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Asymptotic Stability of Multimachine Power Systems

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大規模 電力系統의 安定度 解析에 관한 硏究

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抄 錄

微分方程式으로 주어지는 높은 次數의 動特性 시스템의 安定度를 Lyapunov 理論을 利用하여 解析하였다.

大規模 시스템 自體를 直接 解析함에 있어서는 그의 複雜性과 次數問題가 擡頭되어 매우 곤란하다. 따라서 可能한限 보다 작은 次數의 시스템으로 分割하여 解析하고 이를 다시 合成하여 綜合 解析하였으며 分割된 작은 次數의 시스템에 대해서는 數學行列의 安定特性을 利用한 새로운 形態의 Lyapunov 函數를 使用하였다.

應用의 한 例로써, 3個의 同期發電機를 가진 電力系統에 適用하여 旣存의 方法보다 容易하게 이 시스템의 安定度를 判別할 수 있었다.

I. Introduction

Since large scale systems such as power systems tax the capabilities of most modern computers, we use the decomposition and aggregation method. Making use of the Lyapunov stability and the constructive stability based on the concept of stable matrices, we analyze the stability of the system, where the stability properties of the isolated subsystems are investigated by the constructive method.

The application of the second method of Lyapunov to power systems was introduced by Gless [1] and also by El-Abiad and Nagappan [2] in the same year (1966). For the detailed history of the applications of the second method of Lyapunov to power systems, refer to [3].

The ideas presented in this paper were motivated by [4]. A short summary of this paper is as follows: We first present the constructive stability results by [5] and [6], from which we get computer-generated Lyapunov functions. Next, an aggregated test matrix for the stability analysis of the overall system is determined in terms of the

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qualitative properties of the isolated subsystems and in terms of the interconnecting characteristics. Finally, we apply the above results to a 3-machine power system [7] to show the system is asymptotically stable in some region.

II. Background Material

In this section we summarize some global results from [5] and [6].

We concern ourselves with systems described by equations of the form

$$\dot{x}=F(x) \tag{E}$$

Where $x \in R^n$, $t \in [o, \infty)=R^+$, $\dot{x}=\frac{dx}{dt}$, $F; R^n \to R^n$ and $F(x)=o$ if and only if $x=o$.

System (E) can be rewritten as

$$\dot{x}=M(x)x \tag{E'}$$

where $M(x)$ is chosen so that $M(x)x=F(x)$. Applying Euler's formula to(E'), we obtain

$$x_{k+1}=x_k+h_k M(x_k)x_k \tag{1}$$

where $h_k=t_{k+1}-t_k$, the current step size. If we let S denote the set of all matrices obtained by varying x_k over all allowable values, then we get

$$x_{k+1}=(I_n+h_k M_k)x_k, \quad M_k \in S \tag{2}$$

where I_n denotes the $(n \times n)$ identity matrix.

In [5] and [6] it is shown that if $x=o$ of (2) is stable (globally asymptotically stable) for all sequences $\{h_n\}$, $o<h_n<h'$ for some $h'>o$, then the equilibrium $x=o$ of (E) is stable (globally asymptotically stable).

Let S denote the set of $(n \times n)$ matrices with the property that for every $x \in R^n$ there exists an $M \in S$ such that $F(x)=Mx$. Let $A=[I_n+hS]$ for some $h>o$.

We call a set A of $(n \times n)$ real matrices <u>stable</u> if for every neighborhood of the origin $U \subset R^n$ there exists another neighborhood of the origin $V \subset R^n$ such that for every $M \in A'$, we have $MV \subseteq U$. Here A' denotes the semigroup of A.

In [5] it is shown that the following statements are equivalent : (a) A is stable ; (b) A' is bounded ; (c) There exists a bounded neighborhood of the origin $W \subset R^n$ such that $MW \subseteq W$ for every $M \in A$. (Furthermore, W can be chosen to be convex and balanced.) ; (d) There exists a vector norm $\|\cdot\|_w$ such that $\|Mx\|_w \leq \|x\|_w$ for all $M \in A$ and for all $x \in R^n$.

Let $\alpha \in R$, let $W \subset R^n$, and let $\alpha W=[u \in R^n : u=\alpha w,\ w \in W]$. Since statements (c) and (d) above are related by

$$\|x\|_w=\inf[\alpha : \alpha \geq 0,\ x \in \alpha W] \tag{3}$$

it follows that $\|x\|_w$ defines a Lyapunov function for A, i. e., it defines a function v with the property

$v(Mx) \leq v(x)$ for all $M \in A$ and for all $x \in R^n$.

Next, we call a set of matrices A <u>asymptotically stable</u> if there exists a number $\rho>1$ such that ρA is stable.

In [5] and [6] a constructive algorithm is given to determine whether a set of $m(n \times n)$ real matrices $A=[M_o, \ldots, M_{m-1}]$ is stable by starting with an initial polyhedral neighborhood of the origin, W_o, and by defining a sequence of regions $[W_{k+1}]$ by

$$W_{k+1} \triangleq K[\bigcup_{j=0}^{\infty} M_{k'} W_k], \text{ where } k'=(k-1) \bmod m \tag{4}$$

and where $K[\cdot]$ denotes the convex hull of a set. Now A is stable if and only if

$$W^*=\bigcup_{k=0}^{\infty} W_k \tag{5}$$

is bounded.

In practice, W_o above is usually chosen as simple as possible, i. e., it is chosen as the region defined by

$$E(W_o)=[w_i \in R^n : x_{ii}=1, \quad x_{ij}=0, \quad i \neq j, \quad i=1, \ldots, n]$$

where $w_i^T=(x_{i1},\dots, x_{in})\in R^n$.

We begin by linearizing (E) about the equilibrium x=o of (E). This yields the equation

$$\dot{x}=Jx+F_1(x) \tag{6}$$

where $J=\frac{\partial F}{\partial x}(x)|_{x=0}$ denotes the Jacobian matrix evaluated at x=o and $F_1(x)$ consists of higher order terms in the components of x. Now if the real parts of the eigenvalues of J are negative, then the equilibrium x=o of (E) will be asymptotically stable, and furthermore, the equilibrium x=o of the linearization of (E), given by

$$\dot{x}=Jx \tag{7}$$

will be globally asymptotically stable.

If we follow the procedure stated earlier in this section, then we get the singleton extremal matrix $E(A)=\{I_n+hJ\}$ for some $h>0$. Using the multiplicative semigroup of the set $\{\rho(I_n+hJ)\}$, for some $\rho>1$, we make use of the constructiue algorithm to obtain W^*, starting with an initial convex set W_o. The set W^* determines a norm $v(x) \triangleq ||x||_{w^*}$ which will serve as a Lyapunov function for (7), also for (6), and hence, for (E).

We first note that $\partial W^*=\{x\in R^n : ||x||_*=1\}$. Since W^* is convex, then for any $x\in R^n$ there is an $x'\in \partial W^*$ such that $x=\alpha x'$ for some $\alpha\geq o$. For this α we have $||x||_*=\alpha$.

For further details concerning the above constructive algorithm, the reader should consult [5], [6], and [8].

Ⅲ. Interconnected Systems

Frequently it is possible to view systems described by (E) as interconnected systems of the form

$$\dot{Z}_i=F_i(Z_i)+G_i(x),\ i=1,\dots, l, \tag{Σ_i}$$

where $Z_i\in R^{n_i}$, $F_i : R^{n_i}\to R^{n_i}$, $x\in R^n$ with $x^T=(Z_1^T,\dots,Z_l^T)$ and $n=\sum_{i=1}^{l} n_i$, and $G_i : R^n\to R^{n_i}$. If we define $f(x)^T=(f_1(Z_1)^T,\dots, f_l(Z_l)^T)$ and $g(x)^T=(g_1(x)^T,\dots, g_l(x)^T)$, then ($\Sigma$i) can be rewritten as

$$\dot{x}=f(x)+g(x)\triangleq F(x). \tag{S}$$

We call

$$\dot{Z}_i=F_i(Z_i)\ i=1,\dots, l, \tag{S_i}$$

the isolated subsystems. As in the previous section, (S_i) can be rewritten equivalently as

$$\dot{Z}_i=M_i(Z_i)Z_i \tag{S_i'}$$

where $M_i(Z_i)$ is chosen so that $F_i(Z_i)=M_i(Z_i)Z_i$ for all $Z_i\in R^{n_i}$

Lemma 1[4] : Suppose that for system (S_i') we can find some $h_i'>0$ and some $\rho_i>1$ such that the set

$$\{\rho_i(I_{n_i}+h_i'M_i(Z_i)) : Z_i\in R^{n_i}\} \tag{8}$$

is stable. Let W_i^* denote the convex, balanced set determined by the constructive algorithm for the set(8). Let $v_i(Z_i)=||Z_i||_i$ denote the corresponding norm Lyapunov function. Then the total time derivative along the solutions of (S_i'), $Dv_{i_{(S_i')}}(Z_i)$, can be estimated by

$$Dv_{i_{(S_i')}}(Z_i)\leq-\mu_i v_i(Z_i),$$

where $\mu_i=(1-\frac{1}{\rho_i})(\frac{1}{h_i'})>0$.

Theorem 1[6] : The equilibrium x=0 of (Σ_i) is globally asymptotically stable if the following hypotheses are satisfied : (A−1) The free subsystem (S_i) satisfies Lemma 1 ; (A−2) For (Σ_i) there exist constants $g_{ij}\geq0$ i, j=1, . . . , l, such that

$$||g_i(x)||_i\leq\sum_{j=1}^{l} g_{ij}||Z_j||_j \qquad \text{for all } x\in R^n ;$$

(A−3) The successive principal minors of the $(l\times l)$ test matrix $D=[d_{ij}]$ are all positive=where

$$d_{ij}=\begin{Bmatrix}\mu_i-g_{ii}, & i=j\\ -g_{ij}, & i\neq j\end{Bmatrix}.$$

In particular, if we have $g_i(x)=\sum_{j=1}^{l} A_{ij}Z_j$,

$i=1,\ldots,l$, where $A_{ij}\in R^{n_i\times n_j}$ are constant matrices(independent of x), in this case we can take $g_{ij}=\|A_{ij}\|_{ij}$, where

$$\|A_{ij}\|_{ij}=\max\{\|A_{ij}Z_j\|_i : Z_j\in E(W_j^*)\}.$$

Ⅳ. Application to Power Systems

Consider an n-machine power system in which the absolute motions of the i-th machine is described by the equations.

$$M_i\ddot{\delta}_i+D_i\dot{\delta}_i=P_{mi}-P_{ei},\ i=1,2,\ldots,n \qquad (9)$$

where $P_{ei}=\sum_{j=1}^{n}E_iE_jY_{ij}\cos(\delta_i-\delta_j-\theta_{ij})$ and

δ_i : absolute rotor angle

M_i : inertia coefficient

D_i : damping coefficient

P_{mi} : mechanical power delivered to the i-th machine

P_{ei} : electrical power delivered by the i-th machine

E_i : internal voltage

Y_{ij} : modulus of the transfer admittance between the i-th and j-th machines

θ_{ij} : phase angle of the transfer admittance between the i-th and j-th machines.

In (9), it is assumed that M_i, D_i, P_{mi}, and E_i are constant for all machines. In addition, we assume uniform damping, that is,

$$\frac{D_i}{M_i}=\gamma,\ i=1,2,\ldots,n.$$

We note that the equilibrium of (9) is ($\omega_i=0$, $\delta_i=\delta_i^{\circ}$), where $\dot{\delta}_i=\omega_i$ and δ_i°'s are components of an asymptotically stable equilibrium obtained as solutions of the equations

$$P_{ei}(\delta_i^{\circ})=P_{mi},\ i=1,2,\ldots,n. \qquad (10)$$

Let $\delta_{in}=\delta_i-\delta_n$ and $\omega_{in}=\omega_i-\omega_n$. If we define

$Z_i^T=(\omega_{in},\ \delta_{in}-\delta^{\circ}_{in})$, we get the equations of the form

$$\dot{Z}_i=F_i(Z_i)+G_i(x),\quad i=1,\ldots,n-1 \qquad (\Sigma_i)$$

where

$$F_i(Z_i)=\begin{bmatrix}0 & 1\\ 0 & -\gamma\end{bmatrix}Z_i+\begin{bmatrix}0\\ -1\end{bmatrix}f_i(C^TZ_i),$$

$$C^T=[1 \quad 0],$$

$$G_i(x)^T=[0 \quad g_i(x)],$$

$$f_i(y)=\left(\frac{1}{M_i}-\frac{1}{M_n}\right)E_iE_nY_{in}\cos\theta_{in}[\cos(y+\delta^{\circ}_{in})-\cos\delta^{\circ}_{in}]+\left(\frac{1}{M_i}+\frac{1}{M_n}\right)E_iE_nY_{in}\sin\theta_{in}\cdot[\sin(y+\delta^{\circ}_{in})-\sin\delta^{\circ}_{in}]$$

$$g_i(x)=\frac{1}{M_n}\sum_{\substack{j=1\\ j\neq i}}^{n-1}E_nE_jY_{nj}[\cos(-x_{1j}+\delta^{\circ}_{nj}-\theta_{nj})-\cos(\delta^{\circ}_{nj}-\theta_{nj})]-\frac{1}{M_i}\sum_{\substack{j=1\\ j\neq i}}^{n-1}E_iE_jY_{ij}[\cos(x_{1i}-x_{1j}+\delta^{\circ}_{ij}-\theta_{ij})-\cos(\delta^{\circ}_{ij}-\theta_{ij})]$$

$x_{1i}=C^TZ_i$, and

$x^T=(Z_1^T,\ Z_2^T,\ldots,Z^T_{n-1})$.

As an example, a three-machine system given in [8] will be used. The values of the paramenters are specified as follows: $M_1=0.01$, $M_2=0.01$, $M_3=2.0$, $E_1=1.017$, $E_2=1.005$, $E_3=1.033$, $Y_{12}=0.98\times10^{-3}$, $Y_{13}=0.114$, $Y_{23}=0.106$, $\theta_{12}=86°$, $\theta_{13}=88°$, $\theta_{23}=89°$, $\delta^{\circ}_{12}=5°$, $\delta^{\circ}_{13}=-2°$, $\delta^{\circ}_{23}=-3°$, $\gamma=6$ (instead of $\gamma=100$ which is less realistic). With machine 3 as a reference, two subsystems are formed, where the isolated subsystems are:

$$\dot{Z}_i=F_i(Z_i),\ i=1,2, \qquad (S_i)$$

wheer $Z_1^T=(x_1,\ x_2)$ and $Z_2^T=(x_3,\ x_4)$, and

$$(S_1)\ \begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-6x_2-0.4159\{\cos(x_1+\delta^{\circ}_{13})-\cos\delta^{\circ}_{13}\}-12.0289\{\sin(x_1+\delta^{\circ}_{13})-\sin\delta^{\circ}_{13}\}\end{cases}$$

$$(S_2)\ \begin{cases}\dot{x}_3=x_4\\ \dot{x}_4=-6x_4-0.1911[\cos(x_3+\delta^{\circ}_{23})-\cos\delta^{\circ}_{23}]-11.0579[\sin(x_3+\delta^{\circ}_{23})-\sin\delta^{\circ}_{23}].\end{cases}$$

Applying the constructive algorithm discussed in section II, we get the Jacobian matrix :

$$J_1=\begin{bmatrix} 0 & 1 \\ -12.0292 & -6 \end{bmatrix} \text{ and}$$

$$J_2=\begin{bmatrix} 0 & 1 \\ -11.0581 & -6 \end{bmatrix}.$$

And if we choose the time-step $h_1=h_2=0.1$, then we obtain the asymptotically stable matrices :

$$M_1=\begin{bmatrix} 1 & 0.1 \\ -1.2029 & 0.4 \end{bmatrix} \text{ and}$$

$$M_2=\begin{bmatrix} 1 & 0.1 \\ -1.1058 & 0.4 \end{bmatrix},$$

where $\rho_1=1.38$ and $\rho_2=1.39$ are chosen such that $\rho_i M_i$, $i=1,2$ are stable. Therefore, applying Lemma 1 to this isolated subsystem, we get the stability measure $\mu_1=2.7536$ and $\mu_2=2.8057$.

Next, we will estimate the interconnecting functions $G_i(x)$ by the Lyapunov functions $v_i(x)=\|x\|_i$ generated by W_i^* as discussed in section II. We use the following inequalities :

$$a[\cos(y+\theta)-\cos\theta]\leq|a|\cdot|\sin\theta|\cdot|y|$$

and $|y_1\pm y_2|\leq|y_1|+|y_2|$ for all scalars y_1, y_2, y_3, a, and θ.

Hence, for the interconnecting parts

$$g_1(x)=0.055[\cos(x_3+\theta_1)-\cos\theta_1] -0.1002[\cos(x_1-x_3-\theta_2)-\cos\theta_2]$$

$$g_2(x)=0.0599[\cos(x_1+\theta_3)-\cos\theta_3] -0.1002[\cos(x_1-x_3+\theta_4)-\cos\theta_4],$$

where $\theta_1=86°$, $\theta_2=81°$, $\theta_3=86°$, and $\theta_4=91°$, we get the estimates :

$$G_1(x)=\begin{bmatrix} 0 \\ g_1(x) \end{bmatrix}\leq\begin{bmatrix} 0 & 0 \\ 0.099 & 0 \end{bmatrix}\begin{bmatrix} |x_1| \\ |x_2| \end{bmatrix} +\begin{bmatrix} 0 & 0 \\ 0.154 & 0 \end{bmatrix}\begin{bmatrix} |x_3| \\ |x_4| \end{bmatrix}$$

$$\triangleq A_{11}[|x_1|\ |x_2|]^T+A_{12}[|x_3|\ |x_4|]^T$$

and

$$G_2(x)=\begin{bmatrix} 0 \\ g_2(x) \end{bmatrix}\leq\begin{bmatrix} 0 & 0 \\ 0.16 & 0 \end{bmatrix}\begin{bmatrix} |x_1| \\ |x_2| \end{bmatrix} +\begin{bmatrix} 0 & 0 \\ 0.1002 & 0 \end{bmatrix}\begin{bmatrix} |x_3| \\ |x_4| \end{bmatrix}$$

$$\triangleq A_{21}[|x_1|\ |x_2|]^T+A_{22}[|x_3|\ |x_4|]^T$$

So, $g_{ij}=\|A_{ij}\|_{ij}$ are computed from W_1^* and W_2^* as follows :

$g_{11}=0.099$ $\quad$ $g_{12}=0.099$

$g_{21}=0.16$ $\quad$ $g_{22}=0.1002.$

Now the test matrix D is formed as follows :

$$D=\begin{bmatrix} 2.6546 & -0.099 \\ -0.16 & 2.7055 \end{bmatrix}.$$

Since the successive principal minors of the test matrix D are all positive, by Theorem 1 of Section III, we can conclude that the equilibrium $x=0$ of the power system is asymptotically stable in some region.

V. Conclusion

Using the concept of stable matrices, computer-generated Lyapunov functions were used in the stability analysis of dynamical systems of the form $\dot{x}=F(x)$. Guided by recent developments in stability of large scale systems by Lyapunov functions, the decomposition-aggregation method has been applied to the asymptotic stability analysis of multimachine power systems.

The Lyapunov results given above can yield an estimate for the domain of attraction of the power system(which will be presented later). Also these Lyapunov results could be used to determine parameter sensitivity of power system transient stability, and they can also be used to quickly ascertain any potential emergency conditions based on an existing power system configuration and loading condition. Finally, since the above technique is constructive and graphical in nature, it has the potential of being developed into a powerful on-line tool for dynamic security assessment, and can be used for further analysis to find a similar method for large-scale power systems with non-uniform damping.

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