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Theory and Application of Systems Analysis Techniques to the Optimal Management and Operation of a Reservoir System(Ⅱ)

——저수지 최적 운영을 위한 시스템 해석 기법의 적용——

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Ⅳ. Nonlinear Programming

Nonlinear programming has not enjoyed the popularity that LP and DP have in water resource systems analysis. This was partially due to the fact that the optimization process is usually slow and takes up large amounts of computer storage and time when compared with other methods. The mathematics involved in the nonlinear models is much more complicated that in the linear case and the nonlinear programming unlike DP cannot easily accommodate the stochastic nature of inflows to the system.

However, it is a formulation of the most general mathematical programming and gives the foundations of analysis to the other methods. Nonlinear programming can effectively handle a nonseparable objective function and nonlinear constraints which many programming techniques cannot. Futhermore, it usually includes quadratic programming, geometric programming and separable programs as a special case which can be used iteratively as a master program or as a subprogram in large-scale system problems. Search techniques have also been used in conjunction with simulation in order to evaluate the performance functions of alternative systems (Maass et al., 1962).

An essential preliminary to systems planning and control is a clear statement of objectives. The Water Resource Council's Principles and Standard (1973) established two equally important objective for federal water resources projects: national economic development and environmental quality. Later, it was revised in 1980 to include regional economic development and other social effects. For a system of reservoirs, the number of constraints are large because they deal with similar subsystems, repeated in time or location. Approaches to these large mathematical programs have been aggregation, decomposition and partitioning methods. Therefore, nonlinear programming will gain its practical importance in water resources systems with the developments of computer technology and effctive algorithms for large-scale multi-objective systms optimization (Cohon and Marks, 1975; Haimes, 1977).

A general nonlinear programming problem (of a reservoir system) can be stated as:

$$\min_{x \in X} f(x)$$
$$\text{s.t. } g(x) \geq 0$$

in which x is an n-dimensional vector of decision variables, and $f(x)$ and $g(x)$ are a real-valued and m vector-valued given functions, respectively. The contstraint set X is usually a subset of n-dimensional real space, such as simple upper or lower bounds or nonnegativity conditions. If the objective function $f(x)$ and the constraints $g(x)$ are additively separable, the following separable program will result:

$$\min_{x \in X} \sum_j f_j(x_j)$$
$$\text{s.t. } \sum_j g_{ij}(x_j) \geq 0, \quad i=1, \cdots\cdots, m$$

This special program structure might arise from decomposition in time or space of a reservoir management problem. The separable program becomes the linear programming with the additional assumption of linearity in objectives $f_j\ (x_j)$ and constraints $g_{ij}\ (x_j)$. It becomes a dynamic programming if the constraint

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inequalities are replaced with the recurrence equations of the system dynamic and the appropriate boundary conditions of the system states. Thus, both linear and dynamic programming techniques should be considered as simplified cases of the general nonlinear programming with their associated merits and drawbacks. The choice among these programming methods, i.e., LP, DP or nonlinear programming, depends on the characteristics of the reservoir system, on the availability of data, and on the objective and constraints specified.

There are three mathematical functions associated with the general nonlinear programming formulation. First, the Lagrangian associated with the constrained problem is defined as:

$$L(x,\lambda)=f(x)-\lambda g(x)$$

in which $\lambda(\geq 0)$ is an m-dimensional Lagrange multiplier. Then, a saddle point for the Lagrangian is defined as $(\bar{x},\bar{\lambda})$ such that

$$L(\bar{x},\lambda)\leq L(\bar{x},\bar{\lambda})\lambda\geq 0,$$
$$L(x,\bar{\lambda})\geq L(\bar{x},\bar{\lambda})\forall x\varepsilon X.$$

This condition gives the sufficient condition for optimality but does not give the necessary condition, and thus an optimal solution could exist without any saddle point.

A Kuhn-Tucker point $(\bar{x}\varepsilon X,\ \bar{\lambda}\geq 0)$ can also be defined as:

$$\nabla_x L(\bar{x},\bar{\lambda})=0\cdots\text{stationarity},$$
$$g(\bar{x})\geq 0\cdots\text{feasibility},$$
$$\bar{\lambda}g(\bar{x})=0\cdots\text{complementary slackness}.$$

It need not be even a local minimum but becomes both necessary and sufficient for the optimum of the convex differentiable program satisfying a constraint qualification.

Second, the dual function is defined by the equation

$$\mathcal{L}(\lambda)=\min_{x\varepsilon X} L(x,\lambda),$$

and the dual problem associated with the original (primal) problem requires the maximization of the concave function(λ) over a convex set $\lambda\geq 0$,

$$\max_{\lambda\geq 0}\mathcal{L}(\lambda)$$

The nonlinear duality theorem states that $(\bar{x},\bar{\lambda})$ is a primal dual optimal pair such that $\mathcal{L}(\bar{\lambda})=f(\bar{x})$ if and only if $(\bar{x},\bar{\lambda})$ is a saddle point of the Lagrangian $L(x,\lambda)$. Separable (convex) problems are ideally suited to dual methods, because the required unconstrained minimization of $L(x,\lambda)$ decomposes into small subproblems. Note that $(\bar{x},\bar{\lambda})$ is not a saddle point if $\mathcal{L}(\lambda)$ is not differentiable at $\bar{\lambda}$.

Last, the primal function (or optimal value function) associated with the perturbed problem is defined as;

$$\sigma(Y)\triangleq\min_{X\varepsilon x} f(x)$$
$$\text{s.t.}\, g(x)\geq y$$

The $\sigma(y)$ is a convex function with convex $f(x)$ and concave $g(x)$ on convex x. If $\sigma(y)$ is differentiable $\bar{y}=0$, then $\bar{\lambda}$ is the gradient of $\sigma(y)$ at $\bar{y}=0$. Thus, Lagrange multiplier λ measures sensitivitives of the optimal value function and have interpretations as prices associated with constraint resources. The mathematical properties of th these functions and the convergence theory of iterative algorithms are the most important concepts of analysis in an optimization problem. The convergence theory is concerned with the questions such as whether a given algorithm in some sense yields a solution to the original problem and how fast the algorithm converges to a solution.

Computational methods in nonlinear programming can be divided into the unconstrained and the constrained formulations (Luenberger, 1973). Techniques available for solving the unconstrained problems include the steepest descent (or ascent for maximization) methods, the conjugate direction methods, and the quasi-Newton methods. Most of these techniques require some search techniques such as the Fibonacci search or curve fitting, but they differ in the rule of successive movement directions based on different amounts of derivative information.

The Fletcher-Reeves method for implementing the conjugate gradient algorithm does not require the evaluation of the Hessian matrix $H(x_k)$ at the k^{th} iteration but does require a line searcn. The algorithm is based on the fact that the step size α_k as computed by the pure conjugate gradient algorithm minimizes the objective function in the direction d_k. The algorithm is:

Step 1) Given initial solution x_0, compute the gradient

$\nabla f\ (x_0)$ and set $d_0=-\nabla f(x_0)$

Step 2) Choose α_k to minimize $f\ (x_k+\alpha d_k)$ and set

$$x_{k+1}=x_k+\alpha_k d_k,\ \ d_{k+1}=-\nabla f(x_{k+1})+\beta_k d_k$$

$$\text{where } \beta_k=\frac{\nabla f^T(x_{k+1})\ \nabla f(x_{k+1})}{\nabla f^T(x_k)\ \nabla f(x_k)}$$

Fletcher-Powell method is one of the most powerful quasi-Newton procedures. Central to the method is a symmetric positive definite matrix H_k which is updated at each iteration, and which supplies the current direction of motion d_k by multiplying the current gradient vector $\nabla f\ (x_k)$. The procedure starts with any symmetric positive definite matrix H_0, and any point x_0.

Step 1) Set $d_k=-H_k\nabla f(x_k)$

Step 2) Choose α_k to minimize $f(x_k+\alpha_k d_k)$ to obtain x_{k+1},

$P_k=\alpha_k d_k$, and $q_k=\nabla f(x_{k+1})-\nabla f(x_k)$

Step 3) $H_{k+1}=H_k+\dfrac{P_k P_k^T}{P_k^T q_k}-\dfrac{H_k q_k q_k^T H_k}{q_k^T H_k q_k}$

There are also a numer of unconstrained optimization which do not even require derivatives such as the pattern search of Hooke and Jeeves, Rosenbrock's method, and Powell's method.

Techniques available for solving the constrained problems include primal method (especially for problem with linear constraints), penalty and barrier methods (especially for problem with nonlinear constraints), and dual methods (for convex and/or separable problems). The primal methods consist of feasible direction method, gradient projection method, reduced gradient method, and their variations to handle nonlinear constraints. They require a procedure to obtain an initial feasible solution. Penalty and barrier methods such as the sequential unconstrained minimization technique (SUMT) convert constrained problems to unconstrained problems, and usually need some modification for an accelerated convergence. Computational methods for solving the dual includes the gradient algorithm and the cutting plane method.

Consider a reservoir system problem defined as

$$\min_{l\leq X\leq u}\ f(x)$$

$$\text{s.t. } g(x)=b$$

in which x is a decision vector of release and storage. The equality constraints include the system of continuity equations and the b vector includes reservoir inflows. The vector l and u of the inequality constraints represent lower and upper bounds on decision variables. For example, the lower bounds might be the dead storage for sedimentation plus the minimum downstream requirement for water quality. The upper bounds might be the free board requirement for flood control and the downstream channel capacity for erosion control. The objective function might be long-term water supply and hydropower purposes combined with short-range operational goals or targets. The reservoir inflows are usually assumed to be deterministic in most nonlinear programming approachs and thus it is desirable to include sensitivity capability in any solution algorithm.

With this general nonlinear prdgramming formulation, the penalty and barrier methods could be one of the choices. The inequality constraint can be dealt with using an interior point barrier function and equality constraint can be dealt with using an exterior penality function. Then the augmented objective function becomes

$$v(x;r,t)=f(x)+r\sum_{j=1}^{n}\left(-\frac{1}{x_j-u_j}-\frac{1}{l_j-x_j}\right)+t\sum_{j=1}^{m}[g_j(x)]^2$$

A general minimizing algorithm can be given as mixed interior-exterior SUMT:

Step 1) Make an engineering estimate x_0 of the solution

Step 2) Choose $r_1>0$ and $t_1>0$, and obtain an unconstrained minimum of x_1 of the augmented objective function

Step 3) Continue with $k=2,\cdots$by choosing $r_k<r_{k-1}$ and $t_k>t_{k-1}$ and starting from x_{k-1}, finding an unconstrained minimum point x_k of $V(x\,;\,r,t)$

Step 4) As $r_k\to0$ and $t_k\to+\infty$, if $||x_k-x_{k-1}||$ and $|f(x_k)-f(x_{k-1})|$ are sufficiently small, terminate the process and take x_k as the solution.

The most important issue from practical point of view is the question of how to solve the unconstrained problem in step 2.

If nonlinear terms arising from evaporation losses are linearized, the nonlinear problem can be further simplified with the linear constraint of continuity equations.

$$\min_{l\leq X\leq u} f(x)$$
$$\text{s.t } Ax=b$$

For this nonlinear programming with linear constraints, one of the primal methods or their modification can be used. The Rosen's gradient projection method is motivated by a desire to implement the feasible direction algorithm while not requiring the solution of a linear program at each step. The basic idea is that at feasible point x_k (a feasible solution to the constraints can be found by application of the phase I procedure of linear programming), one determines the active constraints $A_1{}^Tx_k=b_1$ $(A_2{}^Tx_k<b_2)$ and projects the negative gradient onto the the subspace tangent to the surface determined by these constraints. Assuming the lower and upper bounds have been already included into the linear constraints $Ax=b$, one step of the algorithm with a given feasible point x_k is as follows:

Step 1) If A_1 is vacuous, let $P=I$; otherwise, let the projection matrix $P=I-A_1{}^T(A_1A^T)^{-1}A_1$ and $d_k=-P\,\triangle f\;(x_k)$.

Step 2) If $d_k\neq0$, let α_k be an optimal solution to the following line search problem:
min $f(x_k+\alpha d_k)$
s.t. $0\leq\alpha\leq\alpha_{\max}$
where $\alpha_{\max}=\max\{\alpha : x_k+\alpha d_k$ is feasible$\}$.
Update the decomposition and return to step 1.

Step 3) If $d_k=0$, stop if A_1 is vacuous; otherwise,
let $\beta=-(A_1A_1{}^T)^{-1}A_1\triangle f(x_k)$.
If $\beta\geq0$, stop; x_k is a Kuhn-Tucker point.
Otherwise, delete the row from A_1 with the most negative component of β and return to step 1.

Since the set of active constraints changes by at most one constraint at a time, it is possible to calculate one required projection matrix from the previous one by an updating procedure.

Instead of assuming the linearity of equality constraints, both the objective function and constraints might be separated into sums of funcitons of the N individual reservoirs. Then, the general nonlinear programming becomes one of separable problems, probably with some coordinating variables (Haimes, 1977).

$$\min_{l \leq x \leq u} \sum_{i=1}^{N} f_i(x_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} g_i(x_i) \geq 0$$

This separable (convex) problem is ideally suited to dual methods, because the required unconstrained minimization decomposes into small subproblems. The required dual function now is

$$\mathcal{L}(\lambda) = \min_{l \leq x \leq u} \sum_{i=1}^{N} [f_i(x_i) - \lambda g_i(x_i)]$$

which decomposes into the N separate minimization problems

$$\min_{l_i \leq x_i \leq u_i} f_i(x_i) - \lambda g(x_i)$$

The solution of these subproblems can usually be accomplished relatively efficiently, since they are of smaller dimension than the original problem. If a further decomposition of decision variables in time is performed on these subproblems, DP might be used for each time periods in each subproblems (Lasdon, 1970).

The dual objective function in this case can be written as

$$\max_{\lambda \geq 0} \mathcal{L}(\lambda) = \max_{\lambda \quad 0} \min_{l \leq x \leq u} [f(x) - \lambda g(x)]$$

where $f(x) = \sum_{i=1}^{N} f_i(x_i)$ and $g(x) = \sum_{i=1}^{N} g_i(x_i)$. The dual problem is equivalent to the following linear program in the variables z and λ for a given x.

$$\max_{\lambda \leq 0} z$$

$$\text{s.t.} \quad z \leq f(x) - \lambda g(x) \; \forall l \leq x \leq u$$

For this equivalent program, the cutting plane method can start with an initial feasible point x_0.

Step 1) Solve the following master problem to obtain

(z_k, λ_k)

$$\max_{\lambda \leq 0} z$$

$$\text{s.t.} \quad z \leq f(x_j) - \lambda g(x_j), \quad j = 0, 1, \cdots, \quad k-1$$

Step 2) Solve the following subproblem to obtain x_k.

$$\min_{l \leq x \leq u} f(x) - \lambda_k g(x)$$

If $z_k = \mathcal{L}(\lambda_k)$, then stop; λ_k is an optimal dual solution.

Otherwise,i f $z_k > \mathcal{L}(\lambda_k)$, then return to step 1.

Note that the cutting plane algorithm for maximizing the dual can be interpreted as a tangential approximation technique.

Nonlinear programming is an effective technique in reservoir operation, as for as reaching an optimal solution. Basically, there were two method that have been used for reservoir system: the conjugate gradient/gradient projection method extended to handle linear constraints with penalty terms for nonlinear constraints, and the Lagrangian gradient procedure for convex programs. The few articles on the subject confirm that the programs work, but they come up with no solution to the time and storage factors. Overall, therefore, it would appear that nonlinear programming techniques, while effective, are not really viable at the present time unless the separability condition is assumed.

Areas of interest on future research can be directed to the follow aspects of the problem:

○ development in some accelerated procedures of existing methods for faster convergence,

○ incorporation of streamflow sensitivity features within both primal and dual algorithms,

○ extention of the reliability programming to the multifacility reservoir systems problem,

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- o development of effective algorithms for both spatial and temporal decomposition without destroying nonseparability,
- o application and comparison of the cutting plane and the tangential approximation with existing applications, and
- o incorporation of dynamic supply-demand relationships with changes of price in the objective function.