韓國數學教育學會誌<數學教育>
1983. 12. Vol. XXII, No. 1.

Journal of the Korea Society of Mathematical Education
Dec. 1983. Vol. XXII, No. 1.

On a Characterization of Regular Near-rings

Young In Kwon
Gyeongsang National University, Jinju, Korea

A *near-ring* is a set N together with two binary operations "+" and "·" such that (a) $(N, +)$ is a group (not necessarily abelian) (b) $(N, \cdot)$ is a semigroup (c) $\forall n_1, n_2, n_3 \in N$, $(n_1+n_2)\cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ (right distributive law). A subgroup M of a near-ring N with $M \cdot M \subseteq M$ is called a *subnear-ring* of N. A near-ring N is said to be *regular* in the sense of von Neumann [6] if, for every element $n \in N$, there exists an element $m \in N$ such that $nmn = n$. A subnear-ring M of a near-ring N is called a *quasiideal* of N if $NM \cap MN \subseteq M$. The basic reference for near-ring concept is [7]. The elementary properties of the regular near-ring have been given by J.C. Beidleman [1], Steve Ligh [5] and Henry E. Heatherly [3].

In this note we will give a characterization of regular near-ring.

Theorem. *If N is a regular near-ring with identity, the following conditions are equivalent:*

(a) *N is regular.*

(b) *If M is a subnear-ring of N and $MNM \subseteq M$, $MNM = M$.*

(c) *For every quasiideal M of N, $MNM = M$.*

Proof. (a)$\Rightarrow$(b).

Let M be a subnear-ring of N. If $MNM \subseteq M$ and if $m \in M$, then, by the regularity of N, $n = mnm$ for some $n \in N$. Hence $m \in MNM$ and so $M \subseteq MNM$. Therefore $MNM = M$.

(b)$\Rightarrow$(c).

Let M be a quasiideal of N. Then $MN \cap NM \subseteq M$. Since $MNM \subseteq MN \cap NM$, $MNM \subseteq M$ and so, $MNM = M$ by (b).

(c)$\Rightarrow$(a).

Let m be an element in N and let $M = (m)_R \cap (m)_L$ where $(m)_R$ and $(m)_L$ are the right ideal and left ideal of N generated by m respectively.

Then M is a quasiideal of N. Hence $m \in M = MNM \subseteq (m)_R N (m)_L$ by (c). Thus $m = mnm$ for some $n \in N$. Therefore N is regular.

Note. The concept of regular semigroup and quasiideal of a semigroup have been defined analogously by Green [2] and Lajos [4] respectively. An analogous characterization theorem for regular semigroups can be similarly proved.

References

1. J.C. Beidlemen, A note on regular near-rings, *Journal of the Indian Math. Soc.*, **33** (1969),

207-210.
2. J.A. Green, On the structure of semigroups, *Ann. of Math.*, 2 (54) (1951), 163-172.
3. H.E. Heatherly, Regular near-rings, *Journal of the Indian Math. Soc.*, 38 (1974), 345-354.
4. S. Lajos, Generalized ideals in semigroups, *Acta Sci. Math.*, 22 (1961), 217-222.
5. Steve Libh, On regular near-rings, *Math. Japonicae*, 15 (1970), 7-13.
6. J. von Neumann, On regular rings, *Proc. Nat. Actad. Sci. U.S.A.*, 22 (1936), 707-713.
7. G. Pilz, *Near-rings*, North-Holland, Amsterdam, 1977.