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## A Study of Wall Temperature Profiles for a Cryogenic Cylindrical Storage Tank

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저온용 원통형 저장탱크의 벽온도 분포에 관한 연구

손병진 · 박희용 · 이관수 · 김우승

### 초 록

본 논문은 저온용 원통형 저장 탱크를 2층으로 된 적층형 복합재료로 가정하여 각 층의 온도분포를 해석하였다.

이 중공 원통의 외벽에서는 원주방향으로 임의의 열유속을 받고, 주위 온도는 주기적으로 변하며 내외벽에서는 대류가 일어나고, 탱크벽의 초기온도 분포는 임의의 함수라는 가정하에 Fourier cosine 변환과 Green 함수를 도입하여 해석하였다.

### Nomenclature

<p><math>A</math> : Dimensionless amplitude parameter</p> <p><math>A_{1n}, B_{1n}</math> : Constants</p> <p><math>c</math> : Specific heat at constant pressure (J/kg.K)</p> <p><math>G_1, G_2</math> : Initial temperature distribution</p> <p><math>G_{ij}</math> : Green's function</p> <p><math>h_0</math> : Convection heat transfer coefficient in the fluid inside the tank (W/m<sup>2</sup>.K)</p> <p><math>h_3</math> : Convection heat transfer coefficient in the fluid outside the tank (W/m<sup>2</sup>.K)</p> <p><math>H_0</math> : Value defined in eq. (14)</p> <p><math>H_3</math> : Value defined in eq. (14)</p> <p><math>J_m(x)</math> : Bessel function of the first kind of order <math>m</math> and of argument <math>x</math></p> <p><math>Y_m(x)</math> : Bessel function of the second kind of order <math>m</math> and of argument <math>x</math></p> <p><math>k</math> : Thermal conductivity (W/m.K)</p> <p><math>K</math> : Value defined in eq. (14)</p> <p><math>m</math> : Integer</p>	<p><math>N_n</math> : Norm</p> <p><math>q</math> : Heat flux (W/m<sup>2</sup>)</p> <p><math>q^*</math> : Heat flux defined in eq. (3d) (W/m<sup>2</sup>)</p> <p><math>r</math> : Radius (m)</p> <p><math>r'</math> : Dummy variable for radius from reference point</p> <p><math>t</math> : Time (s)</p> <p><math>t'</math> : Dummy variable for time</p> <p><math>T</math> : Temperature in the layer at radius <math>r</math>, angular displacement <math>\theta</math> and time <math>t</math> (K)</p> <p><math>T_0</math> : Temperature of the fluid inside the tank(K)</p> <p><math>T_m</math> : Mean temperature of the surrounding fluid (K)</p> <p><math>T_\infty(t)</math> : Temperature of the surrounding fluid at time <math>t</math> (K)</p> <p><math>u</math> : Excess temperature defined in eq. (5a)</p> <p><math>\alpha</math> : Thermal diffusivity (m<sup>2</sup>/s)</p> <p><math>\gamma_n</math> : Value defined in eq. (14)</p> <p><math>\xi_n</math> : Value defined in eq. (14)</p> <p><math>\eta_n</math> : Value defined in eq. (14)</p> <p><math>\theta</math> : Angular displacement (°)</p> <p><math>\theta'</math> : Dummy variable for angular displacement</p>
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- $\lambda_n$  : Eigenvalue  
 $\rho$  : Density (kg/m<sup>3</sup>)  
 $\bar{\psi}_{in}$  : Eigenfunction  
 $\psi_{in}$  : Transformed eigenfunction  
 $\omega$  : Frequency

#### Subscript

- $i$  : Value of  $i$ -th layer ( $i=1,2$ )

## 1. Introduction

As the uncertainty of oil supply and the price of oil have been increasing, active studies for the development of alternate sources of energy are beginning in many countries, and LNG (Liquefied Natural Gas), one of the alternate sources of energy is expected to be used as the supply source of fuel for power plants and city gas in our country.

Since LNG must be kept at a low temperature (below-162°C) inside the tank, thermal insulation and the selection of cryogenic inner tank materials in contact with LNG are very important.<sup>(1,2)</sup>

The purpose of the insulation is to maintain the cryogenic liquid in the tank with as little boil-off loss as possible due to heat leakage. In most of the typical tanks, boil-off will amount to as little as 0.05% to the tank capacity per day.<sup>(2)</sup>

In view of the importance of cryogenic thermal insulation, there is little research on heat transfer by assuming a cryogenic storage tank as a two-layered composite.

In the modern design, the inner part of the tank in contact with LNG is constituted by a stainless steel membrane composed of a network of orthogonal corrugations allowing for the free thermal contraction of the metal and the external part of the tank is made of a prestressed concrete resisting tank. Between

the steel membrane and the prestressed concrete, there is a rigid supporting insulation such a polyurethane foam.<sup>(3,4)</sup>

In the modern design, since the membrane made of a stainless steel is much thinner and has a higher thermal conductivity than the materials of the external part of the tank and its insulation, the structure of the tank can be considered as a two-layered composite cylinder.

The treatment of problems in diffusion through composite media have been handled by many authors.<sup>(6,7)</sup>

G.P. Mulholland and M.H. Cobble<sup>(6)</sup> use a unique dependent variable substitution and the Vodicka type of orthogonality relationship to solve the temperature distribution in each of  $k$  sections of a composite with internal heat generation and boundary condition of the third kind in composite media having one-dimensional heat flow. J.D. Lockwood and G.P. Mulholland<sup>(7)</sup> use a Fourier integral transformation, a dependent variable substitution and the Vodicka type of orthogonality relationship to solve the temperature distribution in each of  $k$  sections of a hollow laminated composite cylinder with a circumferentially varying external heat flux. The temperature on the inside surface of the composite is an arbitrary function of time.

In the present work, the temperature distribution in the wall of a cryogenic cylindrical storage tank is considered as a two-layered composite, with the convection heat transfer from both an inside and an outside of the wall and with an arbitrary external heat flux varying both with circumferential displacement and with time in the outside, and with periodic variation of outside ambient temperature and initial temperature distribution assumed as an arbitrary function. It is mathe-

matically analyzed with a Fourier's cosine transform and an appropriate Green's function.

**2. Statement of the Problem**

The physical situation and the coordinate system used in the problem to be analyzed are shown in Fig. 1.

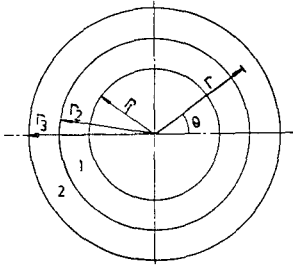


Fig. 1 Physical model and coordinate system.

In the present analysis, the following assumptions are made to solve the heat diffusion equation of a two-layered composite.

- (1) Heat flow is two-dimensional  $(r, \theta)$
- (2) Each layer is homogeneous.
- (3) There is no interfacial thermal contact resistance.
- (4) All of the thermophysical properties are constant; but the thermophysical properties of each layer have different values.
- (5) The ambient temperature outside the outer wall,  $T_\infty(t)$ , oscillates around a mean temperature,  $T_m$ , which is greater than the fluid temperature,  $T_0$ , within the inner wall.

Under the above assumptions, the heat diffusion equation for each layer is mathematically described in the following way

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_1}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_1}{\partial \theta^2} = \frac{1}{\alpha_1} \frac{\partial u_1}{\partial t} \quad \text{in } r_1 < r < r_2, t > 0 \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_2}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_2}{\partial \theta^2}$$

$$= \frac{1}{\alpha_2} \frac{\partial u_2}{\partial t} \quad \text{in } r_2 < r < r_3, t > 0 \quad (2)$$

The boundary conditions are

$$-k_1 \frac{\partial u_1}{\partial r} + h_0 u_1 = 0 \quad \text{at } r = r_1 \quad (3a)$$

$$u_1(r, \theta, t) = u_2(r, \theta, t) \quad \text{at } r = r_2 \quad (3b)$$

$$k_1 \frac{\partial u_1}{\partial r} = k_2 \frac{\partial u_2}{\partial r} \quad \text{at } r = r_2 \quad (3c)$$

$$k_2 \frac{\partial u_2}{\partial r} + h_3 u_2 = h_3 u_\infty(t) + q(\theta, t) = q^*(\theta, t) \quad \text{at } r = r_3 \quad (3d)$$

$$u_1(r, \theta, t) = u_1(r, \theta + 2\pi, t) \quad (3e)$$

$$u_2(r, \theta, t) = u_2(r, \theta + 2\pi, t) \quad (3f)$$

$$\frac{\partial u_1}{\partial \theta}(r, \theta, t) = \frac{\partial u_1}{\partial \theta}(r, \theta + 2\pi, t) \quad (3g)$$

$$\frac{\partial u_2}{\partial \theta}(r, \theta, t) = \frac{\partial u_2}{\partial \theta}(r, \theta + 2\pi, t) \quad (3h)$$

The initial conditions are

$$u_1 = G_1(r, \theta) \quad \text{at } t = 0 \quad (4a)$$

$$u_2 = G_2(r, \theta) \quad \text{at } t = 0 \quad (4b)$$

where

$$u_i = T_i(r, \theta, t) - T_0 \quad (i=1, 2) \quad (5a)$$

$$u_\infty(t) = T_\infty(t) - T_0 \quad (5b)$$

$$T_\infty(t) = T_m + A(T_m - T_0) \cos \omega t, \quad 0 \leq A < 1 \quad (5c)$$

where A is the dimensionless amplitude parameter and  $\omega/2\pi$  is the frequency of oscillation.

**3. Analysis**

In order to analyze the heat diffusion equation by Green's function, the following eigenvalue problems must be solved

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi_{in}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi_{in}}{\partial \theta^2} + \frac{\lambda_n^2}{\alpha_i} \psi_{in}(r, \theta) = 0, \quad i=1, 2 \quad (6)$$

$$-k_1 \frac{\partial \psi_{1n}}{\partial r} + h_0 \psi_{1n} = 0 \quad \text{at } r = r_1 \quad (7a)$$

$$\psi_{1n}(r, \theta) = \psi_{2n}(r, \theta) \quad \text{at } r = r_2 \quad (7b)$$

$$k_1 \frac{\partial \psi_{1n}}{\partial r} = k_2 \frac{\partial \psi_{2n}}{\partial r} \quad \text{at } r = r_2 \quad (7c)$$

$$k_2 \frac{\partial \psi_{2n}}{\partial r} + h_3 \psi_{2n} = 0 \quad \text{at } r = r_3 \quad (7d)$$

$$\psi_{1n}(r, \theta) = \psi_{1n}(r, \theta + 2\pi) \quad (7e)$$

$$\psi_{2n}(r, \theta) = \psi_{2n}(r, \theta + 2\pi) \quad (7f)$$

$$\frac{\partial \psi_{1n}}{\partial \theta}(r, \theta) = \frac{\partial \psi_{1n}}{\partial \theta}(r, \theta + 2\pi) \quad (7g)$$

$$\frac{\partial \psi_{2n}}{\partial \theta}(r, \theta) = \frac{\partial \psi_{2n}}{\partial \theta}(r, \theta + 2\pi) \quad (7h)$$

Multiplying both sides of equations (6) and (7) by the operator  $\int_0^{2\pi} \cos m\theta d\theta$  the following transformed eigenvalue problems are obtained

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{d\bar{\psi}_{in}}{dr} \right) + \left( \frac{\lambda_n^2}{\alpha_i} - \frac{m^2}{r^2} \right) \bar{\psi}_{in}(r, m) = 0 \quad i=1, 2 \quad (8)$$

$$-k_1 \frac{d\bar{\psi}_{1n}}{dr} + h_0 \bar{\psi}_{1n} = 0 \quad \text{at } r=r_1 \quad (9a)$$

$$\bar{\psi}_{1n}(r, m) = \bar{\psi}_{2n}(r, m) \quad \text{at } r=r_2 \quad (9b)$$

$$k_1 \frac{d\bar{\psi}_{1n}}{dr} = k_2 \frac{d\bar{\psi}_{2n}}{dr} \quad \text{at } r=r_2 \quad (9c)$$

where

$$[a] = \left( \begin{array}{cc|cc} J_{m-1}(\xi_n) - \frac{1}{\xi_n}(m+H_0) & Y_{m-1}(\xi_n) - \frac{1}{\xi_n}(m+H_0) & 0 & 0 \\ J_m(\xi_n) & Y_m(\xi_n) & & \\ & J_m(\gamma_n) & -J_m\left(\frac{r_2}{r_3}\eta_n\right) & -Y_m\left(\frac{r_2}{r_3}\eta_n\right) \\ K \left[ \begin{array}{c} J_{m-1}(\gamma_n) \\ -\frac{m}{\gamma_n} J_m(\gamma_n) \end{array} \right] & K \left[ \begin{array}{c} Y_{m-1}(\gamma_n) \\ -\frac{m}{\gamma_n} Y_m(\gamma_n) \end{array} \right] & m \frac{r_3}{r_2} \frac{1}{\eta_n} J_m\left(\frac{r_2}{r_3}\eta_n\right) & m \frac{r_3}{r_2} \frac{1}{\eta_n} Y_m\left(\frac{r_2}{r_3}\eta_n\right) \\ & 0 & -J_{m-1}\left(\frac{r_2}{r_3}\eta_n\right) & -Y_{m-1}\left(\frac{r_2}{r_3}\eta_n\right) \\ & & J_{m-1}(\eta_n) - \frac{1}{\eta_n}(m-H_3) & Y_{m-1}(\eta_n) - \frac{1}{\eta_n}(m-H_3) \\ & & J_m(\eta_n) & Y_m(\eta_n) \end{array} \right) \quad (13a)$$

$$[x] = \begin{pmatrix} 1 \\ B_{1n} \\ A_{2n} \\ B_{2n} \end{pmatrix} \quad (13b)$$

here

$$\xi_n = \frac{r_1 \lambda_n}{\sqrt{\alpha_1}}, \quad \gamma_n = \frac{r_2 \lambda_n}{\sqrt{\alpha_1}}, \quad \eta_n = \frac{r_3 \lambda_n}{\sqrt{\alpha_2}} \quad (14)$$

$$H_0 = \frac{r_1 h_0}{k_1}, \quad H_3 = \frac{r_3 h_3}{k_2}, \quad K = \frac{k_1}{k_2} \sqrt{\frac{\alpha_2}{\alpha_1}}$$

The first two and the fourth of the equation

$$k_2 \frac{d\bar{\psi}_{2n}}{dr} + h_3 \bar{\psi}_{2n} = 0 \quad \text{at } r=r_3 \quad (9d)$$

where

$$\bar{\psi}_{in}(r, m) = \int_0^{2\pi} \psi_{in}(r, \theta) \cos m\theta d\theta \quad (10)$$

The general solution of equation (8) is taken as

$$\bar{\psi}_{in}(r, m) = A_{in} J_m \left( \frac{\lambda_n}{\sqrt{\alpha_i}} r \right) + B_{in} Y_m \left( \frac{\lambda_n}{\sqrt{\alpha_i}} r \right) \quad i=1, 2 \quad (11)$$

In order to determine four coefficients,  $A_{in}$ ,  $B_{in}$  with  $i=1, 2$ , the eigenfunctions  $\bar{\psi}_{in}(r, m)$  given by equation (11) with  $A_{1n}=1$  without loss of generality are inserted into the equation (9).

The resulting system of equations can be represented as a following matrix notation

$$[a][x] = [0] \quad (12)$$

(12) can be used to determine the coefficients.

The coefficients are

$$B_{1n} = - \frac{J_{m-1}(\xi_n) - \frac{1}{\xi_n}(m+H_0) J_m(\xi_n)}{Y_{m-1}(\xi_n) - \frac{1}{\xi_n}(m+H_0) Y_m(\xi_n)} \quad (15a)$$

$$A_{2n} = \frac{D_{A2}}{D} \quad (15b)$$

$$B_{2n} = \frac{D_{B2}}{D} \quad (15c)$$

here

$$D = \left[ Y_{m-1}(\xi_n) - \frac{1}{\xi_n}(m + H_0) Y_m(\xi_n) \right] \cdot \left[ \frac{1}{\eta_n}(m - H_3) Q_m - Q_{m,m-1} \right] \quad (16a)$$

$$D_{A2} = \left[ Y_{m-1}(\eta_n) - \frac{1}{\eta_n}(m - H_3) Y_m(\eta_n) \right] \cdot \left[ P_{m-1,m} - \frac{1}{\xi_n}(m + H_0) P_m \right] \quad (16b)$$

$$D_{B2} = - \left[ J_{m-1}(\eta_n) - \frac{1}{\eta_n}(m - H_3) J_m(\eta_n) \right] \cdot \left[ P_{m-1,m} - \frac{1}{\xi_n}(m + H_0) P_m \right] \quad (16c)$$

Equation (12) has a non-trivial solution when the determinant of [a] is equal to zero. After setting this determinant equal to zero, an expanded form of this determinant is then given as

$$\begin{aligned} & \left[ \frac{1}{\eta_n}(m - H_3) Q_{m-1,m} - Q_{m-1} \right] \cdot \\ & \left[ P_{m-1,m} - \frac{1}{\xi_n}(m + H_0) P_m \right] \\ & + \left[ Q_{m,m-1} - \frac{1}{\eta_n}(m - H_3) Q_m \right] \cdot \\ & \left[ m \left( \frac{r_3}{r_2} \frac{1}{\eta_n} - \frac{K}{r_n} \right) \left( P_{m-1,m} - \frac{1}{\xi_n}(m + H_0) P_m \right) + K \left( P_{m-1} - \frac{1}{\xi_n}(m + H_0) P_{m,m-1} \right) \right] = 0 \end{aligned} \quad (17)$$

for determining the eigenvalue where

$$\begin{aligned} Q_m &= J_m \left( \frac{r_2}{r_3} \eta_n \right) Y_m(\eta_n) - Y_m \left( \frac{r_2}{r_3} \eta_n \right) J_m(\eta_n) \\ Q_{m,m-1} &= J_m \left( \frac{r_2}{r_3} \eta_n \right) Y_{m-1}(\eta_n) - Y_m \left( \frac{r_2}{r_3} \eta_n \right) J_{m-1}(\eta_n) \\ Q_{m-1,m} &= J_{m-1} \left( \frac{r_2}{r_3} \eta_n \right) Y_m(\eta_n) - Y_{m-1} \left( \frac{r_2}{r_3} \eta_n \right) J_m(\eta_n) \\ Q_{m-1} &= J_{m-1} \left( \frac{r_2}{r_3} \eta_n \right) Y_{m-1}(\eta_n) - Y_{m-1} \left( \frac{r_2}{r_3} \eta_n \right) J_{m-1}(\eta_n) \\ P_m &= J_m(\xi_n) Y_m(\gamma_n) - Y_m(\xi_n) J_m(\gamma_n) \\ P_{m,m-1} &= J_m(\xi_n) Y_{m-1}(\gamma_n) - Y_m(\xi_n) J_{m-1}(\gamma_n) \end{aligned} \quad (18)$$

$$\begin{aligned} P_{m-1,m} &= J_{m-1}(\xi_n) Y_m(\gamma_n) - Y_{m-1}(\xi_n) J_m(\gamma_n) \\ P_{m-1} &= J_{m-1}(\xi_n) Y_{m-1}(\gamma_n) - Y_{m-1}(\xi_n) J_{m-1}(\gamma_n) \end{aligned}$$

Expressing the function  $\psi_{in}(r, \theta)$  in terms of the transformed function  $\bar{\psi}_{in}(r, m)$  by using Fourier cosine series, the following equation is obtained

$$\psi_{in}(r, \theta) = \frac{1}{2\pi} \bar{\psi}_{in}(r, 0) + \frac{1}{\pi} \sum_{m=1}^{\infty} \bar{\psi}_{in}(r, m) \cos m\theta \quad (19)$$

The corresponding Green's function  $G_{ij}(r, \theta, t | r', \theta', t')$  of the problem is<sup>(8)</sup>

$$G_{ij}(r, \theta, t | r', \theta', t') = \sum_{n=1}^{\infty} \frac{1}{N_n} \frac{k_j}{\alpha_j} e^{-\lambda_n^2(t-t')} \psi_{in}(r, \theta) \psi_{in}(r', \theta') \quad (i=1, 2, j=1, 2) \quad (20)$$

Where norm  $N_n$  is given as

$$\begin{aligned} N_n &= \frac{k_1}{\alpha_1} \int_{\theta'=0}^{2\pi} \int_{r'=r_1}^{r_2} [\psi_{1n}(r', \theta')]^2 r' dr' d\theta' \\ &+ \frac{k_2}{\alpha_2} \int_{\theta'=0}^{2\pi} \int_{r'=r_2}^{r_3} [\psi_{2n}(r', \theta')]^2 r' dr' d\theta' \end{aligned} \quad (21)$$

Therefore, the temperature distribution of each layer described by Green's function is

$$\begin{aligned} u_i(r, \theta, t) &= \int_{\theta'=0}^{2\pi} \int_{r'=r_1}^{r_2} G_{i1}(r, \theta, t | r', \theta', t') \\ & \left. \int_{t'=0}^t G_1(r', \theta') r' dr' d\theta' + \int_{\theta'=0}^{2\pi} \int_{r'=r_2}^{r_3} G_{i2}(r, \theta, t | r', \theta', t') \right|_{t'=0} G_2(r', \theta') r' dr' d\theta' \\ &+ \alpha_2 \int_{t'=0}^t dt' \int_{\theta'=0}^{2\pi} \frac{G_{i2}(r, \theta, t | r', \theta', t')}{k_2} \left. \int_{r'=r_2}^{r_3} q^*(\theta', t') r_3 d\theta' \right|_{r'=r_2} \end{aligned} \quad (22)$$

Substituting equation (20) into equation (22), and after some modification equation (22) can be rewritten as

$$\begin{aligned} u_i(r, \theta, t) &= \sum_{n=1}^{\infty} \frac{1}{N_n} e^{-\lambda_n^2 t} \psi_{in}(r, \theta) [\bar{G}(\lambda_n) + R(\lambda_n, t)], \quad i=1, 2 \quad (23) \\ \bar{G}(\lambda_n) &= \frac{k_1}{\alpha_1} \int_{\theta'=0}^{2\pi} \int_{r'=r_1}^{r_2} r' \psi_{1n}(r', \theta') G_1(r', \theta') dr' d\theta' + \frac{k_2}{\alpha_2} \int_{\theta'=0}^{2\pi} \int_{r'=r_2}^{r_3} \end{aligned}$$

$$r' \psi_{2n}(r', \theta') G_2(r', \theta') dr' d\theta' \quad (24a)$$

$$R(\lambda_n, t) = \frac{1}{\lambda_n^2} \left\{ e^{\lambda_n^2 t} \int_{\theta'=0}^{2\pi} r_3 \psi_{2n}(r_3, \theta') q^* (\theta', t) d\theta' - \int_{\theta'=0}^{2\pi} r_3 \psi_{2n}(r_3, \theta') q^* (\theta') d\theta' - \int_{t'=0}^t e^{\lambda_n^2 t'} \int_{\theta'=0}^{2\pi} r_3 \psi_{2n}(r_3, \theta') dq^* (\theta', t') d\theta' dt' \right\} \quad (24b)$$

Thus, the temperature distribution of each layer is determined.

#### 4. Numerical Example

Consider the sustained solution where the LNG cylindrical storage tank is subjected to a heat flux,

$$q(\theta, t) = 0$$

at the outside wall of the tank surrounded by a dike as an example.

The sustained temperature difference between the inner wall and the inside fluid is

$$u_1(r_1, t) = \sum_{n=1}^{\infty} \frac{1}{N_n} \psi_{1n}(r_1) \psi_{2n}(r_3) \frac{r_3}{\lambda_n^2} h_3 (T_m - T_0) \left\{ 1 + A \cos \omega t + \frac{Aw}{\lambda_n^4 + \omega^2} (\lambda_n^2 \sin \omega t - \omega \cos \omega t) \right\} \quad (25)$$

The eigen condition is obtained by substituting  $m=0$  into subscript  $m$  in equation (17) and (18), since  $q(\theta, t) = 0$ .

In this case, we are interested in heat flux at the inner wall. The heat flux at the inner wall and the temperature difference between the inside fluid and the inner surface of the wall, are calculated by use of heat flux =  $h_0 \times u_1(r_1, t)$ , equation (25), and the following data.

$$\begin{aligned} \rho_1 &= 32 \text{ kg/m}^3, & \rho_2 &= 2300 \text{ kg/m}^3 \\ c_1 &= 1714.62 \text{ J/kg} \cdot \text{K}, & c_2 &= 920.04 \text{ J/kg} \cdot \text{K} \\ k_1 &= 0.0198 \text{ W/m} \cdot \text{K}, & k_2 &= 1.5102 \text{ W/m} \cdot \text{K} \end{aligned}$$

$$\begin{aligned} \alpha_1 &= 3.59 \times 10^{-7} \text{ m}^2/\text{s} & \alpha_2 &= 7.14 \times 10^{-7} \text{ m}^2/\text{s} \\ T_0 &= 109.15 \text{ K}, & T_m &= 293.15 \text{ K} \\ h_0 &= 100 \text{ W/m}^2 \cdot \text{K}, & h_3 &= 5 \text{ W/m}^2 \cdot \text{K} \\ r_1 &= 31.38 \text{ m}, & r_2 &= 31.56 \text{ m} \\ r_3 &= 32.46 \text{ m} \end{aligned}$$

The effects of the amplitude parameter  $A$  on the heat flux are shown in Fig. 2.

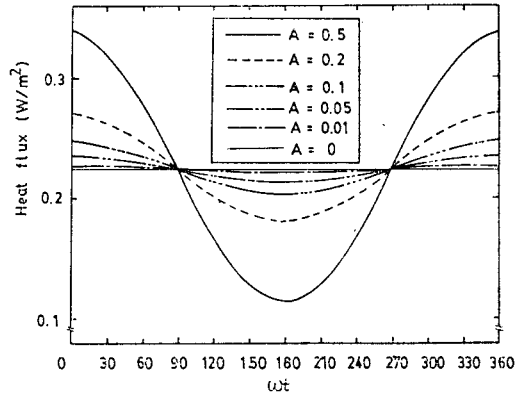


Fig. 2 Effect of parameter A on heat flux at  $h_0=100$  ( $\text{w/m}^2 \cdot \text{k}$ ).

In the numerical example, the eigenvalue is obtained by using the bisection method and incremental-search method from the eigen condition in order to determine the value of equation (25) given by this analysis. The series value of the equation (25) is obtained by summing each term from the first term to the nth term, when the value of the nth term is 1/100000, or the value of

$$\left| \frac{\text{nth term} - (n-1)\text{th term}}{(n-1)\text{th term}} \right| \text{ is } 1/100.$$

#### 5. Conclusion

An analytical method using a Fourier cosine transformation and a Green's function has been developed to determine the temperature distribution in the wall of a cryogenic cylindrical storage tank assumed as a two-layered composite, with the convection heat transfer from both an inside and an outside of the

wall, with an arbitrary external heat flux varying both with circumferential displacement and with time on the outside of the wall, and with the periodic variation of outside ambient temperature and the initial temperature distribution assumed as an arbitrary function.

The results of this study can be applied in the design of double-wall tank because one can predict the boil-off losses and the required days of liquefaction plant operation to completely make up annual boil-off losses by obtaining the amount of heat leak into a tank.

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