; 1.1178
C_{13} (22.0, 60.0)			EC_{13} ; .5790

*error : minimum deviation between input point and output coupler curve

From the given coupler curve shown in Figure 8, the coordinates of points are determined as follows;

$O_A(-20, 30)$,
$C_1(20, 50)$, $C_2(25, 39)$, $C_3(31, 30)$,
$C_4(39, 10)$, $C_5(43-6)$, $C_6(52, -10)$,
$C_7(56, -4)$, $C_8(61, 15)$, $C_9(61\ 27)$,
$C_{10}(59, 40)$, $C_{11}(54, 51)$, $C_{12}(38, 65)$,

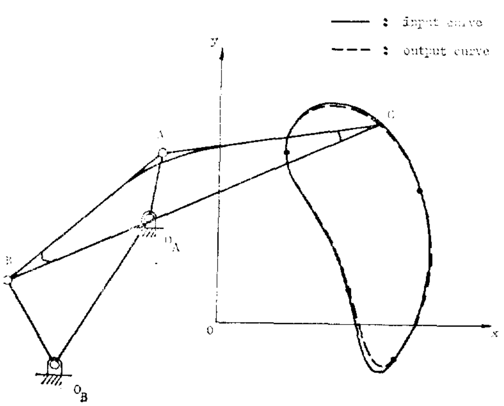

Fig. 9. The synthesized linkage and input and output curves

The coupler curve of the synthesized linkage is compared with the input coupler curve in Figure 9.

Deviation between the input and output curves may decrease by selecting the point O_A properly. But if the desired curve does not satisfy Equation (1), that is, it is not a curve that is generated by a coupler point of the crank and rocker mechanism, deviation may not vanish.

This method can be used when the desired curve is the full sketch curve with four precision points.

5. Conclusions

This method was developed using the conditions that it occurs twice-streched and folded-that the driving link and the coupler link lie on the same straight line during one complete revolution of the driving link, and point B is oscillating about point O_B, and using the matrix theory of linear systems.

The method developed can be used to synthesize a four-bar linkage to generate a prescribed coupler curve using digital computers. The input data for the computer program are the coordinates of points on the given coupler curve. As the number of points are increased, the computer cpu time will be increased. By selecting the minimum data points to describe the desired curve sufficiently, the computer cpu time will be minimized.

References

1. Conte, S.D. and Carl de Boor. 1972. Elementary numerical analysis. 2nd edition. McGraw-Hill, Inc.
2. Freudenstein, F. and G.N. Sandor. 1959. Synthesis of path generating mechanisms by means of a programmed digital computer. Trans. ASME, series B, Journal of Engineering for Industry, Vol.81, pp.159-167.
3. Hartenberg, R.S. and J. Denavit. 1964. Kinematic synthesis of linkages. McGraw-Hill Book Company.
4. Hilderbrand, F.B. 1976. Advanced calculus for application. 2nd edition. Prentice-Hall, Inc.
5. Hinkle, R.T. 1960. Kinematics of machines. 2nd edition. Prentice-Hall, Inc.
6. James M.L., G.M. Smith and J.C. Wolford. 1977. Applied Numerical methods for digital computation. 2nd edition. Happer &. Row Publishers.
7. Shigley, J.E. 1969. Kinematic analysis of mechanisms. 2nd edition. McGraw-Hill, Inc.
8. Suh, C.H. and C.W. Radcliffe. 1978. Kinematics and mechanisms design. John Wiley G. Sons, Inc.

APPENDIX I

Nomenclatures.

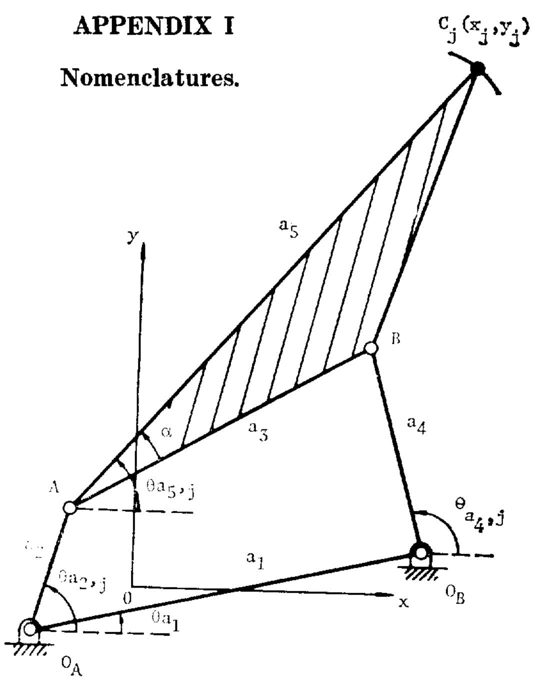

Fig. 10. Four-bar linkage showing nomenclature used

The nomenclatures used in this study are as follows;

A, B = Moving centers

O_A, O_B = Fixed centers

C_j = Coupler point

O_AO_B = Fixed link

O_AA = Driving link

ABC_j = Connecting link

O_BB = Follower link

a_1 = Length of fixed link

a_2 = Length of driving link

a_3 = Length between moving centers A and B

a_4 = Length of follower link

a_5 = Length between moving center A and coupler point C_j

θ_{a_1} = Angular position of fixed link

$\theta_{a_2,j}$ = Angular position of driving link with coupler point C_j

$\theta_{a_4,j}$ = Angular position of follower link with coupler point C_j

$\theta_{a_3,j}$ = Angular position of AC_j with coupler point C_j

α = Angle between AB and AC_j measured counterclockwise from AB

APPENDIX II

Rearranging the Equation (39) by substituing Equations (40)−(45) into Equation (39) yields the following fourth order equation for a_3;

$$A_0 a_3^4 + A_1 a_3^3 + A_2 a_3^2 + A_3 a_3 + A_4 = 0,$$

where

$$A_0 = 0,$$

$$\begin{aligned}
A_1 = {} & 2(p_1r_1+q_1s_1)(r_2(s_4-s_3)+r_3(s_2-s_4)+r_4(s_3-s_2)) \\
& +2(p_2r_2+q_2s_2)(r_1(s_3-s_4)+r_3(s_4-s_1)+r_4(s_1-s_3)) \\
& +2(p_3r_3+q_3s_3)(r_1(s_4-s_2)+r_2(s_1-s_4)+r_4(s_2-s_1)) \\
& +2(p_4r_4+q_4s_4)(r_1(s_2-s_3)+r_2(s_3-s_1)+r_3(s_1-s_2))
\end{aligned}$$

$$\begin{aligned}
A_2 = {} & 2(p_1r_1+q_1s_1)(p_2(s_4-s_3)+p_3(s_2-s_4)+p_4(s_3-s_2)) \\
& \qquad +r_2(q_4-q_3)+r_3(q_2-q_4)+r_4(q_3-q_2)) \\
& +2(p_2r_2+q_2s_2)(p_1(s_3-s_4)+p_3(s_4-s_2)+p_4(s_1-s_3)) \\
& \qquad +r_1(q_3-q_4)+r_3(q_4-q_1)+r_4(q_1-q_3)) \\
& +2(p_3r_3+q_3s_3)(p_1(s_4-s_2)+p_2(s_1-s_4)+p_4(s_2-s_1)) \\
& \qquad +r_1(q_4-q_2)+r_2(q_1-q_4)+r_4(q_2-q_1)) \\
& +2(p_4r_4+q_4s_4)(p_1(s_2-s_3)+p_2(s_3-s_1)+p_3(q_1-q_2)) \\
& \qquad +r_1(q_2-q_3)+r_2(q_3-q_1)+r_3(q_1-q_2))
\end{aligned}$$

$$\begin{aligned}
A_3 = {} & 2(p_1r_1+q_1s_1)(p_2(q_4-q_3)+p_3(q_2-q_4)+p_4(q_3-q_2)) \\
& +2(p_2r_2+q_2s_2)(p_1(q_3-q_4)+p_3(q_4-q_1)+p_4(q_1-q_3)) \\
& +2(p_3r_3+q_3s_3)(p_1(q_4-q_2)+p^2(q_1-q_4)+p_4(q_2-q_1)) \\
& +2(p_4r_4+q_4s_4)(p_1(q_2-q_3)+p_2(q_3-q_1)+p_3(q_1-q_2))
\end{aligned}$$

$$A_4 = 0.$$