<論 文>

用水一廢水 分配模型의 敏感度分析

Sensitivity Analysis of Water Supply-Wastewater Allocation Model

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Abstract

The competiiton for water for municipal, industrial and agricultura and agricultural uses is growing keener as the world food and energy crises are intensifying. It is therefore becoming important how systems engineering techniques can be used to plan effectively for the future development of water supply and waste water management systems in a regional area. The feasible direction method and the out-of-kilter algorithm enable us to find the least-cost mix of alternative allocation networks. The interaction between land use patterns and urban water resources, and the environmental impact of alternative policies are discussed.

요 지

세계 식량 및 에너지 위기의 심화에 따른 여러가지 용수공급의 경합에 맞추어, 시스템공학의 지역적 수자원 개발 계획에 대한 적용이 중요시 되고 있다. 도시 수자원과 토지이용 전략 및 이에 따른 환경 영향에 관한 논의와 함께, 분배 관망의 최소 비용안을 구하기 위한 선형계획 및 비선형계획 방법을 제시하였다. 또한 주어진 가상용수 수요에 대비한 분배 모형의 작성 및 실제전산 처리를 통하여 장래 용수 공급원에 관한 민감도를 분석하였다.

1. Description of the Allocation System

Expanding industrial, agricultural, and recreational activities, and a growing population are imposing increasing demands on conventional water sources. It is therefore becoming important for many municipalities to consider desalted seawater and renovated wastewater as alternate sources. Because of the complexity associated with the analysis of large municipal water supply-wastewater disposal systems, the use of advanced computational procedures involving computer technology is desirable, including appropriate mathematical models describing such systems. The purpose of this study is to exemplify how systems engineering techniques can be used to plan effectively for the future development of water supply and wastewater management systems in a regional area.

The basic elements of the water supply and wastewater disposal systems can be illustrated as a network of interconnected water supply sources and water demands. (Fig. 1)' The water sources include surface water, ground water, seawater, blackish water and wastewater. The water demands include municipal, industrial, irrigational water and ground water recharge. Since all of these elements are

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interrelated, the selection of alternatives for the minimization of costs cannot be done separately for each element, but must be evaluated in the context of an overall municipal water supply and wastewater disposal system.

The quality aspects of water supplies and wastewater discharges which relate the type of water source and its treatment to specific water demand quality requirements can be defined as two classes: (1) those concerned with discharges of wastewater and receiving water quality; and (2) those concerned with the quality required for specific uses of water. Since the water use requirements for any city depend upon numerous factors, they necessitate an in-depth investigation for each specific case.

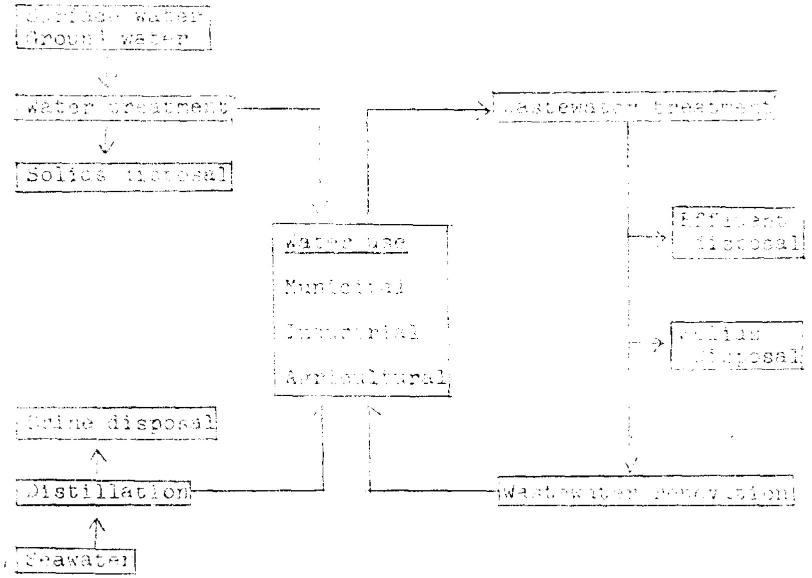

Fig. Elements of the water supply and wastewater disposal system
Source : 〔Reddle, 1970〕

2. Development of the Network Model

Most of the existing water resource supply and demand models involve the resource allocation in a framework of least-cost conjunctive use of available supplies. The basic difference in available models is the solution algorithm. A necessary prerequisite to use one of the available mathematical models is quantifying the objective function and the constraints on alternative courses of action.

The objective function can be derived from the process cost and the piping cost of each source of water. The process cost function includes first costs, amortized capital, and second costs, operation and maintenance as a function of water production rate. The process cost function may include technology adjustment multipliers and a time horizon interest rate adjustment multiplier. Young, 1970 The piping cost can be estimated on a per mile basis as a function of flow rate. These cost functions must take into account local as well as regional differences related to such factors as labor rates, productivity, and materials costs.

The upper bound constraints define the amounts of water that the alternative sources can ultimately supply. In the absence of storage, the safe yield of a river is its lowest dry weather flow; with full development of storage, the safe yield approaches the mean annual flow. The maximum yield of a well can be estimated by calculating the specific capacity and the maximum available drawdown. Fair, 1971

The lower bound constraints difine the amounts of water demand of projected population which can be estimated from the logistic S-curve, comparison method or correlation studies.

The competition for water for municipal, industrial, agricultural and energy uses is growing keener as the world food and energy crises are intensifying Thus, engineering judgement should be applied in defining the model structure which includes specification of a flow and locational basis for all major elements such as the possible treatment facilities, conveyance routes, sources and demands. Particularly, a better coordination of water resources policies and land use policies is required because of the competition between land and water uses.

The interactions between land use patterns and urban water resources including the relationship to the whole watershed depends on the distribution of point, non-point water demand and source of wastewater in population center and open spaces throughout the watershed. The amount and location of open spaces and their locations with respect to the water resources is significant to the quantity and quality of surface and ground water.

3. Solution Method of the Problem

Given water demands and sources, the strategy is to develop a network of possible water supplies linked by pipeline that permit the optimum sources of water to be tapped and transmitted to the desired location. Because the economies of scale and price variation of the objective function are nonlinear, a nonlinear program subject to linear constraints with 0～1 matrix is formed as follows:

$$\min f(X) \tag{1}$$

$$\text{s.t. } a_{ij}x_j=b_i \quad i=1,2,\cdots,m \tag{2}$$

$$x_j\geq 0 \quad j=1,2,\cdots,n \tag{3}$$

where x_j are decision variables (flow rates in the pipeline) and b_i are the the constraints on the system (amount of water demands and sources).

One method for solving the nonlinear program (1)～(3) is the method of feasible directions [Luenberger, 1973]. To start execution of the program, a uniform start, which distributes the water demand evenly amongst the decision variables, can be used to speed up the computation. From the initial position the feasible direction of gradient vector can be established by solving the following linear programming:

$$\min \nabla f(X^k)d \tag{4}$$

$$\text{s.t. } A(X^k+d)\geq b \tag{5}$$

$$X^k+d\geq 0 \tag{6}$$

The next step is solving the following line search for the step size:

$$\min f(X^k+3d^k) \tag{7}$$

This process iterates until the minimum point on the response surface is located.

To account for the nonlinear cost functions and to formulate the input data in the correct format, an iteration procedure was developed which uses a preprocessing program and a recosting program, inconjunction with the network program[Weddle, 1970]. The preprocessing program selects the unit cost equation applicable to the activities represented by each network arc and calculates the unit cost for each arc using the developed cost functions based on the specified flow rate. The recosting program coverts the result from network program to the total annual cost and calculate the percent change in the total network costs between the first and second iterations through the model.

Then the objective function for the water resource management system can be expressed as a minimum-cost flow problem for the product of unit costs and flows in all arcs, i.e.,

$$\min \sum_{(i,j)\in A} c_{ij}x_{ij} \tag{8}$$

where A is the set of directed arcs (i, j). Flow continuity must be maintained at each node j contained in the set of nodes N, which may be expressed as

$$\sum_{j\in N} x_{ij}-\sum_{j\in N} x_{ij}=0 \quad \forall j\in N \tag{9}$$

Upper and lower bound constraints on the flows in a given arc are defined as

$$l_{ij}\leq x_{ij}\leq b_{ij} \quad A(i, j)\in A \tag{10}$$

Alternative water management policies may be investigated by appropriate change of (10).

A computationally efficient method for solving the network flow problem (8)~(10) is the out-of-kilter algorithm [Ford, 1962]. Starting from the source node, node prices for the water or wastewater in the direction of flow are computed at each node on the flow pattern and the network program determines the arc cost. If the overall solution is feasible and optimality conditions in any arc are satisfied, the flow in that arc is considered as in-kilter. If an arc is out-of-kilter, the flow in that arc is then changed by using a flow circulation technique. This process is repeated until all arcs in the network are being in-kilter, in which case the solution is feasible and represents the minimum cost.

In the development of the model structure, a new parallel arc to the existing facility arc can be specified to take into consideration the expansion of existing facilities. Dummy nodes can be used to consolidate alternative routings of proposed facilities via parallel arcs into single arcs. Treatment requirements which relate the type of water source and its treatment to specific water demand quality requirements can be incorporated into the network structure by providing several nodes in a treatment arc. An example of a very simple network is shown in Fig. 2 to illustrate arcs and nodes.

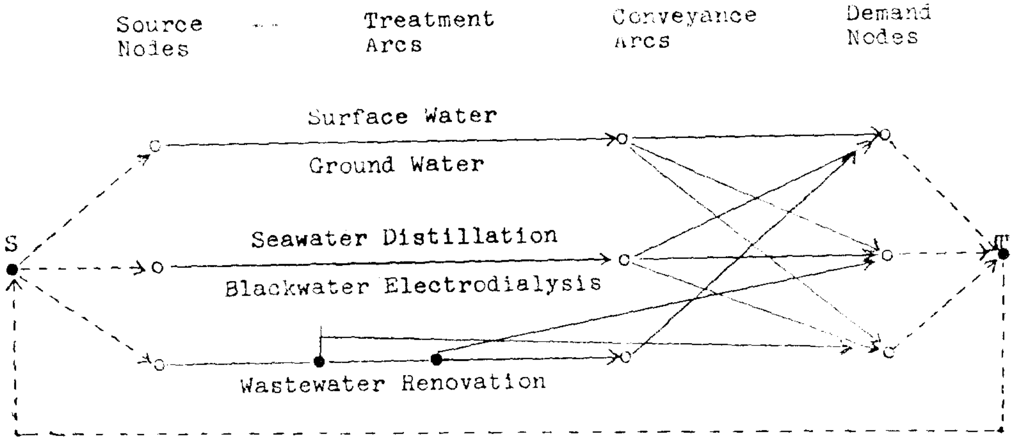

Fig. 2. Simple netwprk of water supply-wastewater disposal syatem Source : [Weddle, 1970]

4. Evaluation of the Alternatives

The network model can be used for evaluating alternative plant locations and conveyance routes, obtaining parametric information for the appropriate cost equations and observing the effect on the system solution either including or eliminating certain treatment processes. But the least-cost water supply-wastewater disposal system may not meet all objectives in a certain municipal area. Therefore alternative water management policies should be delineated to explore the costs and benefits of meeting the other, environmentally related, objectives.

The decomposition of overall water resource management system networks and detailed subsystem optimization prior to synthesizing alternatives are desirable in order that the development of alternatives for decision making can be focused on different sets of objectives and resource allocation policies. For evaluating environmental impact, several iterations on the least-cost allocation model coupled with simulation models will be needed. The cost-effectiveness of alternatives relative to quantifiable costs and benefits should be analyzed prior to the rational selection of the recommended plan.

Therefore, the use of mathematical models does not preemp the decision-maker rather the model clears the air of complexity and enables the decision-maker to focus on the key factors. Although inputs are fed into the model and outputs are generated, the analysis demonstrates the need for judgement and an interrelationship between the planner and the decision-maker. Appreciation of this fact will enhance the acceptance of analytical tools, and will make them more efficient and effective.

Sensitivity Analysis of the Model

*A*1. Based on the previous general descriptions of water supply-wastewater allocation models, a sensitivity analysis is performed for a nonlinear programming model subject to linear constraints based on the following network representation.

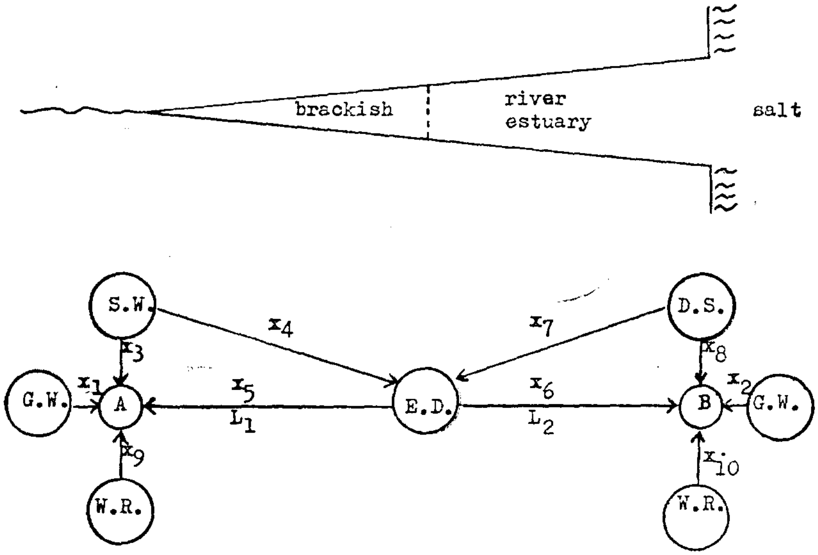

Fig. A1. Schematic diagram of flows (x_i : amount of allocated water in MGD)

Then given water demands and sources what would be the mix of water supply sources in 2020?

*A*2. The estimated amount of water from each source for these respective areas(City A and City B) is described below with the sensitivity analysis in mind.

Surface water: A yield of 1,000MGD can be supplied by reservoir development in the upper watershed. Through weather modification, 660MGD of water can be added to the fresh water supply.

Ground water: Wells in the City A area are capable of supplying 143MGD of water and wells in the City B area are capable of yielding 647MGD. The present pattern of the water supply does not use wells.

Electrodialysis: The optimum location of the plant and the necessary transmission lines need to be determined.

Desalination: Two types of process are the plant with distillation alone and the plant with nuclear power generation.

Wastewater Reclamation: However a large psychological barrier to this recycle process exists, it is feasible to recycle from 25% up to as much as 50% of the water initially used.

The demand in the region for the year 2020 is 200(MUN.)+1100(IND.)=1300MGD for City A and 210(MUN.)+60(IND.)=270 MGD for City B. A demand management program could reduce the municipal demand by 10% for each area.

The objective function is the annual cost of the process costs and the piping costs. Each of these costs includes the capital cost and O&M cost with pricing adjustment weights. This cost structure permits to study the sensitivity of interest change and technological changes.

A3. Mathematical formulation for this general setting is as follows.

$$\min.\ \sum_{i=1}^{10} (f_{1i}\cdot u_i\cdot a_i x_i^{bi}+f_{2i}\cdot c_i x_i^{di})+\sum_{i=4}^{7} L_i(g_{1i}\cdot v_i\cdot \alpha_i x_i^{\beta i}+g_{2i}\gamma_i x_i^{\delta i})$$

$$\begin{aligned} \text{s.t.}\ & x_1+x_3+x_5+x_7+x_9=\mathrm{MUN}_A+\mathrm{IND}_A \\ & x_2+x_4+x_6+x_{10}=\mathrm{MUN}_B+\mathrm{IND}_B \\ & x_3+x_4 \qquad\qquad \leq \mathrm{SUP.} \\ & \qquad 0\leq x_i\leq 1_i \end{aligned}$$

where $f_{1i}, f_{2i}, g_{1i}, g_{2i}$; pricing adjustment weights.

u_i, v_i; capital recovery factor.

$a_i, b_i, c_i, d_i, \alpha_i, \beta_i, \gamma_i, \delta_i$; cost coefficients.

L_i; length of transmission lines in miles.

1_i; upper limit of water quantity allocated.

Table A1 is a summary containing the derived coefficients for each process and piping costs

Process	$a_i(\alpha_i)$	$b_i(\beta_i)$	$c_i(\delta_i)$	$\gamma_i(\delta_i)$	Notes
Surface water	0.00222	2.83	0.00235	2.51	T=100Yrs, i=4%
Electrodialysis	115,000	0.85	160,000	0.90	T=30Yrs, i=6%
	x0.37		x0.32		for low E.D.
Desalination	184,000	0.85	132,000	0.95	T=30Yrs, i=6%
	127,000	0.85	76,000	0.87	for nuclear
Ground Water	4,710	1.0	8,430	1.0	T=25Yrs, i=4%
Wastewater Reclamation	69,200	0.715	92,500	0.885	T=25Yrs,i=5%
Pipes	2,500	0.62	220	0.417	T=50Yrs, i=6%

Table Al. Dedrive coefficients in cost formula

The solution algorithm chosen from the computer library is a gradient projection method using Lagrange multipliers. To start execution of the program, a uniform start which distributes the water demand evenly among the decision variables; a conventional start using S.W., G.W. and wastewater reclamation only; and a exotic start using electrodialysis and desalination only is used. Note that the optimum always occurs at the boundary point since the objective is a convex functions.

A4. Using this methodology, the entire spectrum of alternatives is capable of being analyzed.

(Interest Rates)

Using 4% interest rate for the conventional processes and 6% for the exotic processes, the minimum total cost is about 12 million dollars. But for uniform interest rate of 6%, the cost is increased about 2 million dollars and for 4% the cost is decreased about 3 million dollars. Even if the plan of water sources for each cities is not changed, we can see that the total cost could be changed as much as 20% using uniform interest rates. Note that the plan doesn't include any water transmission scheme between two cities.

(Electrodialysis Location-Basic Solution)
By changing from high electrodialysis to low, the total cost decrease to 9 million dollars. The source of water for City A change from S.W.+G.W.+W.R. to S.W.+G.W.+E.D. but for City B no change occurs from G.W. only. Note that the location change corresponds to the uniform interest rate of 4% in total costs.

(Desalination Method)
Nothing changes between the D.S. alone process and the D.S. process with nuclear power generation. This is due to the relative high cost of this process compared with the cost of G.W. for City B.

(Wastewater recycling)
Nothing changes between 25% of recycling and 50% as the psychological factor of this process develops. This change is not enough to alter the basic solution which does not include the wastewater reclamation process.

(Without of Ground Water)
By not using the ground water supply sources, the total cost doubles that of basic solution. Then the supply sources for City B changes to S.W.+E.D.+W.R. with 30% recycling. Even if with this change, the desalination does not enter the solution.

(Weather Modification)
The total cost decreases by 1.8 million dollars for the additional 270MGD of City B water supply. We can think this cost reduction as an alternative cost of weather modification. We can see here that the surface water is the major water supply source for this regional settings.

(Demand Management)
The total cost decreases by 0.8 million dollars using 10% reduction of municipal demands for each city. Thiscost reduction comes from the electrodialysis which is the most expensive source of water for City A.

(Technological Changes)
Using the following weights for the exotic sources we can see from corresponding cost reduction that O. and M. costs are much more important than capital costs.

0.1 million dollars with $u_i=0.5$ and $v_i=1.0$
2.4 〃 1.0 〃 0.5
2.5 〃 0.5 〃 0.5

5. Conclusions

The basic elements of the water supply and wastewater disposal systems can be illustrated as a network of interconnected water supply sources and water demands. To find the least cost mix of alternative allocation networks, a nonlinear program subject to linear constraints is formed. The solution algorithm chosen for a sensitivity analysis of the model is a gradient projection method using Lagrange multipliers. With the derived coefficients for each process and piping costs, the entire spectrum of alternative management policies can be analyzed using the methodology.

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