유한 저어널 베어링에서 점탄성의 영향

The Elastico-Viscous Effect in Finite Journal Bearings

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요 약

점탄성유체 유동에 대하여 일반적으로 U. C맥스웰 모델 또는 올드로이드 B모델이 사용되지만 이러한 모델들은-매우 큰 전단율 영역인 윤활문제에서 믿기 힘든-수직응력의 크기가 전단율의 제곱에 비례함을 나타내므로, 본 연구에서는 수직응력 계수들(ψ, ψ_2)이 가정될 수 있는 Criminale-Ericksen-Filbey 모델이 사용되었다. 2차 수직응력계수는 다른 문제들에서와 같이 무시되었으며 Weissenberg수가 포함된 특수레이놀즈식이 유도되었다.

이 모델의 속도분포는-2차원 약한점탄성 유체에 대하여 증명된 바와 같이-뉴우톤 유체와 같이 가정되었다. 유도된 특수레이놀즈 식은 Weissenberg수를 1까지 계산되었으며 그 결과 점탄성유체가 유한저어널 베어링에서 유리한 것으로 나타났지만 그 차가 미소하여 일반베어링에서 점탄성 윤활유의 영향이 무시됨을 보였다.

Nomenclature

D	:	diameter of journal
$e_{ik}^{(N)}$	:	Nth rate of strain tensor
h	:	lubricant film thickness
L	:	bearing length
p, P*	:	dimensional and non-dimensional isotropic pressure
p_{ik}	:	stress tensor
p'_{ik}	:	extra stress tensor
U	:	journal rotational velocity
u,v,w	:	rotational, axial and film thickness directional velocity
x,y,z	:	rotational, axial and film thickness directional coordinate
δ	:	radial clearance
ε	:	eccentricity ratio
η_0	:	viscosity
θ^*	:	angular coordinate
λ_1, λ_2	:	relaxation time, retardation time
ψ_1, ψ_2	:	the first and the second normal stress coefficient
ω	:	angular speed of journal
ω_{ik}	:	vorticity tensor
$\frac{\partial}{\partial t}$	:	absolute derivative
$\frac{D}{Dt}$	:	classical material derviative
$\frac{\&}{\&t}$	:	corotational (or Jaumann) derivative
$\frac{g}{gt}$	:	codeformational (or convected) derivative

1. Introduction

The use of additives as modifiers of mineral lubricant performance, the modified oils exhibit non-Newtonian behaviour which does not exist in normal mineral oils. The

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main physical properties, which are different from the Newtonian counterpart are shear thinning phenomenon in steady shear, normal stress effect (elastic effect) in steady shear, etc.. This paper is concerned with the elastic effect. The Weissenberg rod climbing and the die swell phenomena are the well known examples of the elastic effect.

The elastico-viscous behaviour of the non-Newtonian liquid was extensively studied since the late 1940's and there are a lot of rheological models. But, even within the experimental range (maximum shear rate is about $1000s^{-1}$) there are still some deviations between the analytical results of various models and the experiments. Up to the presnt time, most of theoretical papers which deal with rheology use the generalized Maxwell model, the Oldroyd B model, or sometimes the Criminale-Ericksen-Filbey model or others derived from the kinematic theory of polymer solution.

The analytical studies of non-Newtonian lubrication started in the early 1960's. Tanner(1) solved the lubrication of infinite rollers by the Maxwell model, assuming small extra normal stresses. The result was negative for non-Newtonian liquid. Davies(2) solved the infinite journal bearing by the second order-, third order fluid model and by the 4 constants Oldroyd model. The result was positive, but for the Oldroyd model he limited his analysis to small eccentricity. Normally, bearings run for stable operation at an eccentricity ratio from 0.7 to 0.95 (Barwell(3)). Harnoy(4) solved the finite journal bearing by the Oldroyd B model, considering the stress relaxation effect without normal stresses. The result was positive for non-Newtonian liquid.

The reason why many researchers neglected the normal stress effect was that, because of bearing geometry, the Weissenberg number should be very large in order to affect the load capacity, otherwise the normal stress effect would be neglible. Our idea is to choose a proper model which seems to be more realistic than the conventional Maxwell or Oldroyd model, to simplify the model by approximations normally used in journal bearing calculation, and to show how much the Weissenberg number will influence finite journal bearing performances.

2. Experimental and Theoretical Background

Experiments show that the variation of the normal stress with the shear rate is a very complex function, and up to now, experiments only deal with small shear rate. Therefore it is very hard to say what the value will be in the journal bearing case, and it is still in controversy. Typical normal stress values within experimental range are represented in figures 1 and 2 (Boger (5)). Separan (Polyacrylamide) solutions in corn syrup (which is a completely Newtonian liquid) are highly elastic, with a Weissenberg number about 5

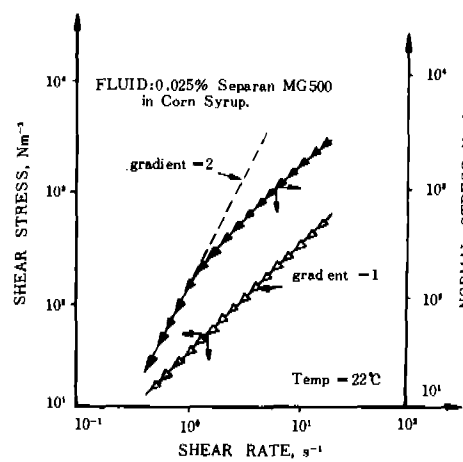

Fig. 1. Shear and first normal stress difference measurements for a 0.05 percent Separan MG500 in corn syrup solution.

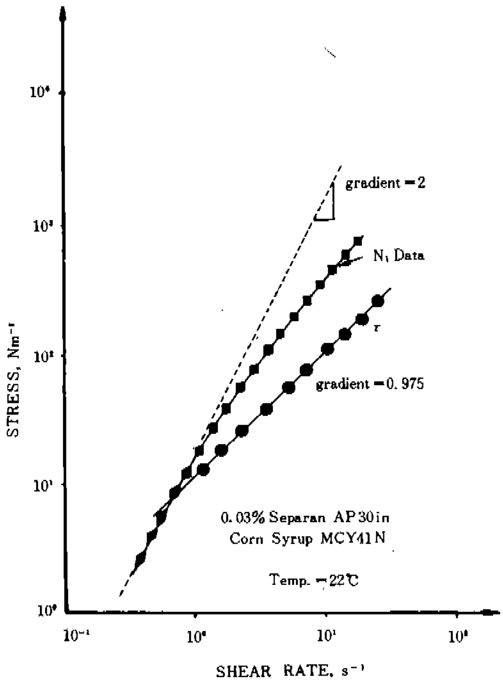

Fig. 2. Shear and first normal stress difference measurements for 0.03 percent Separan AP30 in corn syrup solution.

to 10 at 10 s^{-1} shear rate. For silicone oils the number is very small, and the normally used modified oils (e.g. engine oils) are even less elastic (Boger (5)).

The gradient in the figures of stresses is normally decreasing with increasing shear rate, so at high shear rate the Weissenberg number may not be large. Therefore it may be beneficial to analyse the lubrication problem with small Weissenberg number.

When we try to solve non-Newtonian flow problems, first thing to do is to select a proper rheological model. The problem is if we choose a simple model then the flow pattern is quite different from the real flow. If we choose a more complex model then the flow pattern will be better, but it leads to much more computational problems, which make the model difficult or even impossible to calculate. So, up to now, most famous model for theoreticians considering elastic effect is the Maxwell model, or Oldroyd B model. Actually the Maxwell model (eq. 1 and 2) is the simplest one with two constants. The Oldroyd B model has three constants (eq. 1 and 3).

$$P_{ik} = -p\delta_{ik} + p'_{ik} \quad \cdots\cdots (1)$$

$$p'_{ik} + \lambda_1 \frac{g p'^{ik}}{g t} = 2\eta_o e^{(1)ik} \quad \cdots\cdots (2)$$

$$p'^{ik} + \frac{g p'^{ik}}{g t}\lambda_1 = 2\eta_o \left(e^{(1)ik} + \lambda_2 \frac{g e^{(1)ik}}{g t}\right) \cdots (3)$$

For engineers who want to solve polymer processing problems, the "CEF" (eq. 1 and 4) or others based on the kinetic theory of dilute polymer solution are well known.

$$p'_{ik} = 2\left(\eta_o e_{ik}^{(1)} - 1/2\psi \frac{\psi e_{ik}^{(1)}}{1\psi t} + (\psi_1 + 2\psi_2) e_{ij}^{(1)} e_{jk}^{(1)}\right) \quad \cdots\cdots (4)$$

The lubrication problem can be distinguished from "normal" flow problems by two points:

a) extremely high rate of shear: up to 10^6 s^{-1}, while most of rheological problems are in the range of 10-1000 s^{-1}.
b) bearing gap geometry is very thin and long: the ratio of thickness and length is about 0.001, while it is about 0.1-1 for the other cases.

If we consider steady shear flow, the Maxwell and the Oldroyd model show that the normal stress is proportional to the square of the shear rate, and the shear stress is proportional to the rate of shear (for example, a Weissenberg number 1 at 10^3 s^{-1} will be 10^3 at 10^6 s^{-1}, which is too high to be realistic). Even though the exact value of the Weissenberg number at extremely high rate of shear is not known, the number is rather small

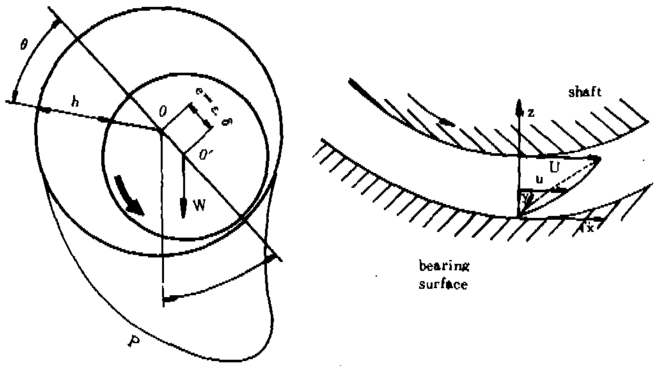

Fig. 3. Bearing geometry

within the experimental range. Therefore we think that the CEF model, in which we can assume ψ_1 and ψ_2 is more suitable for lubrication problems.

The journal bearing geometry is outlined in figure 3, and the following assumptions are normally used in the bearing analysis (Cameron (6) .

$$w=0,\quad \frac{\partial u}{\partial z} \gg \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y};\ \frac{\partial v}{\partial z} \gg \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \quad \cdots\cdots\cdots\cdots (5)$$

In order to simplify the procedure, we can assume the flow kinematics of the CEF and Newtonian fluids to be identical, which is proved for two dimensional slightly elastic flows by Tanner (7). This assumption is also used by Tadmor (8). The rate of strain tensor e_{ik} and the vorticity tensor ω_{ik} are

$$e_{ik}=\frac{1}{2}\begin{pmatrix} 0 & 0 & \frac{\partial u}{\partial z} \\ 0 & 0 & \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & 0 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

$$\omega_{ik}=\frac{1}{2}\begin{pmatrix} 0 & 0 & -\frac{\partial u}{\partial z} \\ 0 & 0 & -\frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & 0 \end{pmatrix} \quad \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

Substituting the foregoing experessions into the CEF equation 4 gives

$$\begin{pmatrix} P'_{xx} & P'_{xy} & P'_{xz} \\ P'_{yx} & P'_{yy} & P'_{yz} \\ P'_{zx} & P'_{zy} & P'_{zz} \end{pmatrix} = \eta_o \begin{pmatrix} 0 & 0 & \frac{\partial u}{\partial z} \\ 0 & 0 & \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & 0 \end{pmatrix}$$

$$+\left(\frac{1}{2}\psi_1+\psi_2\right)\begin{vmatrix} \left(\frac{\partial u}{\partial z}\right)^2 & -\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right) & 0 \\ \left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right) & \left(\frac{\partial v}{\partial z}\right)^2 & 0 \\ 0 & 0 & \left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2 \end{vmatrix} - \frac{1}{2}\psi_1$$

$$\begin{vmatrix} -\left(\frac{\partial u}{\partial z}\right)^2 & -\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right) & 0 \\ -\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right) & -\left(\frac{\partial v}{\partial z}\right)^2 & 0 \\ 0 & 0 & \left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2 \end{vmatrix}. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

The second normal stress coefficient is negative and probably about 1/10 to 1/4 of the magnitude of ψ_1 (Bird(9)). Therefore we neglect this coefficient ψ_2, and the stress components for the assumed flow kinematics and for $e_{ik}^{(1)}$ will be:

$$\left.\begin{array}{ll} p'_{xx}=\psi_1\left(\frac{\partial u}{\partial z}\right)^2 & p'_{xy}=\psi_1\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right) \\ p'_{yy}=\psi_1\left(\frac{\partial v}{\partial z}\right)^2 & p'_{yz}=\eta_0\frac{\partial v}{\partial z} \\ p'_{zz}=0 & p'_{zx}=\eta_0\frac{\partial u}{\partial z} \end{array}\right\} \quad (9)$$

The equations of motion without inertia terms are

$$\frac{\partial\left(\psi_1\left(\frac{\partial u}{\partial z}\right)^2\right)}{\partial x}+\frac{\partial\left[\psi_1\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right)\right]}{\partial y}+\frac{\partial\left(\eta_0\frac{\partial u}{\partial z}\right)}{\partial z}=\frac{\partial p}{\partial y} \quad \cdots\cdots (10a)$$

$$\frac{\partial\left[\psi_1\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right)\right]}{\partial x}+\frac{\partial\left[\psi_1\left(\frac{\partial v}{\partial z}\right)^2\right]}{\partial y}+\frac{\partial\left(\eta_0\frac{\partial v}{\partial z}\right)}{\partial z}=\frac{\partial p}{\partial y} \quad \cdots\cdots (10b)$$

The rate of shear is (Cameron(6))

$$\left.\begin{array}{l} \frac{\partial u}{\partial z}=\frac{1}{2\eta_0}\frac{\partial p}{\partial x}(2z-h)+\frac{U}{h} \\ \frac{\partial v}{\partial z}=\frac{1}{2\eta_0}\frac{\partial p}{\partial y}(2z-h) \end{array}\right\} \quad \cdots\cdots (11)$$

and the derivatives of the shear rate:

$$\left.\begin{array}{l} \frac{\partial^2 u}{\partial x\partial z}=\frac{1}{2\eta_0}\frac{\partial^2 p}{\partial x^2}(2z-h)+\frac{\partial p}{\partial x}\left(-\frac{\partial h}{\partial x}\right)+U\frac{-\frac{\partial h}{\partial x}}{h^2} \\ \frac{\partial^2 u}{\partial y\partial z}=\frac{1}{2\eta_0}\frac{\partial^2 p}{\partial x\partial y}(2z-h) \\ \frac{\partial^2 u}{\partial z^2}=\frac{1}{\eta_0}\frac{\partial p}{\partial x} \\ \frac{\partial^2 v}{\partial x\partial z}=\frac{1}{2\eta_0}\frac{\partial^2 p}{\partial x\partial y}(2z-h) \\ \frac{\partial^2 v}{\partial y\partial z}=\frac{1}{2\eta_0}\frac{\partial^2 p}{\partial y^2}(2z-h) \\ \frac{\partial^2 v}{\partial z^2}=\frac{1}{\eta_0}\frac{\partial p}{\partial y} \end{array}\right\} \quad \cdots\cdots (12)$$

In equatation 11 and 12 terms with (2z-h) change their sign through the film, so we can neglect these components compared to the other expressions. Therefore equation 10 becomes

$$\frac{\partial\left[\psi_1\left(\frac{\partial u}{\partial z}\right)^2\right]}{\partial x}+\frac{\partial\left(\eta_0\frac{\partial u}{\partial z}\right)}{\partial z}=\frac{\partial p}{\partial x} \quad \cdots (13a)$$

$$\frac{\partial\left(\eta_0\frac{\partial v}{\partial z}\right)}{\partial z}=\frac{\partial p}{\partial y} \quad \cdots\cdots (13b)$$

If we choose Weissenberg number as N, assumming the constant number througout the bearing surface, the equation 13 a becomes

$$N\frac{\partial}{\partial x}\left(\eta_0\frac{\partial u}{\partial z}\right)+\frac{\partial}{\partial z}\left(\eta_0\frac{\partial u}{\partial z}\right)=\frac{\partial p}{\partial x} \quad \cdots\cdots (14)$$

Using the following parameters to non-dimensionalized ratio $\left(\frac{\partial^2 u}{\partial x\partial z}\right)/\left(\frac{\partial^2 u}{\partial z^2}\right)$ it becomes as equation 17

$$\left.\begin{array}{l} h=\delta(1+\varepsilon\cos\theta^*) \\ x=r\theta^* \\ p=p^*\eta\,\omega\left(\frac{r}{\delta}\right)^2 \end{array}\right\} \quad \cdots\cdots (15)$$

$$\frac{\frac{\partial^2 u}{\partial x\partial z}}{\frac{\partial^2 u}{\partial z^2}}=\frac{\delta\varepsilon\sin\theta^*\left(\frac{1}{2}\frac{\partial p^*}{\partial\theta^*}+\frac{1}{(1+\varepsilon\cos\theta^*)}\right)}{\frac{\partial p^*}{\partial\theta^*}}$$

$$=B(\theta^*,Y) \quad \cdots\cdots (16)$$

$$\frac{\partial^2 u}{\partial z^2}=\frac{1}{\eta_0(1+NB)}\frac{\partial p}{\partial x} \quad \cdots\cdots (17)$$

Equation 17 is entirely the same as in the Newtonian case except η_0 which is substituted by η_0 (1+NB). Substitute equation 13 b and 17 into the continuity equation, we get equation 18. For the Newtonian case N is O, which leads to the well known Reynolds equation. We propose equation 18 as a modified Reynolds equation dealing with the elastic effect for non-Newtonian oils.

$$\frac{\partial}{\partial x}\left(\frac{h^3}{(1+NB)}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)$$

$$=6\eta_0 U\frac{\partial h}{\partial x} \quad \cdots\cdots (18)$$

Let us rearrange equation 18 by using equation 15 and y = ry*.

$$\frac{\partial^2 p^*}{\partial\theta^{*2}}\frac{1}{1+NB}+\frac{\partial^2 p^*}{\partial y^{*2}}-\left\{\frac{N}{(1+NB)^2}\frac{\partial B}{\partial\theta^*}\right.$$

$$\left.+\frac{3\varepsilon\sin\theta^*}{(1+NB)(1+\varepsilon\cos\theta^*)}\right\}\frac{\partial p^*}{\partial\theta^*}$$

$$+\frac{6\varepsilon\sin\theta^*}{(1+\varepsilon\cos\theta^*)}=0 \quad \cdots\cdots (19)$$

The corresponding finite difference equation is:

$$p(i,j)\left\{\frac{1}{(1+NB(i,j))}\frac{2}{G^2}+\frac{2}{Y^2}\right\}$$

$$=p(i+1,j)\left\{\frac{1}{(1+NB(i,j))G^2}-C1(i,j)\frac{1}{2G}\right\}$$

$$+p(i-1,j)\left\{\frac{1}{(1+NB(i,j))G^2}+C1(i,j)\frac{1}{2G}\right\}$$

$$+(p(i,j+1)+p(i,j-1))\frac{1}{Y^2}+\frac{6\varepsilon\sin\theta^*}{(1+\varepsilon\cos\theta^*)^3}$$

$$\cdots\cdots (20)$$

where

$$C1(i,j)=\frac{1}{1+NB(i,j)}\left\{\frac{N}{1+NB(i,j)}\right.$$

$$\left.\frac{B(i+1,j)-B(i-1,j)}{2G}+\frac{3\varepsilon\sin\theta^*}{1+\varepsilon\cos\theta^*}\right\}$$

3. Computation procedure

The elastico-viscous effect paramenter B(θ^*,Y) becomes large for small p* values (e.g. small L/D and eccentricity), and the eq. 20 becomes unstable. This numerical problem can be solved by changing the convective terms to forward or backward differences. The computation is done in successive steps:

a. Pressure field:

1. calculate the Netwonian pressure field by finite difference relaxation method, for a given bearing geometry
2. find the value B(θ^*,Y) of equation 16
3. calculate the new pressure field by equation 20
4. calculate a non-Newtonian B(θ^*,Y) value with new pressure field
5. compare step 4 and 2, if the difference is larger than 0.1%, send 4 values to step 3 for recalculating the pressure field. Otherwise call performance subroutine.

b. Bearing parameters:

Calculate total load, friction, attitute angle and the coefficient of fricition by using performance subroutine.

4. Results and Discussion

Figure 4 and 5 show the pressure distribution for ε = 0.8 and 0.2 with Weissemberg No. 1. The values are little higher than the Newtonian ones, but the differences are extremely small, so it is impossible to distinguish the curves in a diagram. The pressure difference ($(P_{non\text{-}Newtonian}-P_{Newtonian})/P_{Newtonian}$) are nearly zero for = 0.8, and about 1% for L/D = 0.1, = 0.2 and 0.2% for L/D = 0.25, = 0.2.

The difference is larger for small L/D and eccentricity, but this range is out of practical interest. Even the Weissenberg No. 1 is not so small, this paper shows that the effect of non-Newtonian lubricant (by consider-

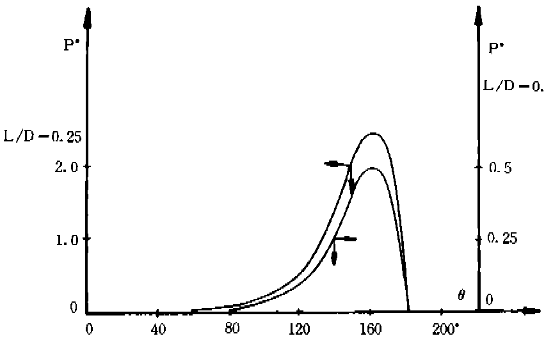

Fig. 4. Pressure distribution vs. angle (in the mid plane)

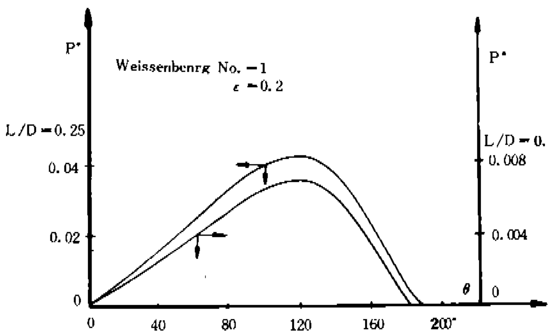

Fig. 5. Pressure distribution vs. angle (in the mid plane)

ing elastico-viscous effect) is negligible for finite journal bearing performances.

Acknowledgements

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