# Compensation of Image Motion Effect Through Augmented Transformation Equation

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## Abstract

Degradation of image caused by relative motion between the object and the imaging system (like a camera with its platform) is detrimental to precision photogrammetry. Principal modes of relative motion are identified. The discussion is, however, concentrated on the systematic motions, translatory and rotatory. Various analogical approaches of compensating for the image motion are cited. An analytical-computational approach is presented. This one considers the relationship of transformation bet ween the image and the object, known as the collinearity condition. The standard forms of collinearity condition equations are presented. Augmentation of these equations with regard to both translatory and rotatory motions are expounded. With ever increasing use of high speed computers (as well as analytical plotters in the realm of photogrammetry), this approach seems to be more costeffective and seems to yield better precision in the long run than other approaches that concentrate on analogical corrections to the image itself.

## Introdution

The necessity of dealing with motion-degraded images has been always with us. In the recording of images, degradation occurs when there is a relative motion between the image and the recording system. This relative motion can be due either to the movement of the camera or of the object. In any case, due to such a movement, the image of a spot would become a blur or a streak. The magnitude of such degradations could, however, be ignored in most aerial mapping cases. In recent years, in aerospace applications with demanding precisions or various sorts, requirements have been created to eliminate and/or compensate for such degradations. Similar, although somewhat different, applications related to any other type of high-speed photography would also cause similar mensural problems. The science and technology for dealing with the problem have advanced and numerous techniques have been devellopped during the last several decades. All such techniques have some merits, yet the cost-effectiveness of each procedure stays relative depending on the agency, country, time and other circumstances. The author presents one approach which is strictly computational and thus would be universal but would be usable only analytic-

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ally, either on-line at an analytical plotter or off-line at any high speed computer. With increasing capabilities of computer technology, its potential would increase under all circumstances.

Such image motions happen during the opening of the camera shutter due to three causes:

- —Velocity (i.e., translatory motion) of the camera along with its platform (e.g., aircraft or space vehicle) relative to the object;
- —Rotatory movement of the camera relative to the object; and
- —Vibration and other causes of random nature, due to varying air densities, atmospheric turbulence, terrain ruggedness, etc.

Such movements between the camera and the object are always relative. The effects, however, are similar.

Very subtle random motions generally cause no problem except where very high resolution is necessary. A fast shutter would minimize the effects and a stabilized mount may help damp out the random motions. On the other hand, image motion can never be uniform across the image plane of a camera. One can attempt to cancel an average systematic value of image motion but can not simply compensate for the higher order motions produced by ground's topographical changes or vibrations of the camera. The generation of a variant nature of image motion due to camera-Object distance in spite of linear camera motion is illustrated in Fig 1. This effect would be random, quite in keeping with the randomness of the surface photographed.

Some researchers (e.g., Granger, E. M., see pp. 161~165, NASA, 1968) have considered that in a dynamic system the recorded image is the result of a convolution of the image with a four-dimensional image motion function. Such images, they consider, can be restored on a four-dimensional correction space, two dimensions representing the sample points on the image while the other set of dimensions is the spatial frequency. This kind of approach would necessitate that the suitably weighted image be Fourier-transformed and multiplied by a correction filter function. Even the studies needed to understand such complexities in a particular imaging system would much reduce the cost-effectiveness of the compensation.

Several computational techniques for correcting blurred images have been developpped and are known to be used in military intelligence related areas. These are basically electronic restoration techniques based on analog detection and processing systems used in such applications.

Disciplines other than photogrammetry may tackle the problem quite differently. For example, despite image degradation resulting in average seeing motions on the order of 2 seconds of arc, an astronomer obtains directional precisions as great as 0.002 second of arc, at the cost of large bodies of data. A photogrammetrist can not usually afford the luxury of such observational redundancy. Analog or digital procedures of deconvolution based upon star images formed by the same telescope during the same viewing interval are known to have been developped. This would be impossible with object details of divers shapes and sizes. In the domaine of photogrammetry, we have been able to perform analogical image motion compensation to cancel an average value of the motion to leave residuals, which may often be beyond the acceptable limits.

Initial corrective approaches in photogrammetry have been with regard to the use of the camera. Camera magazines have been designed to provide film movement during exposure by using an Image Motion Compensation (IMC) device. Since the advent of jet aircrafts and the use of fine-grain, high-resolution and slow-speed photographic emulsions, the need for IMC became more profound. We have noted three basic methods of accomplishing IMC: (a) By moving the platen-film assembly; (b) By moving the lens cone (tilting around the nodal point); and (c) By using a focal plane shutter. Sometimes the third method is combined with the first or the second. Devices such as electro-mechanical V/h sensor cam correlators have been successfuly used. [Note: Here V is the ground speed of the aircraft which carries the camera and h is the flying height above a certain datum in the terrain].

These IMC device related approaches essentially consider the corrections in two parts: (1) Those due to linear motions, and (2) Those due to rotational motions (see for example, Kawachi, 1965). Such approaches, from the computational point of view, are applicable in refining the photo-coordinate data. However, the formulas developped for this purpose in the 1960's do not interconnect the image and the object, point by point, in a direct sense. This possibility exists in using the collinearity condition equations. Some form of augmentation of the collinearity equations, therefore, could be computationally feasible. This could, however, be used on-line into developping an analogical (mechanical or optical) compensation device. On the other hand, by using a computer with sufficient memory and appropriate software, this would become extremely cost-effective and efficient because this would not require any additional equipment and would be universal in its application. The rationale of this approach is described in the following.

## The Collinearity Condition

As illustrated in Fig 2, the locations of a point on a photograph (p) and the corresponding point on ground or object (P) with regard to the perspective center (nodal point) in the camera lens are represented by the vectors $\vec{r}$ and $\vec{R}$ respectively. They can be described in two separate but mutually associated three-dimensional coordinate systems, $x$-$y$-$z$ and $X$-$Y$-$Z$ of the photograph and the ground, respectively. In the photographic coordinate system, considering the perspective center as origin, the photograph being a plane surface, $z=-f$ can be considered as a constant. Considering that a light ray travels along a straight line, one can establish a relationship of transformation:

$$\vec{r}=k\cdot M\cdot\vec{R} \qquad (1)$$

where $k$ is a scalar multiple (scale factor);
and $M$ is the rotation martix defining one system with respect to the other.

The location vectors $\vec{r}$ and $\vec{R}$ are as follows:

$$\vec{r}=\begin{bmatrix} x_p-x_0 \\ y_p-y_0 \\ O-f \end{bmatrix} \text{ and } \vec{R}=\begin{bmatrix} X_p-X_0 \\ Y_p-Y_0 \\ Z_p-Z_0 \end{bmatrix}$$

where $x_p, y_p$ are the photo-coordinates of point p;
$f$ is the focal length (calibrated) of the camera;
$x_0, y_0$ are the photo-coordinates of the principal point;
$X_p, Y_p, Z_p$ are the ground coordinates of point P;
and $X_0, Y_0, Z_0$ are the ground coordinates of the perspective center.

Furthermore, $M$ is a 3×3 orthogonal matrix and is made up of three independent rotations around the three mutually perpendicular axes ($\omega$ around $X$, $\phi$ around $Y$ and $\kappa$ around $Z$, see Fig 2). It is usually expressed in one of the following forms:

$$M=M_\kappa M_\phi M_\omega=\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \cos Xx & \cos Yx & \cos Zx \\ \cos Xy & \cos Yy & \cos Zy \\ \cos Xz & \cos Yz & \cos Zz \end{bmatrix} \quad (2)$$

where $M_1, M_2$ and $M_3$ are the row matrices expressing the $M$;

$M_\kappa, M_\phi$ and $M_\omega$ are the rotation matrices for $\kappa, \phi$ and $\omega$ rotations, respectively;

$m_{11}, m_{12}$, etc. are the elements of the $M$ matrix;

and $\cos Xx, \cos Yx$, etc. are the cosines of the space angles between the respective $x, y, z$ axes of the system to be transformed into and the $X, Y, Z$ axes of the system to be transformed.

Various authors have selected different directions and sequences for such rotations. Nonetheless, the numerical values of the nine elements of $M$ are identical, regardless of the choice of the angles involved therein. Because of the peculiar nature of the rotation matrix $M$, its inverse and its transpose are identical.

The matrix multiplication indicated in Eq 1 may be carried out by the components along rows and columns. If the first and second rows are divided by the third row and the resulting expressions are multiplied by $-f$, one obtains the "collinearity condition" equations with respect to the two photocoordinates:

$$\left. \begin{aligned} x=(x_p-x_0) &= -f\frac{(X_p-X_0)m_{11}+(Y_p-Y_0)m_{12}+(Z_p-Z_0)m_{13}}{(X_p-X_0)m_{31}+(Y_p-Y_0)m_{32}+(Z_p-Z_0)m_{33}} \\ y=(y_p-y_0) &= -f\frac{(X_p-X_0)m_{21}+(Y_p-Y_0)m_{22}+(Z_p-Z_0)m_{23}}{(X_p-X_0)m_{31}+(Y_p-Y_0)m_{32}+(Z_p-Z_0)m_{33}} \end{aligned} \right| \quad (3)$$

These two equations imply that the two vectors, $\vec{r}$ and $\vec{R}$ are collinear i.e., the object point ($P$), the perspective center (or, the exposure station, 0) and the image point ($p$) lie on the same straight line.

Let us consider that the location vector $\vec{R}$ (OP in Fig 2) subtends angles $\alpha, \beta$ and $\gamma$ with the $X$, $Y$ and $Z$ axes, respectively. These define the direction cosines of this vector in the $X$-$Y$-$Z$ system. In Eq3, the quantities $(X_p-X_0)$, $(Y_p-Y_0)$ and $(Z_p-Z_0)$ are proportional to the direction cosines of the vector. Therefore, with proper substitutions one may write the collinearity equations also in the following forms:

$$(x_p-x_0) = -f\frac{m_{11}\ \cos\alpha+m_{12}\ \cos\beta+m_{13}\ \cos\gamma}{m_{31}\ \cos\alpha+m_{32}\ \cos\beta+m_{33}\ \cos\gamma}$$

$$(y_p-y_0) = -f\frac{m_{21}\ \cos\alpha+m_{22}\ \cos\beta+m_{23}\ \cos\gamma}{m_{31}\ \cos\alpha+m_{32}\ \cos\beta+m_{33}\ \cos\gamma} \quad (3a)$$

The condition equations in this form are useful in certain problems where scale of the object is of no concern, e.g., in astronomical applications.

## Augmentation of Collinearity Equations

The collinearity condition equations (Eqs 3 or 3a) may be written, for the sake of brevity, in the following forms:

$$\left. \begin{aligned} (x_p-x_0) &= -f\{M_1(\underline{X}_p-\underline{X}_0)/M_3(\underline{X}_p-\underline{X}_0)\} \\ (y_p-y_0) &= -f\{M_2(\underline{X}_p-\underline{X}_0)/M_3(\underline{X}_p-\underline{X}_0)\} \end{aligned} \right\} \quad (4)$$

where $f, x_p, x_0, y_p, y_0$ are as defined before:

$M_i$ is the ith row of the general rotation matrix $M$:

$\underline{X}_p$ is a vector of object space coordinates, i.e., $\underline{X}_p=[X\ Y\ Z]_p^T$

and $\underline{X}_0$ is a vector of coordinates representing the perspective center (camera station), i.e., $\underline{X}_0=[X\ Y\ Z]_0^T$

In relative terms, the effect on the image of the movement of an object point with respect to a stationary camera would be the same if the object remains stationary while the camera moves. The movement may be consdered to be of two kinds, translatory and rotatory. The augmentation of the collinearity equations can be performed better in two parts, along linear and angular motions. The following are presented without considering any of one type of motion over the other. However, they may be commutative in some cases and consecutive others. They may, therefore, be applied in consideration of the specifics of the situation.

### A. Case of translatory motion

With regard to Eqs 4, considering that the camera moves uniformly along a straight path, let us suppose that one end of the resulting blur is exposed at $\underline{X}_0=\underline{X}'_0$ (yielding photo coordinates, $x'_p$ and $y'_p$) while the other end is exposed at $\underline{X}_0=\underline{X}''_0$ (yielding photo coordinates, $x''_p$ and $y''_p$), all for the same object point represented by $\underline{X}_p$. Assuming that the camera moves with a velocity $V$ in three-dimensional space, $\underline{X}''_0-\underline{X}'_0=V\cdot\Delta t$ where $\Delta t$ is the exposure time responsible for creating the blur. Therefore, expressions for $x''_p$ and $y''_p$ can also be written (using Eqs 4) alternatively,

$$(x''_p-x_0)=-f\{M_1(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)/M_3(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)\}$$
$$(y''_p-y_0)=-f\{M_2(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)/M_3(\underline{X}_p-\underline{X}'_0-V\cdot\Delta)\} \quad (5)$$

The components of the image displacement causing the blur are as below:

$$\Delta x=x''_p-x'_p=-f\{M_1(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)/M_3(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)\}+f\{M_1(\underline{X}_p-\underline{X}'_0)/M_3(\underline{X}_p-\underline{X}'_0)\}$$
$$\Delta y=y''_p-y'_p=-f\{M_2(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)/M_3(\underline{X}_p-\underline{X}'_0-V\cdot\Delta t)\}+f\{M_3(\underline{X}_p-\underline{X}'_0)/M_3(\underline{X}_p-\underline{X}'_0)\} \quad (6)$$

If desired, the image velocities along the two directions on the photo ($x$ and $y$) due to the translatory motion of the perspective center can be obtained from the above:

$$v_x=\Delta x/\Delta t \text{ and } vy=\Delta y/\Delta t \quad (7)$$

### B. Case of rotatory motion

Considering, as before, that $\Delta t$ is the exposure epoch during which the camera undergoes a rotatory motion to cause the blur and that the total rotation contains three components, $\omega, \phi$ and $\kappa$, one can express the augmented form of the rotation matrix $(M)$:

$$N=M_{(\kappa+\dot{\kappa}\cdot\Delta t)}\ M_{(\phi+\dot{\phi}\cdot\Delta t)}\ M_{(\omega+\dot{\omega}\cdot\Delta t)} \quad (8)$$

This, in view of Eq 2 and by virtue of the 'double angle formulas', gives:

$$N=\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}=M_\kappa M_{\dot{\kappa}\cdot\Delta t}M_\phi M_{\dot{\phi}\cdot\Delta t}M_\omega M_{\dot{\phi}\cdot\Delta t} \quad (9)$$

where $\dot{\kappa}, \dot{\phi}$ and $\dot{\omega}$ are the respective angular velocities (radians per second).

The general expressions for image coordinates of a moving system respect to a defined epoch are, then (see also Eps 5):

$$(x_p-x_0)=-f\{N_1(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)/N_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)\}$$
$$(y_p-y_0)=-f\{N_2(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)/N_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)\} \quad (10)$$

In a way similar to the case of translation (Eqs 6 obtained from Eqs 5), one can obtain expressios for $\Delta x$ and $\Delta y$, by differentiating which with respect to $\Delta t$, arrives at the following expressions for the image velocities from rotational motions:

$$v'_x=\dot{x}_p=-f\{[\mathring{N}_1(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)-N_1V][N_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)]-[\mathring{N}_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t-N_3V][N_1(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)]\}\cdot[N_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)]^{-2} \quad (11a)$$

and

$$v'_y=\dot{y}_p=-f\}[\mathring{N}_2(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)-N_2V][N_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)]-[\mathring{N}_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)-N_3V][N_2(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)]\}\cdot[N_3(\underline{X}_p-\underline{X}_0-V\cdot\Delta t)]^{-2} \quad (11b)$$

where $\dot{N}_1 = dN_1/d\Delta t$ and $\dot{N}_2, \dot{N}_3$ are obtained similarly.

Equations 11 are without any restricition as to the initial orientation (directional attitude) of the camera, the terrain form, the time interval or the magnitude of rates of movements. The only restriction remains in the assumption that rates are constant. By way of expanding the matrices $N$ and $\mathring{N}$ and by using specific values, it can be demonstrated that Eqs 11 would, by degeneration and approximation, yield the values and expressions as obtained by Kawachi (1965).

It is evident that this approach tends to correct the direction of the optical ray (i.e., the rotation matrix $M$ with all its components), whereas all other IMC approaches, analogical or computational, are meant to correct the image directly (i.e., the $x$ and $y$ photo coordinates). This aspect makes it adaptable to various complexites. Consider the problem faced sometimes in photogrammetry, when the object moves in one direction (say$x$) at a known constant speed while the camera having a focal-plane shutter moves at a known but non-linear speed in another direction (say $y$). In such a case, each of the collinearity equations for $x$ and $y$ can be appropriately modified to do the needful.

With worldwide availability of high-speed computers and their increasing use, such modifications/augmentations of transformation equations with regard to specific points on the photograph can be performed with success. This has been successfully applied to solve problems related to moving camera platform as in an artificial satellite. There seems to be no reason why this could not be used in problems related to high speed photography of any kind.

## References

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NASA (1969), *Evaluation of Motion-degraded Images*, Proceedings of a seminar, December 3~5, 1968 at Cambridge, Mass., USA. NASA publication No. SP-193, Office of Technology Utilization, NASA, Washington, D.C., 192pp.

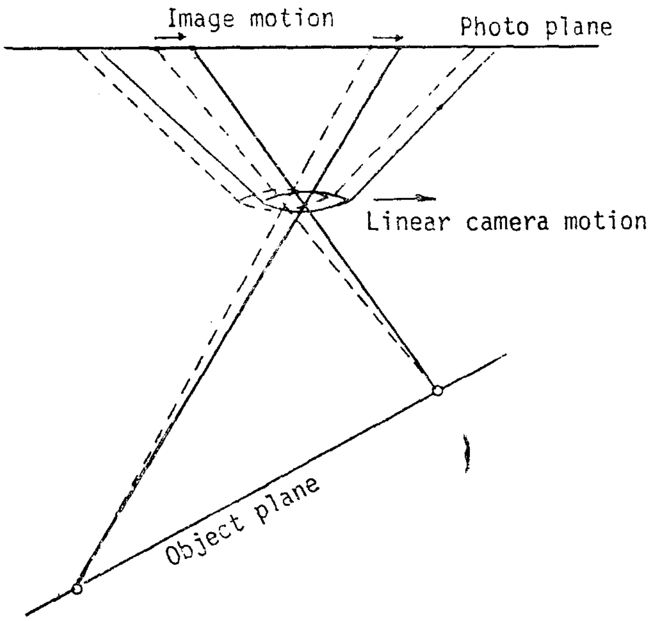


Figure 1:
Generation of variant image motion (in spite of linear camera motion)

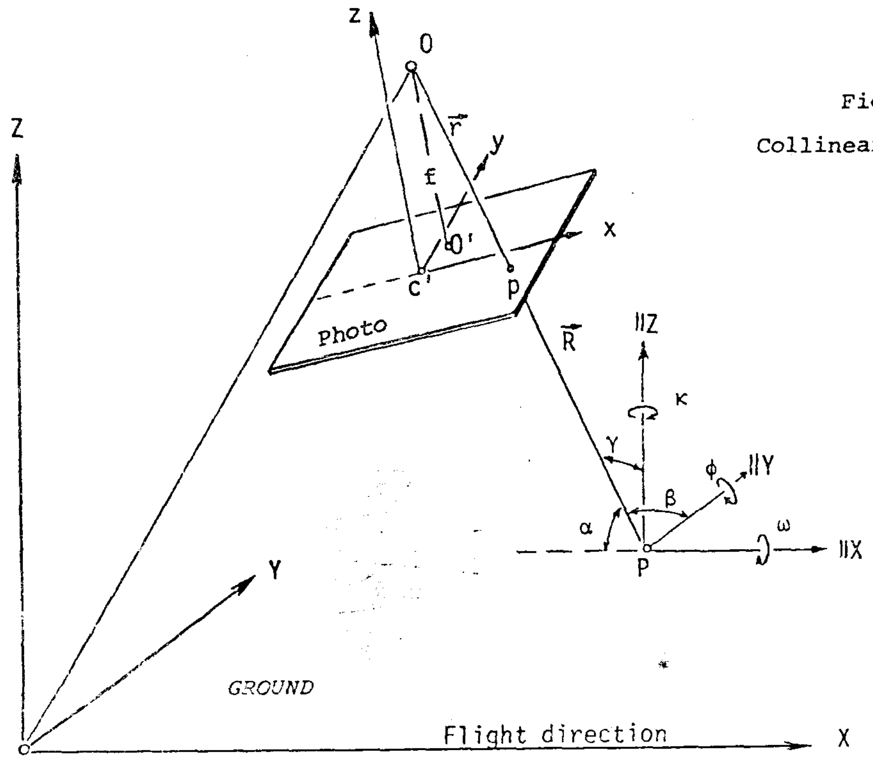


Figure 2:
Collinearity situation