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A New Simple Technique for View Factor Computation

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간단한 복사 형상계수 계산 방법

조 성 환

초 록

복사 형상계수를 계산하는 새롭고도 간단한 수치적 방법이 개발되었다. 유한선분 적분법은 윤곽 적분을 이용하며, 윤곽은 유한한 수의 선분으로 구성된 것으로 가정한다. 미소면적으로부터 유한면적까지의 복사 형상계수는 적분로 상의 절점의 좌표값에만 관계되며, 전자 계산기에 쉽게 프로그램될 수 있다. 가우스의 적분을 이용하여 두 유한 면적사이의 복사 형상계수를 구한다. 미소 면적에서 원판까지, 두개의 평행원판 사이, 및 두개의 직사각형 사이의 복사 형상계수를 구하여 엄밀해와 비교하여 유한선분 적분법의 정확성이 우수함을 보였다.

단위구와 단위 정사각형에서 타원체까지의 복사 형상계수의 값도 구하였다.

Nomenclature

A	: Area
a	: Major or minor axis of ellipsoid in x-direction
b	: Major or minor axis of ellipsoid in y-direction
C	: Contour of integration
c	: Major or minor axis of ellipsoid in z-direction
f	: Factor of integration for area integral
$F_{dA_1-A_2}$	: Radiative view factor from a differential area to a finite area
$F_{A_1-A_2}$	: Radiative view factor between two finite areas
$\hat{i},\hat{j},\hat{k}$	: unit vectors in x,y, and z-directions, respectively
$l,m,n,$	: Directional cosines
M	: Number of Gaussian quadrature points
N	: Number of nodal points on contour C
$\overline{N}$	: Normal vector to the surface of ellipsoid
$\overline{P}$	: Position vector of a point on the surface of ellipsoid
r	: Distance between dA_1 and a point on contour C
s	: Scalar parameter defined by equation (4)
u,v	: Parameters defined by equation (A2)
w	: Gaussian weight of integration
x,y,z	: Space coordinates

Subscripts

1	: Body 1
2	: Body 2

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1. Introduction

Many analytic and numerical studies on radiative view factors have been reported [1-5]. These studies, however, are not applicable for general shapes. For example, Chung and Naraghi [3] developed a formular to compute radiative view factor from a sphere to an axisymmetric body. Their method cannot be applied to a non-axisymmetric body.

In this paper a new simple numerical technique for radiative view factor computation is developed. This technique, the finite line integral method, is applicable for general shapes and easily programmed for a digital computer.

2. The Finite Line Integral Method

Radiative view factor from a differential area to a finite area can be computed by the contour integral developed by Sparrow [2].

$$F_{dA_1-A_2}=l_1\oint_C \frac{(z_2-z_1)dy_2-(y_2-y_1)dz_2}{2\pi r^2}$$
$$+m_1\oint_C \frac{(x_2-x_1)dz_2-(z_2-z_1)dx_2}{2\pi r^2}$$
$$+n_1\oint_C \frac{(y_2-y_1)dx_2-(x_2-x_1)dy_2}{2\pi r^2} \quad (1)$$

The line of integration C is the contour of A_2 which can be seen directly from dA_1. Symbols are given in Nomenclature.

By translation and rotation of coordinate system, it is always possible to make the origin of the coordinate system to be at dA_1, and z-axis coincides with the normal direction to dA_1, in which case $x_1=y_1=z_1=0$; $l_1=m_1=0$, and $n_1=1$. Then equation (1) becomes

$$F_{dA_1-A_2}=\oint_C \frac{ydx-xdy}{2\pi(x^2+y^2+z^2)} \quad (2)$$

Subscript 2 has been omitted in the right hand side of equation (2) for convenience.

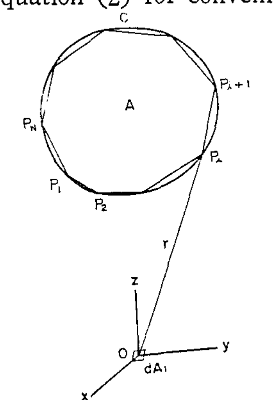


Fig. 1 Finite line contour approximation.

Now the contour C is assumed to be composed of a finite number of straight lines(Fig. 1). This is a similar approximation used in the finite element method. Let two ends of a straight line be P_i (x_i, y_i, z_i) and $P_{i+1}(x_{i+1}, y_{i+1}, z_{i+1})$, then equation (2) can be written as

$$F_{dA_1-A_2}=\sum_{i=1}^{N}\int_{P_i}^{P_{i+1}} \frac{ydx-xdy}{2\pi(x^2+y^2+z^2)} \quad (3)$$

The coordinates of a point $P(x,y,z)$ on the line P_iP_{i+1} can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}+s\begin{pmatrix} x_{i+1}-x_i \\ y_{i+1}-y_i \\ z_{i+1}-z_i \end{pmatrix},\quad 0\leq s\leq 1 \quad (4)$$

Substituting equation (4) into equation (3), and integrating it from $s=0$ to $s=1$, one obtains the finite line integral.

$$F_{dA_1-A_2}=\sum_{i=1}^{N} \frac{y_i x_{i+1}-x_i y_{i+1}}{2\pi d}$$
$$\left\{\tan^{-1}\left(\frac{b+c}{d}\right)-\tan^{-1}\left(\frac{b}{d}\right)\right\}$$

where

$$a=x_i^2+y_i^2+z_i^2$$
$$b=x_i(x_{i+1}-x_i)+y_i(y_{i+1}-y_i)+z_i(z_{i+1}-z_i)$$
$$c=(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2+(z_{i+1}-z_i)^2$$
$$d=\sqrt{ac-b^2}$$

Here the radiative view factor $F_{dA_1-A_2}$ is given as a function of coordinates of N end points of straight lines which are assumed to make the contour C. When $z_i<0$ and/or $z_{i+1}<0$, all or part of the line $\overline{P_iP_{i+1}}$ cannot be seen directly from dA_1. In this case the point on $\overline{P_iP_{i+1}}$ for which $z=0$, can be found by interpolation, and integration should be performed to that point.

3. Radiative View Factors Between Two Finite Bodies

Radiative view factor between two finite bodies, $F_{A_1-A_2}$, is defined as [1]

$$F_{A_1-A_2}=\frac{1}{A_1}\int_{A_1} F_{dA_1-A_2}dA_1 \quad (7)$$

Using values of $F_{dA_j-A_2}$ obtained by equations (5) and (6) and Gaussian quadrature, equation (7) can be approximated as follows:

$$F_{A_1-A_2}=f\sum_{j=1}^{M}F_{dA_j-A_2}w_j \qquad (8)$$

where M is the number of quadrature points. The Gaussian weight w_j depends on the position of dA_j, and f is a factor of integration, which depends on the geometry of A_1.

4. Error Analysis

Radiative view factors are computed by a digital computer with seven digit single precision using equations (5) and (8). Equation (5) is exact when the contour C is actually composed of N straight lines, *i.e.*, when A_2 is a polygon or a polyhedron.

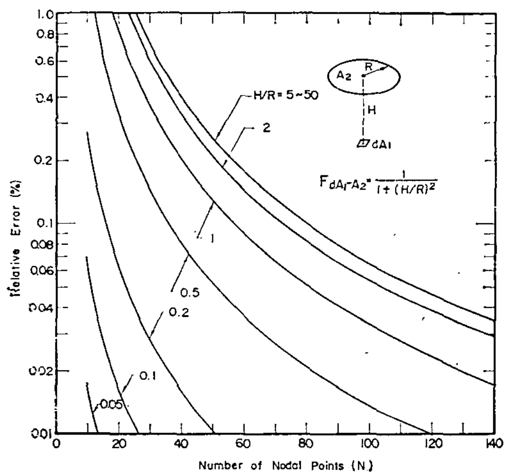


Fig. 2 Relative errors in radiative view factors from a differential area to a circular disk.

Fig. 2 shows the relative errors as functions of the number of nodal points for radiative view factors from a differential area to a unit circular disk. Relative errors are smaller when the distance from dA_1 to A_2 is smaller, for which case the view factor $F_{dA_1-A_2}$ is larger. Error decreases with the number of nodal points, N, as expected. When $N>100$, relative error is less than 0.1%. Roundoff error becomes important when the number of nodal points is excessively large.

When radiative view factors between two finite areas are computed by equation (8), errors can occur from two sources. One is from the finite line integral, the other from the Gaussian quadrature. Tables 1 and 2 show radiative view factors between two parallel and perpendicular rectangles, respectively. In these cases errors are due to the Gaussian quadrature only, since the contour C is composed of four straight lines. Numerical computation is performed with $M=8\times8$ and $M=16\times16$. Accuracy is seen to be excellent in Table 1 for parallel rectangles even when $M=8\times8$. Errors are somewhat large in Table 2 when $M=8\times8$. Accuracy in using Gaussian quadrature depends on the integrand, and in general can be increased with the number of quadrature points M.

Fig. 3 shows relative errors in radiative view factors between two coaxial parallel circular disks. Relative errors are not sensitive to the number of Gaussian quadrature points in this case.

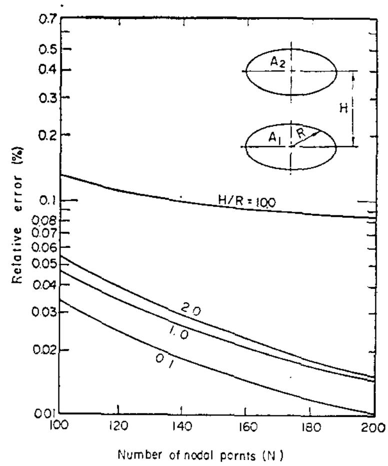


Fig. 3 Relative errors in radiative view factors between two coaxial parallel circular disks.

Table 1 Radiative view factors between two parallel rectangles.

X/H	Y/H	Exact solution	Present solution $M=8\times8$	Present solution $M=16\times16$
1.0	1.0	0.199825	0.199825	0.199821
1.0	5.0	0.359167	0.359164	0.359162
1.5	0.5	0.146415	0.146414	0.146411
1.5	1.0	0.252258	0.252256	0.252253
1.5	2.0	0.364046	0.364043	0.364042
2.0	0.1	0.035143	0.035143	0.035143
2.0	0.5	0.165269	0.165269	0.165266
2.0	1.0	0.285875	0.285873	0.285872
2.0	2.0	0.415253	0.415249	0.415250
5.0	0.5	0.205860	0.205861	0.205856
5.0	2.0	0.529931	0.529928	0.529927
5.0	5.0	0.690245	0.690241	0.690224
10.0	0.5	0.220810	0.220799	0.220806
10.0	1.0	0.386382	0.386370	0.386379
10.0	2.0	0.573376	0.573367	0.573372
10.0	5.0	0.753627	0.753624	0.753600

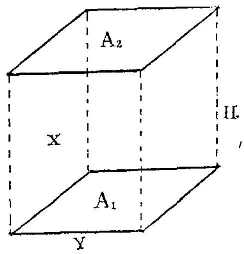
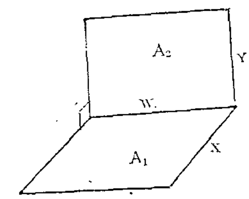


Table 2 Radiative view factors between two perpendicular rectangles.

X/W	Y/W	Exact solution	Present solution $M=8\times8$	Present solution $M=16\times16$
1.0	1.0	0.200044	0.200044	0.220042
1.0	5.0	0.246899	0.246900	0.246897
1.5	0.5	0.102713	0.102686	0.102710
1.5	1.0	0.148216	0.148188	0.148212
1.5	2.0	0.182863	0.182836	0.182859
2.0	0.1	0.021878	0.022089	0.021876
2.0	0.5	0.078650	0.078612	0.078647
2.0	1.0	0.116426	0.116387	0.116423
2.0	2.0	0.149300	0.149261	0.149296
5.0	0.5	0.032232	0.032158	0.032227
5.0	1.0	0.049380	0.049301	0.049374
5.0	2.0	0.068095	0.068017	0.068089
5.0	5.0	0.088102	0.088024	0.088096
10.0	0.5	0.016175	0.016210	0.016164
10.0	1.0	0.024921	0.024776	0.024910
10.0	2.0	0.034910	0.034757	0.034899
10.0	5.0	0.047768	0.047615	0.047756



5. Radiative View Factors From a Unit Sphere to an Ellipsoid

In order to show the capability of the finite line intgral method, radiative view factors from a unit sphere to an ellipsoid are computed. Chung and Naraghi[3] have reported radiative view factors from a sphere to an axisymmetric ellipsoid, *i.e.*, a spheroid. Their method, however, cannot be applied to non-axisymmetric ellipsoids.

In order to check the accuracy of the finite line integral method, radiative view factors between two spheres are computed, and the results are compared with reported values of Ref. [3]. Axisymmetry is used in the numerical computation, where view factor from a quarter of sphere 1 to a half sphere 2 is

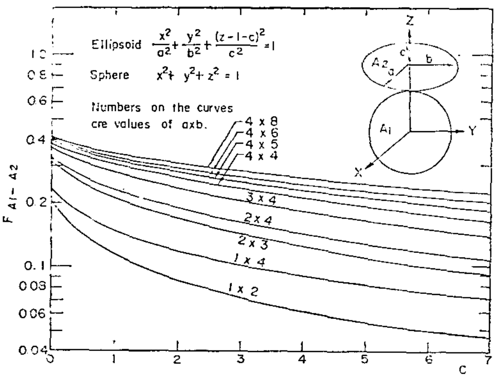


Fig. 4 Radiative view factors from a unit sphere to an ellipsoid contacting each other.

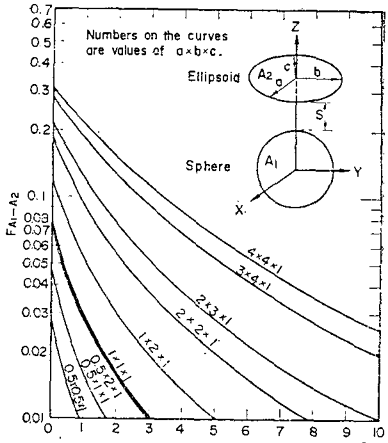


Fig. 5 Radiative view factors from a unit sphere to an ellipsoid

actually computed, and then the result is multiplied by 2. Accuracy is seen to be excellent from Table 3. Detail of computing the coordinates of nodal points on contour C is given in Appendix.

Fig. 4 shows radiative view factors from a unit sphere to ellipsoids in contact with the sphere. Fig. 5 shows view factors from a unit

Table 3 Radiative view factors between two spheres, $F_{A_1-A_2}$.

R_2/R_1	S/R_1	Ref. [3]	$N=80$ $M=16$	$N=80$ $M=8$	$N=41$ $M=16$	$N=41$ $M=8$
0.1	0.0	0.29930E-2	0.29926E-2	0.29954E-2	0.29913E-2	0.29941E-2
	0.9	0.67028E-3	0.67028E-3	0.67024E-3	0.66977E-3	0.66973E-3
	3.9	0.10103E-3	0.10099E-3	0.10096E-3	0.10093E-3	0.10089E-3
	8.9	0.25063E-4	0.25044E-4	0.25041E-4	0.25031E-3	0.25027E-4
0.2	0.0	0.93167E-2	0.93155E-2	0.93169E-2	0.93116E-2	0.93130E-2
	0.8	0.26892E-2	0.26884E-2	0.26891E-2	0.26864E-2	0.26871E-2
	3.8	0.40425E-3	0.40414E-3	0.40423E-3	0.40383E-3	0.40392E-3
	8.8	0.10026E-3	0.10022E-3	0.10018E-3	0.10015E-3	0.10011E-3
0.5	0.0	0.34351E-1	0.34347E-1	0.34346E-1	0.34335E-1	0.34334E-1
	0.5	0.17147E-1	0.17144E-1	0.17140E-1	0.17132E-1	0.17129E-1
	3.5	0.25322E-2	0.25312E-2	0.25303E-2	0.25294E-2	0.25284E-2
	8.5	0.62697E-3	0.62680E-3	0.62704E-3	0.62631E-3	0.62656E-3
1.0	0.0	0.75587E-1	0.75578E-1	0.75579E-1	0.75554E-1	0.75554E-1
	1.0	0.29590E-1	0.29584E-1	0.29582E-1	0.29564E-1	0.29556E-1
	3.0	0.10211E-1	0.10209E-1	0.10205E-1	0.10201E-1	0.10197E-1
	5.0	0.51555E-2	0.51542E-2	0.51554E-2	0.51503E-2	0.51515E-2

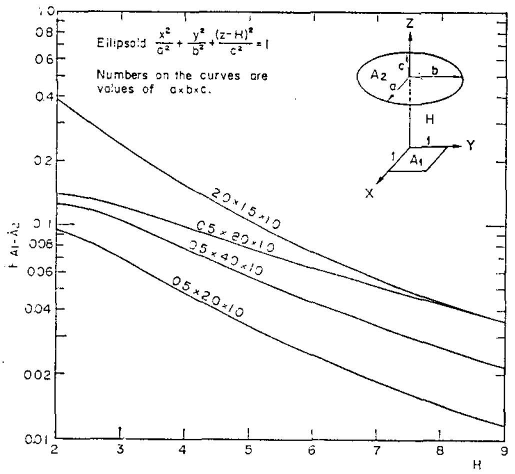


Fig. 6 Radiative view factors from a unit square to an ellipsoid.

sphere to various ellipsoids as functions of space between the two bodies. Numbers on the curves are values of major and minor axes of the ellipsoids. Values for axisymmetric ellipsoids, $a=b$, are compared with Figs. 11 and 12 of Ref. [3]. Errors are hard to detect from the scale shown.

Radiative view factors from a unit square to an ellipsoid are also given in Fig. 6.

6. Conclusion

A new simple numerical procedure is developed to compute radiative view factors from a differential area to a finite area. This technique can be easily programmed for a digital computer. Basic concept used in the finite line integral method is to assume the contour of receiving body to be composed of finite number of straight lines. This technique is exact if the receiving body is a polygon or a polyhedron.

Gaussian quadrature is used for radiative view factors between two finite bodies. Several numerical examples are given to check the accuracy of the method. Accuracy depends on the number of nodal points, and can be increased by increasing the number of nodal points.

Radiative view factors from a unit sphere to ellipsoids are obtained. For axisymmetric ellipsoids, the results agree well with reported values of Ref. [3]. Radiative view factors from a unit square to ellipsoids are also given.

7. References

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Appendix

Contour of integration for radiative view factor from a differential area to an ellipsoid

Radiative view factor from a differntial area at the origin of the coordinate system to an ellipsoid

$$\frac{(x-r_x)^2}{a^2}+\frac{(y-r_y)^2}{b^2}+\frac{(z-r_z)^2}{c^2}=1 \tag{A1}$$

where $r_z \leq c$, is considered. The contour of integration C is composed of the points, P, on the surface of the ellipsoid, where normal vector to the surface is orthogonal to the position vector OP.

Using two parameters u and v, equation (A1) can be rewritten as follows.

$$\left.\begin{aligned} x&=r_x+a \sin u \cos v \\ y&=r_y+b \sin u \sin v \\ z&=r_z+c \cos u \end{aligned}\right\} \qquad \text{(A2)}$$

where $0 \leq u \leq \pi$, $0 \leq v \leq 2\pi$. An outward normal vector to the surface of the ellipsoid is

$$\bar{N}=\frac{\sin u \cos v}{r_x}\hat{\imath}+\frac{\sin u \sin v}{r_y}\hat{\jmath} + \frac{\cos u}{r_z}\hat{k} \qquad \text{(A3)}$$

Position vector $\bar{P}$ of a point on the surface of the ellipsoid is

$$\bar{P}=(r_x+a \sin u \cos v)\hat{\imath}+(r_y+b \sin u \sin v)\hat{\jmath}+(r_z+c \cos u)\hat{k} \qquad \text{(A4)}$$

From the condition $\bar{N}\cdot\bar{P}=0$ at a point on the contour C, following equation is obtained.

$$\left(\frac{r_x}{a}\cos v+\frac{r_y}{b}\sin v\right)\sin u+\frac{r_z}{c}\cos u +1=0 \qquad \text{(A5)}$$

For a given value of v, the value of u can be obtained from equation (A5).

$$\sin u=\left\{-g+\sqrt{g^2+\left(\frac{r_z^2}{c^2}-1\right)\left(g^2+\frac{r_z^2}{c^2}\right)}\right\}\Big/\left(g^2+\frac{r_z^2}{c^2}\right) \qquad \text{(A6)}$$

where

$$g=\frac{r_x}{c}\cos v+\frac{r_y}{b}\sin v \qquad \text{(A7)}$$

The condition $\sin u \geq 0$ for $0 \leq u \leq \pi$ has been used in equation (A6).

Vales of u and v are substituted into equation (A 2) to obtain coordinates of nodal points on C. When normal vector to the differential area is different to z-direction, rotation of the coordinate system is applied to obtain new coordinates of the nodal points.