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演修講座

# THEORY AND APPLICATION OF SYSTEMS ANALYSIS TECHNIQUES TO THE OPTIMAL MANAGEMENT AND OPERATION OF A RESERVOIR SYSTEM(1)

—저수지 최적 운영을 위한 시스템 해석 기법의 적용—

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## I. Introduction

Economic development of a river basin is governed by among other factors, the quantity and quality of the water guaranteed to prospective users. The water use includes water supply, hydropower, flood protection, navigation, pollution control and water-based recreation. To match water resources with these uses, reservoir systems and groundwater pumping facilities can be built and operated. The economics of a reservoir network is a function not only of construction costs but also of the strategy adopted to operate the reservoirs. The operating policy needs to account not only for the uncertainty in the streamflows but also for conflicting objectives of the reservoir systems.

In the past decade, two methods of solution have received much attention—stochastic simulation and mathematical optimization; the one emphasizing sophisticated reproduction of inflow sequences with simplified operation, the other emphasizing detailed operation models with generally simplified inflow sequences. The simulation does not guarantee an optimal solution to the problem and the operation policy from the optimization is an approximation of the optimal strategy for the real system. The objective of this study is to review the state of the art of theories and applications of systems analysis techniques to the optimal management and operation of the reservoir system.

The analysis of a complex water resources system may involve thousands of decision variables and model parameters. Once the objectives and constraints have been identified, the problem lends itself to solution techniques developed in the fields of operations research and management science. With the aid of high speed computers, solutions can be obtained to provide optimum decision alternatives which can be used by water resources managers to assist their decision making. Many successful applications of optimization techniques are made in reservoir studies, mostly for planning purposes, but there may still exist a gap between the theories and applications, particularly in the area of real-time reservoir operation. Enormous economic gains can be realized with only a modest improvement in operating and managing the reservoir systems. Therefore, it is extremely important and timely to review and evaluate the state of the art of both theories and applications in this field.

A critical review and in depth analysis will be made on optimization techniques in order to assess their merits and drawbacks. The choice of method should depend on the characteristics of the system, on the availability of data, and on the objectives and constraints specified. Special attention will be made to their applicability to the operational problems of reservoir systems. Recommendations on future research needed to advance the state of the art in the field of reservoir management and operation will be made.

## II. Reservoir System Models

### 1. Systems Approach

A reservoir system is an organized or complex whole : an assemblage or combination of reservoirs in series

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and/or parallel forming a complex unit or unitary whole. physical water quantity subsystem and biological water quality subsystem of reservoir(s) are interconnected to a total water resources system in a hierarchical framework. Essentially, any reservoir system can be stripped down to four basic components: inputs (controllable and uncontrollable), state determined process, output (physical or economic), and environment (physical or legal).

Systems approach can be viewed as actually constituting not one phase but a three-phase process, that is, systems analysis/design, systems testing and implementation, systems monitoring and evaluation with any feedback loops. This systems cycle is closely linked with the management functions of planning, organization and direction, and control, respectively. Many systems analysis of the reservoir system in the public sector, particularly those using the benefit-cost or the cost-effectiveness analysis, focus mainly on the planning phase. Management process of forecasting, decision-making, and action is carried on at two levels of hierarchy for long-term strategic goals and short-term operational targets.

## 2. Mathematical Models

A reservoir model is a simplified representation of the relevant aspects of an actual reservoir system or process. The value of any kind of model depends entirely on how well it represents what it is supposed to represent for the purpose for which it is intended. There are numerous ways (physical or mathematical) to construct a model of an object or reservoir system, and such models differ considerably in degree of abstraction, ease of construction, and ease of manipulation. As models become more abstract, it becomes correspondingly more difficult to construct them to cover specifics, but this difficulty is the price for their greater flexibility.

Mathematical models in a reservoir system can be defined in terms of their purpose and types of the variables. The models may be either descriptive or normative in purpose. A descriptive model (for analysis and forecasting) describes things as they are and a normative or optimization model (for design and control) indicates how they should be. The variables of a model are either deterministic or probabilistic. Deterministic models are used under condition of certainty and probabilistic models are made under conditions of uncertainty. Most of the mathematical models involve some degree of simulation. Simulation models of reservoir storage are predictive in purpose and usually have some probabilistic variables.

## 3. Reservoir Management and Operation

An essential preliminary to project planning and control is a clear statement of objectives. The Water Resources Council's Principles and Standards (1973) established two equally important objectives for federal water resources projects: national economic development (NED) and environmental quality (EQ). Later, it was revised in 1980 to include regional economic development (RED) and other social effects (OSE). For a reservoir system, project purposes include use of the system for water supply and drought, hydropower and flood control, and other water quality purposes. Operational goals or targets might be minimization of deviations from predetermined standards or plans. Objectives or purposes of any reservoir system must be verifiable in order to facilitate management controls. (A review of multi-objective programming by Cohon and Marks (1975) evaluates techniques of weighting method, constraint method and surrogate worth tradeoff method).

The constraints of a reservoir system include environmental, legal and hydrologic limitations. The environmental and legal constraint are usually in the form of upper or lower bounds on reservoir storage or release variables. The hydrologic budget equations of water balance for a reservoir have the following form for each time period: change of storage = inflow - release - losses where losses include evaporation which is dependent on both reservoir storage and release. In addition to the economic uncertainty in the parameters of the objective

function, the inflow term gives hydrologic uncertainty in the constraint equations. For a system of reservoirs these constraints will be large because it deals with similar subsystems, repeated in time or location, with the subsystems having storage terms in common. The uncertainty problem has been treated with the chance-constraint formulation or stochastic programming in planning situations and the forecasting techniques in real-time operations. Approaches to solving large mathematical programs have been decomposition, partitioning, aggregation, or successive approximation methods.

### III. Dynamic Programming

Dynamic programming (DP) is used extensively in the optimization of water resource systems (Buras, 1966). The popularity and success of this technique can be attributed to the fact that the nonlinear and stochastic features which characterize a large number of water resource systems can be translated into a DP formulation. In addition, it has the advantage of effectively decomposing highly complex problems with a large number of variables into series of subproblems which are solved recursively. Dynamic programming, a method formulated largely by Richard Bellman (1957), is a procedure for optimizing a multistage decision process.

In the analysis of many operational problems, it is convenient to consider the idea of a system with a number of possible discrete states  $x_n$  at each stage  $n$  which can follow any of a number of relevant processes (Figure). The movement of the system is controlled by a decision maker who, at each stage  $n$ , uses one of a set of feasible decisions  $d_n$  and which make a sequence of transitions  $x_{n-1} = t_n(x_n, d_n)$  between the states. As the system proceeds it generate a sequence of returns  $r_n(x_n, d_n)$ , which will usually be a net benefit or a cost. The decision maker wishes to find the sequence of decisions which in some sense optimizes a function  $f_n(x_n, d_n)$  of returns generated when the system starts in that state and a particular policy is followed.

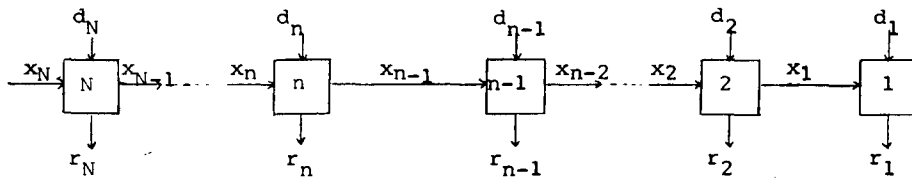


Figure. Discrete-time Dynamic

In determining optimal releases from a reservoir, the states may represent the amount of water stored in the reservoir in certain month and the decision may represent how much water to release from the reservoir during the current month. But, it is not unusual to find that a problem can be formulated in more than one way and part of the art of DP lies in deciding the most efficient formulation for the problem on hand. For example, stages may represent different points in time or in space, and states may be continuous rather than discrete. The key feature of DP application is that it usually be identified as a serial or progressive directed network for an operation or planning problem, respectively (Hastings, 1973). From every state a terminal state is reached in some predetermined number of stages for serial problems, but it is reached in not more than a predetermined number of stages for progressive problems.

When the returns are independent and additive with the discounting factor  $r = (1+i)^{-1}$ , the recurrence relation is

$$f_n(x_n) = \max_{d_n} [r_n(x_n, d_n) + r \cdot f_{n-1}(x_{n-1})]$$

where  $x_{n-1} = t_n(x_n, d_n)$ ,  $i$  is the discount rate, and  $f_0(x_0)$  is given for all terminal states. This is based on the principle of optimality enunciated by Richard Bellman: An optimal set of decision has the property that

whatever the first decision is, the remaining decisions must be optimal with respect to the outcome which results from the first decision. To decompose a general problem into stages with decisions required at each stage, the value of every state should satisfy the separability condition and the monotonicity condition (Nemhauser, 1966). The validity of DP regarding separability can be extended by increasing the number of states, but this extension is achieved at the cost of increased computation. For example, reservoir inflows are defined as a state in addition to reservoir storage when the inflows are Markovian rather than random in the transition equation. If the returns are additive, multiplicative or of the minimax or maximin variety it is sufficient to check that the returns are independent.

Where there is no special reason for choosing either backward or forward formulation, the previous backward recurrence is normally used. The procedure of making first a backward and then a forward pass is convenient especially in problems involving with time, as it gives the optimal policy in chronological order; in stochastic problems backward recurrence is essential, since each stage depends on the results of the former stage. But, forward recurrence is advantageous when a deterministic problem has to be solved several times with different planning horizons. This may occur because a plan is periodically reviewed or where the appropriate horizon is unknown and a sensitivity analysis is undertaken. The value table can be extended forward in time without repeating previous calculations.

Constraints which restrict only the state or decision space are advantageous in (discrete) DP because they reduce the amount of computation. By contrast, state and decision space constraints can cause considerable procedural difficulties for other optimization techniques. However, when DP is applied to a multiple reservoir system, the usefulness of the technique is limited by the so-called curse of dimensionality which is strongly a function of the number of state variables. For computational efficiency, problems should have no more than a few state variables at a time. All methods of dimensionality reduction involve decomposition into subsystems and the use of iterative procedures. Also, there are difficulties in developing stochastic DP for a multiple reservoir system where serial and cross correlations are prevalent between streamflows. These difficulties could be partially overcome with the combined use of deterministic DP and simulation.

On formulating problems for solution by DP it is advisable to follow a standard procedure. One of the advantages of following a formal procedure is that every term and condition is defined and the scope and effect of possible modifications are more readily apparent. The following procedure for problem formulation is cost of increased computation. For example, reservoir inflows are defined as a state in addition to reservoir storage when the inflows are Markovian rather than random in the transition equation. If the returns are additive, multiplicative or of the minimax or maximin variety it is sufficient to check that the returns are independent.

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1. Define the appropriate terms: stage, state, decision, return function, optimal value function, together with the symbol used to denote the relevant quantity including specific definitions of their arguments. In case of difficulty, try to draw a directed network diagram, either serial or progressive.
2. Write down the appropriate equations: recurrence relation, either backward or forward; transition equation.
3. Note the appropriate boundary conditions or terminal values, and inequalities determining the range of the stage, state and decision variables.
4. Check that the validity conditions (both separability and monotonicity) are satisfied.

For a reservoir system, studies are typically divided into planning studies and operation studies. Operation studies are further divisible into short-range and long-range operation. For short-range operation, one may regard both the water demand and the water inflow as deterministic. For the long-range, the stochastic nature of the inflows has to be taken into account. This can be done either by probabilistic consideration or by synthetically generating equally likely streamflows. Dynamic programming is extremely well suited by its nature to handle the following types of problem: Progressive directed network for capacity expansion problem, deterministic DP for short-range operation, and stochastic DP for long-range operation.

Dynamic programming (DP) has the desirable properties to handle adaptive, nonlinear and stochastic problems of a reservoir system. DP is specifically applicable to reservoir management functions of planning and operation which can be represented as either progressive or serial directed network problem. Among the assumptions of monotonicity and separability in the DP model, the separability condition of the objective function limits some model applications and thus an artful choice of stage and state is required. In addition to the multipurpose and multifacility nature of a large reservoir system, considerations of reliability and real-time operation has aggravated the so-called "curse of dimensionality" and thus numerous efforts have been attempted to alleviate this problem during the past decade.

In the deterministic operation, two lines of development can be observed which is based on either incremental DP (IDP) or differential DP (DDP). IDP has been extended to IDP with successive approximation (IDPSA) and then to multilevel IDP (MIDP); DDP has been extended to discrete DDP (DDDP) and then to constrained DDP (CDDP). DP-LP, multi-objective DP (MODP) and Progressive Optimality Algorithm are another lines of approach primarily to the reservoir release scheduling or planning problems. Both CDDP and Progressive Optimality Algorithm have a rather unique feature that the state space do not have to be discretized.

In the stochastic operation, stochastic DP (SDP) and sequential stochastic optimization (SSO) are applicable for the stationary policy and the real-time operation, respectively. The Monte Carlo DP (MCDP) and the chance-constrained DP approaches have the important advantage of being able to assess the reliability or risk associated with a design or planning policy. The application of these probabilistic formulation to a multi-purpose multifacility reservoir system is either time-consuming or difficult to achieve. The techniques used in handling the dimensionality problem in deterministic DP might be applied to reduce these problems.

Based on these review of DP to reservoir systems, the following five areas of future research are recommended for an incorporation of DP in the overall system approach:

- continuation of research in incremental DP procedures to include risk or reliability and multiobjectives,
- development of research in differential DP procedures to facilitate nondifferentiable objective functions,
- continuation of research in real-time operations to incorporate adaptive forecasts in an interactive mode,
- development of research in decomposition or partitioning to facilitate nonseparable objective functions,
- initiation of research in the semi-Markov programming for multiyear reservoir operation with an econometric forecast.

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