論 文
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最適有効 및 無効電力配分을 위한 新 앨고리즘

A New Algorithm for Optimal Real and Reactive Power Dispatch

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요 약

本 論文은 電力系統의 経済的 運用을 위한 最適 有効 및 無効電力配分을 위한 새로운 알고리즘을 提示한다. 從來의 方法과는 달리, 有効電力最適化過程 (E. L. D. 問題)뿐만 아니라 無効電力最適化過程 (電壓無効電力制御問題)에서도 最適化目的函數로서 全發電費用을 取한 點이 현저한 特徵이라하겠으며, 有効電力最適化過程에서의 制御變數로서는 各 發電所의 有効出力을, 그리고 無効電力最適化過程에서의 그것은 各 無効電力源投入量과 各 變壓器탭設定値를 取하고, 各 制御變數의 動作上下限値와 各 母線電壓의 許容上下限値는 不等 制限條件이 된다.

從來의 B－定數法을 脫皮하고, 이 目的으로 새로이 變形된 그라디언트 프로젝션法에 依하여 計算効率이 向上된 實用的 解法을 提示하였다.

Abstract

This paper presents a new method for optimal real and reactive power dispatch for the economic operation of a power system. Unlike the usual approach of minimizing the transmission loss, this method minimizes the total production cost not only for the real power optimization problem, but also for the reactive power optimization. The control variables are real power generation of units for real power optimization, and reactive power generation of units, other var sources, and the transformer tap settings for reactive power optimization. The constraints are the operating limits on these control variables and the limits on the bus voltages.

Methematical models are developed to represent the sensitivity relationships between dependent and control variables for both real and reactive power optimization modules, and thus eliminate the use of B-coefficients. In order to handle many functional inequality constraints, a modified version of the gradient projection method is developed for optimization procedure, and has shown a remarkable advantage in computation efficiency.

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1. Introduction

The optimal operation of power system is required to precede the optimal planning of facilities or devices for the system, and they, in general, consist of generating plants, reactive-power compensation or voltage-regulation devices, and transmission networks. Since the optimal power flow method was first introduced by Dommel nd Tinney,[1] many articles have appeared in the literature on this method.[2)-4)] The optimal power flow problem is to minimize the fuel costs, system losses, or some other appropriate objective function, while maintaining an acceptable system performance in terms of limits on generator real and reactive power outputs, outputs of compensating devices, transformer tap settings, or bus voltage levels, etc. When total fuel cost is minimized, the optimal power flow results in an economic dispatch, and in addition to optimally allocating real power generation to units, determines the reactive power output of generators and other var sources as well as transformer tap settings.

In order to handle the large scale problems of this nature, the idea of P-Q decomposition was applied to the optimal power flow,[5)-7)] where the problem is decomposed into the real power optimization problem (P-problem) and the reactive power optimization problem (Q-problem). The P-problem is to minimize the production cost under the assumption that system voltages are held constant, and the Q-problem is to minimize the transmission loss under the assumption that real power generation is held constant. Due to the loose coupling between two problems, the sequential optimization of these provides a considerable advantage over the simultaneous optimization of all control variables. It should also be noted that a number of articles are devoted to the Q-problem alone for the optimum control of reactive power flow.[8)-10)]

The optimization problem is a general nonlinear programming problem with nonlinear objective functions and nonlinear functional equaltiy and inequality constraints. A typical approach is to augment the constraints into objective function by using the Lagrange multiplers[11)] and/or penalty functions, and to minimize the augmented objective function by using one of the optimizaton schemes, such as the steepest descent algorithm,[1),7)] or the sequential unconstrained minimization technique (SUMT).[6)] Other approaches are the use of linear programming approximation to the non-linear problem,[8),10)] or the use of the quadratic approximation to the objective function in order to apply the quadratic programming technique.[12)] Due to the size of the problem as well as the large number of functional inequality constraints, improvement on computational efficiency has been the thrust of most works.

This paper presents a new method for optimal real and reactive power dispatch for economic operation of a power system. The method is based upon three modules coupled to each other. First, the P-optimization module, which is equivalent to the conventional economic load dispatch, optimally allocates the real power generation among generators. The second module, Q-optimization module, optimally determines the reactive power output of generators and other var sources are well as transformer tap settings. the load flow module is used to make fine adjustments on the results of P- and Q-optimization modules. The algorithm developed and presented here has the following distinctive features and advantages:

(1) Unlike the usual approach of minimizing the transmission loss,[6),7),10)] the total production cost (fuel cost) is minimized not only for the P-optimization module, but also for the Q-optimization module. This approach unifies two decoupled optimization problems into one reference framework and avoids the switching

of objective functions from one to another.

(2) Like the economic dispatch for real power, the Q-optimization also allows an economic and efficient division of real power generation between generators. This additional benefit is, of course, due to our choice of the objective function mentioned above.

(3) In order to handle many functional equality and inequality constraints, a modified version of the gradient projection method[13] is developed for optimization procedure, and has shown a remarkable advantage in compution efficiency and reliability. This method avoids the augmentation of constraints into the objective function, thus eliminating the need for computing a large number of Lagrange multipliers which is common in other method.[1),6),7)]

(4) Mathematical models are developed to represent the sensitivity relationships between dependent and control variables for both optimization modules, and thus eliminate the use of B-coefficients.

(5) Mathematical models are developed with a great care in order to preserve the sparsity of Jacobians, and thus the sparsity is fully utilized in computing the sensitivity matrices.

(6) Since the Q-optimization module determines the optimum reactive power allocation, all buses, except the swing bus, are made to be the Q-type bus in the load flow module, where reactive power (output from the Q-optimization module) is specified instead of voltage magnitude. The swing bus voltage is not fixed, but is determined optimally by the Q-optimization module.

2. Formulation and Decomposition of the Problem

The optimal real and reactive power dispatch problem is mathematically defined as

Minimize

$$C = f(P_{sg}, Q_{sgc}, n) \quad (1)$$

subject to

$$\left.\begin{array}{l} \underline{P}_{sg} \leq P_{sg} \leq \overline{P}_{sg} \\ \underline{Q}_{sgc} \leq Q_{sgc} \leq \overline{Q}_{sgc} \\ \underline{n} \leq n \leq \overline{n} \\ \underline{V} \leq V(P_{sg}, Q_{sgc}, n) \leq \overline{V} \\ g(P_{sg}) = 0 \end{array}\right\} \quad (2)$$

where

P_{sg}: vector of real power of generators including the swing bus generator

Q_{sgc}: vector of reactive power of generators including the swing bus generator, and other reactive power compensating devices

n: vector of off-nominal tap settings of LTC

V: bus voltage vector

g: supply and demand balance equation

$\overline{P}_{sg}$, $\overline{Q}_{sgc}$, $\overline{n}$, $\overline{V}$: vector of the upper limits of P_{sg}, Q_{sgc}, n, V, respectively

$\underline{P}_{sg}$, $\underline{Q}_{sgc}$, $\underline{n}$, $\underline{V}$: vector of the lower limits of P_{sg}, Q_{sgc}, n, V, respectively

Here, the objective function f(•) is the total power production cost, or more specifically the total summation of generator fuel costs. It is noted that V is the dependent variable depending on control variables P_{sg}, Q_{sgc}, and n. If some other constraints such as line flow limitation, maximum allowable voltage angle differences between two buses are required to be considered, they can also be added to the above constraints.

For the computational efficiency the original problem in Eqs. (1) and (2) can be decomposed into the following two subproblems:

(1) P-Optimization Subproblem

Minimize

$$C_p = f_p(P_{sg}) \tag{3}$$

subject to

$$\left.\begin{array}{l} \underline{P_{sg}} < P_{sg} < \overline{P_{sg}} \\ g(P_{sg}) = 0 \end{array}\right\} \tag{4}$$

where

$f_p(\cdot)$: the total summation of generator fuel costs which is the function of P_{sg}

(2) Q-Optimization Subproblem

Minimize

$$C_q = f_q(Q_{sgc}, n) \tag{5}$$

Subject to

$$\left.\begin{array}{l} \underline{Q_{sgc}} \leq Q_{sgc} < \overline{Q_{sgc}} \\ \underline{n} \leq n \leq \overline{n} \\ \underline{V} \leq V(Q_{sgc}, n) \leq \overline{V} \end{array}\right\} \tag{6}$$

where

$f_q(\cdot)$: the total summation of generator fuel costs which is the functions of Q_{sgc} and n.

It is noted that in the P-optimization module Q_{sgc} behaves as dependent variables, while in the Q-optimization P_{sg} is dependent variable.

Since the P- and Q-optimization modules are mutually dependent, and each gives, pproximate optimum values in the intermediate stages, the two modules are alternately iterated until an optimum is obtained and each module is immediately followed by the load-flow calculation module which makes fine adjustments on the approximate results as shown in Figure 1.

As mentioned previously, one of the striking differences of the presented method is that, like the P-optimization module, the Q-optimization module also uses the total fuel cost $f_q(\cdot)$ as the objective function. It is obvious that adopting $f_q(\cdot)$ as the objective function would be more realistic and reasonable than using transmission losses as we do in conventional approaches[6),7),10)]

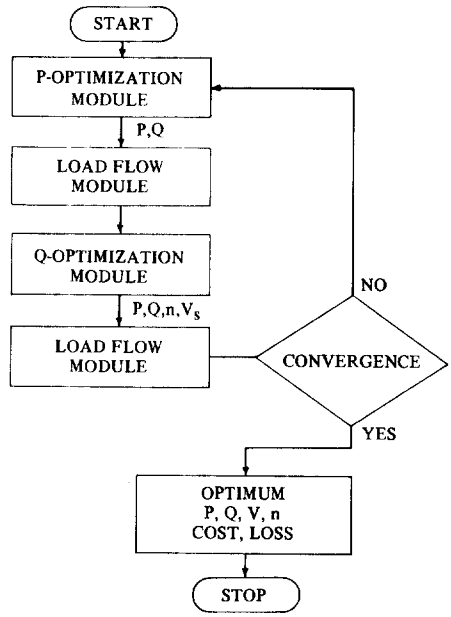

Fig. 1. Computational sequence of three modules.

sine minimizing the power production cost is more economical than minimizing system losses in the cases that the fuel costs required to produce the same quantity of power are different among generating units.

The costs function in Eq. (1) is given by the total summation of generator fuel costs which can be expressed as the quadratic function of generating power P_k for all $k \in G$:

$$C(P_{sg}) = \sum_{k \in G} (a_k + b_k P_k + c_k P_k^2) \tag{7}$$

where

G: set of indices of generator buses including the swing bus,

and it can be expanded in the Taylor series as

$$C(P_{sg} + \Delta P_{sg}) = \sum_{k \in G} [(a_k + b_k P_k + c_k P_k^2) + (b_k + 2c_k P_k)\Delta P_k + c_k \Delta P_k^2 \tag{8}$$

Subtracting Eq. (7) from Eq. (8), we get

$$\Delta C(\Delta P_{sg}) = \sum_{k \in G} [(b_k + 2c_k P_k)\Delta P_k + c_k \Delta P_k^2] \tag{9}$$

or in matrix form

$$\Delta C(\Delta P_{sg}) = \beta_P \Delta P_{sg} + \Delta P_{sg}^T \gamma_P \Delta P_{sg} \tag{10}$$

where

$\beta_P \triangleq [b_1+2c_1P_1, b_2+2c_2P_2, \ldots, b_m+2c_mP_m]$

$\gamma_P \triangleq dia\ [c_1, c_2 \ldots, c_m]$

m: number of generator buses

An important point associated with the cost function is the fact that Eq. (10) is directly usable in the P-optimization module since ΔPsg itself is the decision variable in that module, while in the Q-optimization module ΔP_{sg} should be expressed by a linear combination of the Q-optimization control variables ΔQ_{sgc} and Δn in order to transform Eq. (10) into a function of ΔQ_{sgc} and Δn as described later.

Another important thing that should be mentioned here is that the method presented here utilizes a modified version of the gradient projection method specially developed for the use in the P- and Q-optimization modules. Since this version of the gradient projection method is efficient and powerful in handling many functional equality and inequality constraints and also exhibits finite and reliable convergence characteristics in attaining an optimum, it has shown, through this study, remarkable advantages over other mathematical programming techniques in solving the above real and reactive power optimization problems.

3. P-optimization Module

The nodal difference equation for the bus powers can be expressed in matrix form as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J & J_n \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \\ \hline \Delta n \end{bmatrix} \quad (11)$$

where

J: Jacobian matrix with respect to voltage angles and magnitudes

J_n: Jacobian matrix with respect to off-nominal tap settings

ΔP: vector of chages in bus real powers

ΔQ: vector of changes in bus reactive powers

$\Delta\delta$: vector of changes in bus voltage angles

ΔV: vector of changes in bus voltage magnitudes

Δn: vector of changes in off-nominal tap setting of LTC.

The Jacobian J in Eq. (11) is partitioned as

$$\begin{bmatrix} \Delta P_s \\ \hline \Delta P_g \\ \Delta P_\ell \\ \hline \Delta Q_s \\ \hline \Delta Q_g \\ \hline \Delta Q_c \\ \Delta Q_{\ell'} \end{bmatrix} = \begin{bmatrix} & J_{11} & \\ \hline & J_{12} & \\ \hline & & \\ \hline & J_{13} & \\ \hline & & \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \hline \Delta\delta_g \\ \Delta\delta_\ell \\ \hline \Delta V_s \\ \hline \Delta V_g \\ \hline \Delta V_c \\ \Delta V_{\ell'} \\ \Delta n \end{bmatrix} \quad (12)$$

where

s,g,c: indices for swing bus, other generator buses, and reactive-power compensating device buses, respectively

ℓ,ℓ': indices for all load buses and the load buses excluding reactive power compensating device buses, respectively

Although more exact decomposition can be realized by setting that

$$\Delta P_\ell = \Delta V = \Delta\delta_s = \Delta Q_{\ell'} = \Delta n = 0 \quad (13)$$

the condition $\Delta Q_{\ell'} = 0$ can be shown to destroy the sparsity property of the Jacobian matrices. For that reason and considering the fact that the calculated values of $\Delta Q_{\ell'}$ in the P-optimization module are close to zero, the condition $\Delta Q_{\ell'} = 0$ is relaxed.

The use of conditions of Eq. (13) into Eq. (12) yields the following mutual dependency among real generating powers:

$$\Delta P_s = J_A\ \Delta P_g \quad (14)$$

It is noted that the relation in Eq. (14) replaces the conventional supply and demand balance equation based on the B-coefficient method.

Consequently, from Eq. (10) the P-optimization subproblem in Eqs. (3) and (4) can be redefined as

Minimize

$$\Delta C_P = \beta_P \Delta P_{sg} + \Delta P_{sg}^T \gamma_P \Delta P_{sg} \tag{15}$$

subject to

$$\left.\begin{array}{l} [1 \mid -J_A]\, \Delta P_{sg} = 0 \\ \underline{\Delta P}_{sg} \leq \Delta P_{sg} \leq \overline{\Delta P}_{sg} \end{array}\right\} \tag{16}$$

where

$$\underline{\Delta P}_{sg}\ \underline{\Delta P}_{sg} - P_{sg} \text{ and } \overline{\Delta P}_{sg}\ \overline{\Delta P}_{sg} - P_{sg}$$

After the optimum solution to the P-optimization subproblem is found by using a modified version of the gradient projection method, both the real and reactive powers are updated from P_g, Q_{gc} to $P_g + \Delta P_g$ and $Q_{gc} + \Delta Q_{gc}$. It can be also shown that ΔQ_{gc} is calculated by the following equation:

$$\Delta Q_{gc} = J_B \Delta P_g \tag{17}$$

4. Q-optimization Module

In order to obtain the formulas for the Q-optimization module, J is partitioned, as shown in Eq. (18).

$$\begin{bmatrix} \Delta P_s \\ \Delta P_g \\ \hline \Delta P_\ell \\ \hline \Delta Q_{sgc} \\ \hline \Delta Q_{\ell'} \end{bmatrix} = \begin{bmatrix} & J_{21} & J_{22} \\ \hline & & \\ \hline & J_{23} & J_{24} \\ \hline & & \end{bmatrix} \begin{bmatrix} \Delta\delta_s \\ \Delta\delta_g \\ \hline \Delta\delta_\ell \\ \hline \Delta V \\ \hline \Delta n \end{bmatrix} \tag{18}$$

The sensitivity matrix J_n can be obtained by differentiating the nodal power equations with respect to the off-nominal tap setting values n. In the case when a transofrmer tap n_{pq} is installed one the p side of a line between buses p and q as shown in Figure 2, the sensitivity coefficients

Fig. 2. Convention of off-nominal tap of LTC.

J_{22} and J_{24} constitute J_n.

Although more exact decomposition can be realized by setting that

$$\Delta Q_{\ell'} = \Delta\delta_s = \Delta\delta_g = \Delta P_\ell = 0 \tag{19}$$

But the condition $\Delta P\ell = 0$ is relaxed in order to preserve the sparsity property of the Jacobian matrices, and instead of that the approximate condition $\Delta\delta_e = 0$ is used since these values are negligibly small. The dependent variables for the Q-optimization module, ΔP_s, ΔP_g and ΔV, can be obtained in terms of the control variables, ΔQ_{sge} an Δn, as

$$\begin{bmatrix} \Delta P_s \\ \Delta P_g \end{bmatrix} = \begin{bmatrix} J_C \end{bmatrix} \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \tag{20}$$

$$\Delta V = \begin{bmatrix} J_D \end{bmatrix} \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \tag{21}$$

Consequently, from Eq. (10) the Q-optimization sub-problem in Eqs. (5) and (6) can be redefined as

Minimize

$$\Delta C_q = \beta_q \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} + \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \gamma_Q \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \tag{22}$$

subject to

$$\begin{array}{l} \underline{\Delta Q}_{sgc} \leq \Delta Q_{sgc} \leq \overline{\Delta Q}_{sgc} \\ \Delta \underline{n} \leq \Delta n \leq \bar{n} \end{array} \tag{23}$$

or

$$\Delta \underline{V} < \begin{bmatrix} J_D \end{bmatrix} \begin{bmatrix} \Delta Q_{sgc} \\ \Delta n \end{bmatrix} \le \Delta \overline{V}$$

where

$\Delta \underline{V} \triangleq \underline{V} - V$ $\quad\quad \Delta \overline{V} \triangleq \overline{V} - V$

$\Delta \underline{Q}_{sgc} \triangleq \underline{Q}_{sgc} - Q_{sgc}$ $\quad\quad \Delta \overline{Q}_{sgc} \triangleq \overline{Q}_{sgc} - Q$

$\Delta \underline{n} \triangleq \underline{n} - n$ $\quad\quad \Delta \overline{n} \triangleq \overline{n} - n$

$\beta_Q \triangleq \beta_P J_C$ $\quad\quad \gamma_Q \triangleq J_C^T \gamma_P J_C$

After the optimum solution to the Q-optimization module, both the real, reactive powers and the tap settings are updated from P_g, Q_{gc} and n to $P_g+\Delta P_g$, $Q_{gc}+\Delta Q_{gc}$ and $n+\Delta n$, respectively. It is noted that Q_s need not be updated since the load-flow calculation immediately following gives more exact updated value for that.

5. Load-flow Calculation Module

The above P-and Q-optimization subproblems are solved by using the modified gradient projection method assuming the approximated linearized constraints as given in Eqs. (16) and (23). Their optimum values thus are not exact. Therefore, it is necessary to use the load-flow calculation module in order to make fine adjustments on those optimum values. But the load-flow calculation module is slightly different from the conventional one in the fact that all the buses excluding the swing bus are regarded as P-Q assigned buses irrespective of actual bus types, and the swing bus voltage is assigned optimally by the preceding optimization module.

Finally it is noted that there is no need to update the Jacobian matrix for the load flow calculation purpose, since the matrix also is taken

Table 1. Data and initially assumed values for the 6-bus sample system (100MVA Base).

(A) Operating Conditions and Assumed Initial Values

Bus Number	Bus Voltage Magnitude	Bus Voltage Phase	Full Loads P_ℓ	Full Loads Q_ℓ	Power Output P_{gs}	Power Output Q_{sgc}
1	1.05	0.0	0.000	0.000	0.000	0.000
2	1.10	0.0	0.000	0.000	0.500	0.341
3	1.00	0.0	0.550	0.130	0.000	0.000
4	1.00	0.0	0.000	0.000	0.000	0.000
5	1.00	0.0	0.300	0.180	0.000	0.000
6	1.00	0.0	0.500	0.050	0.000	0.000

(B) System Paramteters

Line Number	From Bus Number	To Buss Number	Line Impedance R	Line Impedance X	Tap Setting
1	1	6	0.123	0.518	–
2	1	4	0.080	0.370	–
3	4	6	0.097	0.407	–
4	6	5	0.000	0.300	1.025
5	5	2	0.282	0.640	–
6	2	3	0.723	1.050	–
7	4	3	0.000	0.133	1.100

(C) Assumed Fuel Cost Coefficients

Unit Number	First Study a	First Study b	First Study c	Second Study a	Second Study b	Second Study c
1	0	1	0	0	1	0.05
2	0	1	0	0	1	0.01

used was the same 6-bus sample system used by Mamandur and Chenoweth,[10] which is shown in Figure 3, and the data for the system is given in Table 1.

as the objective function. However, the second study is aimed to present a more realistic power dispatch by rising the fuel cost coefficients as shown in Table 1(C).

The result on convergence characteristics of the first study is in Table II and shows that the total transmission loss and the total power generation decrease monetonically until the optimum is found in four iterations.

It is important to note that the final transmission loss is 8.115[MW] which is smaller than 8.93[MW], the result obtained by Mamandur and Cheonoweth.[10] The computer results of all system variables for this case are summarized in Table 3. Again, to make a comparison with the results in Reference 10), the same study is made for 1/2 load level. Table 3 shows that the transmission loss is 1.111[MW] as compared to 2.24

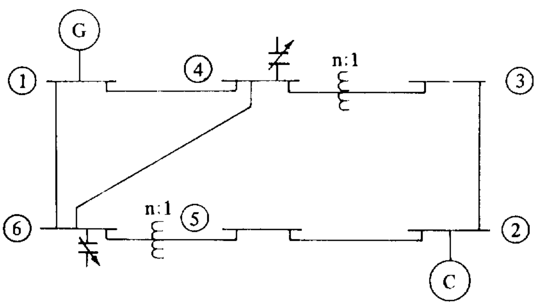

Fig. 3. The 6-bus network for simulation

Table 2. Convergence characteristics in both study cases

Iteration		First Study Loss MW	(Full Load) Cost p.u.	Second Study Loss MW	Cost p.u.
Initial		10.828	145.828	10.828	150.670
1	a	11.112	146.112	11.840	151.554
	b	10.079	145.079	10.761	150.134
2	a	9.682	144.681	11.323	150.539
	b	10.434	145.434	11.403	150.839
3	a	10.860	145.860	11.763	151.023
	b	11.010	146.010	8.644	147.765
4	a	10.429	145.429	8.644	147.765
	b	8.115	143.115	8.644	147.765

a = after the P-optimization
b = after the Q-optimization

[MW] of Reference 10)

The second study is the general case of optimal real and reactive power dispatch, and for this purpose the fuel cost coefficients listed in Table 1(C) are used. Here, it is noted that the transmission loss is slightly increased (from 8.115[MW] to 8.644[MW] compared to the first case study in order to minimize the total fuel cost rather than the total power (which is the case of the first study). In this case the total cost in Tables 2 and 3 represents the fuel costs in per unit, while the same represents the total power generations for the first study.

The result on convergence characteristics for the second study is compared with the first study in Table 2 and shows that the optimum is found in three iteration. To compare other system variables with the first study, the results are tabulated in Table 3.

The simulation was performed on the NEC AS/9000N system. Time per iteration for the first study was about 0.061 second compared to 0.208 second of Reference 10). An important thing to be added is that the presented method uses two iptimizations (P-and Q-optimization) per iteration, while the method in Reference 10) uses only one optimization per iteration, and yet the new method is much faster than the other. This suggests a bright promise of using the new method for on-line application to the optimal load flow calculations. Time per iteration for the second study was about 0.103 second which is a slight increase from the first study case. Since in the second case the objective function is no longer linear, but a quadratic (nonlinear) function, the optimization takes longer time within the gradient projection algorithm.

7. Conclusions

It is concluded from this study that the new methodology presented here has distinct advantages summarized below.

Table 3. Summary of results for two study cases.

Variable	Limits		First Study-Full Load		First Study-Half Load		Second Study-Full Load	
	Lower	Upper	Initial	Final	Initial	Final	Initial	Final
P_1(MW)	10.	100.	95.828	99.131	45.557	49.149	95.829	87.205
P_2(MW)	10.	100.	50.000	43.984	25.000	19.462	50.000	56.438
Q_1(MVAR)	-20.	100.	36.541	38.011	4.621	7.989	36.541	28.254
Q_2(MVAR)	-20.	100.	34.100	23.385	21.600	15.796	34.100	39.075
Q_{C4}(MVAR)	0.	5.0	0.000	5.000	0.0	1.645	0.000	1.141
Q_{C6}(MVAR)	0.	5.5	0.000	5.500	0.0	1.239	0.000	0.335
n_4	0.900	1.100	1.025	0.976	1.025	1.069	1.025	1.051
n_7	0.900	1.100	1.100	0.998	1.100	1.063	1.100	1.088
V_1(P.U.)	1.00	1.10	1.05	1.02	1.05	1.00	1.05	1.07
V_2(P.U.)	1.00	1.15	1.10	1.07	1.10	1.01	1.10	1.15
V_3(P.U.)	0.90	1.00	0.86	0.91	0.94	0.91	0.86	0.90
V_4(P.U.)	0.90	1.00	0.96	0.92	1.03	0.97	0.96	0.99
V_5(P.U.)	0.90	1.00	0.90	0.90	1.00	0.90	0.90	0.93
V_6(P.U.)	0.90	1.00	0.94	0.90	1.02	0.90	0.94	0.98
Number of Iterations			4		3		3	
Cost Function (P.U.)			145.828	143.115	70.557	68.611	150.670	147.765
Loss (MW)			10.878	8.115	3.057	1.111	10.828	8.644

1. More reasonable performance measure – In contrast with most of the conventional methods which adopt the total fuel costs as the performance measure for real power optimization and the total transmission loss as that for reactive power optimization, the method presented here utilizes the same performance measure (the total fuel costs) for both optimizations, thus resulting in the adoption of more realistic and reasonable performance. It should be noted that minimizing the power production cost is more economical than minimizing system loss if the fuel costs required to produce the same quantity of power are different among generating units. In other words, the minimization of loss is equivalent to the minimization of power generation for a given load, but not to the fuel cost minimization.
2. More economical reallocation of real powers in the case of reactive power optimization – In contrast with most of the conventional methods which, for reactive power optimization, fix all nonswing bus real powers and change only the swing bus real power to absorb the reduction in transmission losses, the Q-optimization method presented here optimally reallocates all generator real powers because of the performance measure being defined as the total fuel cost.
3. Non-fixed swing-bus voltage – In some of the conventional algorithms the swing-bus voltage remains fixed in the reactive power optimization procedure, but the method presented here optimally determines the swing-bus voltage, along with any other bus voltages.

4. More accurate algorithm – The conventional B-coefficients for transmission loss formula are functionally replaced by the more accurate sensitivity matrix which is successively updated during the iteration.
5. Fast computation – The method presented here, in spite of a lot of matrix operations, still preserves the highly-sparse characteristics of system Jacobian matrices, and thus makes it possible to use the optimally-ordered triangular factorization technique which allows for handling of a large power system with faster computaton.
6. Application of gradient projection method – A modified version of the gradient projection method developed for this study is shown to be efficient and highly reliable for finding an optiumum in either optimization process.
7. Possibility for on-line applications – The fast and reliable characteristics in computation mentioned above present the future possibility for its on-line applications, such as the economic load dispatch, reactive power-voltage control, online optimal load flow, etc.

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