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<**Original**>

Optimal Constant Feedback Control of Flow-Induced Vibration in Bluff Structures

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유체의 흐름에 의해 야기되는 구조물 진동의 최적 제어

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초 록

유동장(流動場)에 구조물이 놓여 있을 때 유체의 운동과 구조물 진동의 상호작용으로 비선형 자려 진동(self-excited vibration)을 일으키는 경우가 많다. 본 논문에는 이러한 현상으로 야기되는 구조물의 불안정한 진동을 없애기 위한 방법으로의 일환으로 최적 진동 제어기를 설계하였으며, 설계 방법과 시뮬레이션 결과를 자세히 언급하였다.

1. Introduction

Many engineering structures are exposed to excitation of fluid flow in many forms. Strong oscillation of the structure can be excited when vortex shedding frequency coincides with its natural frequency of vibration. This usually occurrs over the relatively small range of the flow velocity. For the flow velocity greater than that of the vortex-excited vibration, there is another form of flow-induced vibration termed "galloping oscillation".

This form of self-excited oscillation results from unsteady aerodynamic force acting on the structure when its motion interacts with the flow field. The aerodynamic force properties are such that aerodynamic lift forces induced by a small structural motion act in the direction of the motion, thus producing negative damping type forces. This type of aerodynamically induced vibration has long been observed in such structures as tall buildings(1~3), slender towers and stacks(4), transmission line(5), circular saw(6), and turbine disc and blade(7).

In recent years considerable research works have been done to identify the instability mechanism through analytical and experimental works(1~9). Due to these works the instability mechanism and phenomenon have been identified but suppression method to circumvent this problem has not yet been fully inve-

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stigated. One possible approach to stabilize the flow-induced instability is by the use of activ control system which is designed to sense the structural motions and to generate a corrective control force acting on the structure. This type of the active control concept has been applied to action control of building structures subjected to stochastic windforce excitation(10,11). In these studies the wind force has been assumed to be independent of the structural motion, so that the interaction phenomenon between the structural motion and wind flow has not been created. As compared with this situation, the galloping oscillation results from the interaction between the two and thus exhibits nonlinear oscillation characterisaics due to a nonlinear relationship between the angle of attack and the aerodynamic force: As the structural motion occurrs, this changes the instantaneous flow direction relative to the structure i.e. angle of attack, thus resulting in variation in the aerodynamic force. This galloping mechanism often leads to destructive vibration which has been observed for many years from many of engineering structures.

In this study an active control of a vibratory system exhibiting such galloping oscillation is considered. A design procedure for optimal constant feedback controller is presented to suppress the vibration and based upon the minimization of a quadratic performance representing the system vibratory energy.

2. Optimal Controller Design

Fig. 1 shows the geometry of a vibrating structural cross section in a uniform flow. When an elastic structure moves with velocity $\dot{x}$ perpendicular to a two dimensional flow field of velocity, v, the aerodynamic force acting on the body is generated by relative

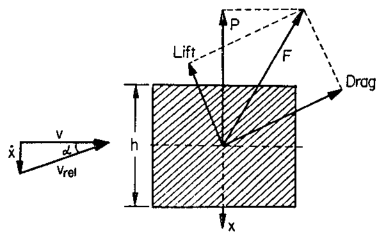

Fig. 1 Structure section and flow geometry.

flow velocity v_{rel}. This lateral force is nonlinear function of angle of attack $\alpha=\tan^{-1}\dot{x}/v$ and given by

$$p=\frac{1}{2}C_p(\alpha)\rho hlv^2 \quad (1)$$

where $C_p(\alpha)$ is the lateral force coefficient, ρ is the air density and h and l are the side length and axial length of the section, respectively. The force coefficient is usually described by a power series(1).

$$C_p(\alpha)=a_1\left(\frac{\dot{x}}{v}\right)-a_2\left(\frac{\dot{x}}{v}\right)^3-a_3\left(\frac{\dot{x}}{v}\right)^5 \quad (2)$$

where terms beyond $\left(\frac{\dot{x}}{v}\right)^5$ are neglected.

Using the same dynamic model as used in references(1~3), the differential equation governing the oscillation is

$$m\ddot{x}+c\dot{x}+kx=\frac{1}{2}C_p\rho hlv^2+u \quad (3)$$

where in this model the system was assumed to be a single degree of freedom and nonaerodynamic damping viscous. In the above x is the vibration displacement, m is the vibrating mass, c is the damping coefficient, k is the spring coefficient, and u is the control force to be acted upon the structure. Introducing the dimensionless quantities,

$$X=\frac{x}{h},\ \beta=\frac{c}{2m\omega},\ \eta=\frac{\rho h^2 l}{2m},$$

$$U=\frac{u}{hm\omega^2},\ V=\frac{v}{\omega h},\ \tau=\omega t$$

where ω is the system natural frequency, and defining state variables $x_1=x$ and $x_2=\dot{x}$, equation (3) can be written in the state variable form:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_1 + \eta a_1 \left\{ \left(V - \frac{2\beta}{\eta a_1} \right) x_2 - \left(\frac{a_2}{a_1 V} \right) x_2^3 - \left(\frac{a_3}{a_1 V^3} \right) x_2^5 \right\} + U \quad (4)$$

Equation (4) is of the well-known form of a weakly nonlinear system due to smallness of the term ηa_1 and thus represents a nonlinear control system. When the influence of terms containing higher degree of $x_2 = \dot{x}$ vanishes, the character of a vibratory motion depends only on the sign of the coefficient at x_2, i.e., on the sense of the total damping. If the coefficient is positive, the zero position $x=0$ is stable. If it is negative, the zero position is unstable and Eq. 4 describes self-excited oscillations, starting when

$$V > V_{crit} = \frac{2\beta}{\eta a_1}$$

wheres V_{crit} denotes the critical flow velocity. If the above condition is fulfilled, Eq.4 describes self-excited oscillations. The vibration amplitude in this case can grow infinitely if the higher order terms x^3 and x^5 in Eq. (4) are neglected, However, if these nonlinear terms are included, the amplitude can be finite, reaching a steady amplitude called "limit cycle oscillation amplitude". It can be shown that due to the nonlinearity a finite steady amplitude of limit cycle oscillation occurrs and this steady amplitude depends primarily upon the wind velocity, the aerodynamic coefficient and damping factor. As the wind velocity increases, the steady amplitude curve approaches an asymptotic straight line

$$A_{steady} = kV \quad (5)$$

where k is a constant. This indicates that the amplitude of steady state oscillation increases with increasing flow velocity. Another feature of the weakly nonlinear system is that the oscillation amplitude grows very slowly towards that of the steady state. The phenomena stated above are characteristic of a weakly nonlinear vibratory system. The analysis of this system can be done by employing Bogoliubov and Krylov method and can be found in the references(1)~(3).

A natural goal for the vibration suppression is to minimize the system energy with smaller control force. A quadratic performance index for this purpose may be chosen as

$$J = \frac{1}{2} \int_0^{\tau f} (x^T Q x + U^2) d\tau \quad (6)$$

Where τf is a specified final time, and the weighting matrix Q is chosen to have the following form:

$$Q = diag\ \{1, 1\}$$

The first terms in Eq. (6) represent the potential energy and the kinetic energy of the system, and the second term in the performance index is included to constrain the control force in minimizing the vibration energy.

This nonlinear regulator problem is not easily solved and normally leads to an open-loop controller which is often ineffective for a practical implementation. One popular approach to this problem is to specify a fixed feedback configuration and to optimize with respect to the free parameters, retaining the nonlinear system description.

If a state variable feedback configuration is used then the control law is given by

$$U = -kx = -[k_1, k_2]x \quad (7)$$

where k are a constant feedback gain vector. The problem is then to find the gain parameters which minimize the performance index in equation(6). Several works discussing the optimization technigue to obtain the optimal feedback gain include quasilinearization method[9], the method of successive substitutions[10], Newton-Raphson method[11] and gradient minimization method[12]. Although these numerical solution methods provide an efficient base of computing the optimal parameter, subroutine ZXMIN based on the Harwell library VAlOA[16] appears to yeld fast convergence rate and to require small data-storage: This routine minimizes a function of N variables using a quasi-Newton method. In the following simulation study this routine is used to compute the optimal feedback gain k_1 and k_2 from equations (4)~(7).

3. Control Results and Conclusions

The pertinent data of a flow-induced structural model given in equation (3) were taken from reference[1]. The experimental test model used was made of *a* 2.54 cm by 2.54 cm aluminum square section. These data are as follows:

$\beta=0.00152$, $\eta=0.000922$, $\omega=55.6$rad/sec., and the aerodynamic force coefficient obtained from wind tunnel test are:

$a_1=3.11$, $a_2=16.8$, $a_3=208$.

To compute the optimal feedback gain k_1 and k_2, the control law in (7) was substituted into equations (4) and (6). The final time $\tau f=10$ was used for the optimization and integration of the equation (4) was done using the fourth order Runge-Kutta method with step step $\Delta\tau=0.1$ and with the initial condition $x_1(0)=0.01$ and $x_2(0)=0$.

Fig. 2 shows responses of the structural motion without the control force input. In Fig. 2 uncontrolled response is illustrated for two different flow velocities but for the some initial conditions $x=0$ and $\dot{x}=0.01$ Fig. 2 (*a*) is the case of $V=2.0$, while Fig.2 (*b*) that of $V=3.0$. These velocities are much higher than the critical velocity $V_{crit}=1.06^1$: The V_{crit} is defined as the flow velocity above which the

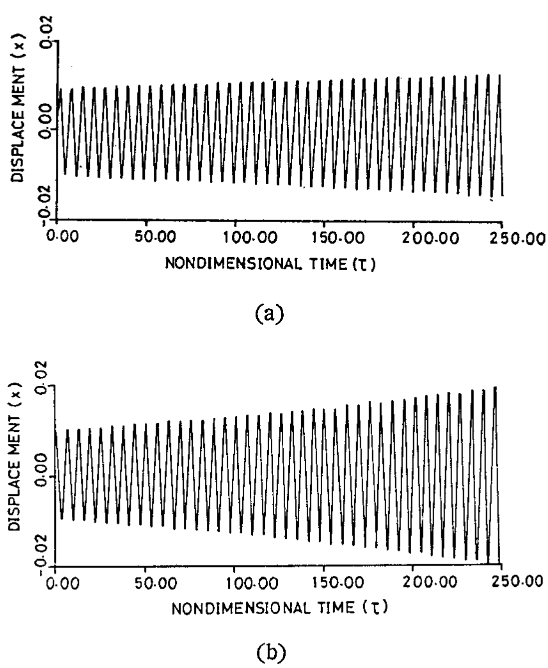

Fig. 2 Uncontrollecl response.

structural motion becomes unstable. Regardless of the flow velocity, the amplitudes grow very slowly towards some steady state amplitudes of limit cycle oscillation. Another observation is that, increasing the flow velocity from $V=2$ to $V=3$ the steady state amplitude is also increased. As discussed before, these behaviors are characteristic of the weakly nonlinear system. Figures 3(*a*) and 3(*b*) compare the controlled responses for aribitrary gain values $k_1=1.414$ and $k_2=0.474$ and for the optimal gain values, respectively: The optimal values were obtained to be $k_1=0.474$ and $k_2=1.414$, whereas the arbitrary gain values were chosen as $k_1=1.414$ and $k_2=0.474$: The k_1 value was arbitrary increased from the optimal one $k_1=0.474$ and the k_2 value was also arbitrary decreased from the optimal one $k_2=1.414$. The nonoptimal response shown in Fig. 3 (*a*) shows stable but oscillating response at initial stage, and the oscillation lasts up to nondimensional time $\tau=$

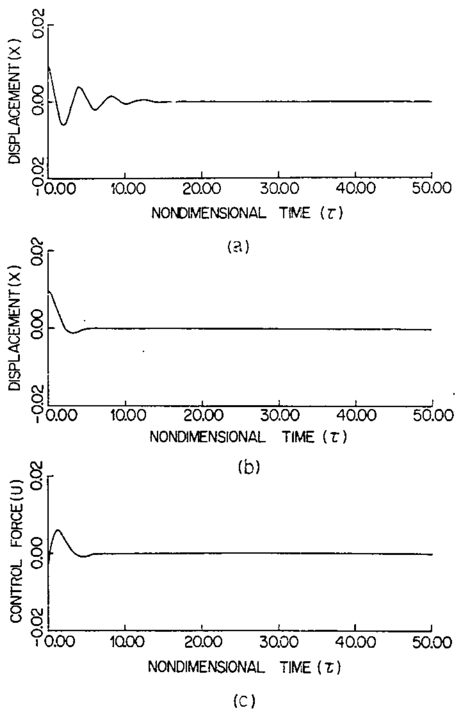

Fig. 3 (a) Nonoptimal response at $V=3.0$
(b) Optimal response at $W=3.0$
(c) Optimal control force input

15. As compared with this the optimal control force input effectively suppresses otherwise unstable motion of the structure within a very short time. It requires approximately one $\tau=5$, which corresponds to 0.09 sec. The same trend has been found even for much higher flow velocity, $V=10$. This indicates that with the optimal control the structural motion can be stabilized even when the low velocity is increased up to the value times higher than the critical speed. In Fig. 3(c) the corresponding optimal control input force in shown. Maximum input force is approximately $U=0.008$ which corresponding optimal control input force in shown. Maximum input force is approximately $U=0.008$ which corresponds to 0.35N for the experimental model considered in Reference 1.

Variation of the optimal feedback gains with the flow velocity was investigated. The optimal values have been found to remain almost unchanged regardless of the flow velocity, although the results are not shown here. This indicates that, although the nonlinear aerodynamic force causes the unstable motion, it is small as compared the optimal control force. Thus, it is expected that the flow velocity does not alter the feedback gain appreciably. This result makes the feedback implementation problem easier. since varing the feedback gain during the controlled period is not an easy problem in practical situations, if the gains depend upon the flow velocity.

Based upon the simulation results presented above, it can be concluded that active control force applied to bluff structures interacting with the flow can stabilize otherwise unstable motion and that optimal choice of the controller gain parameters leads to the effective suppression of the unstable motion within a very short time. The optimal feedback gains are found to remain almost unchanged for a wide range of flow velocities but these are expected to be initial condition dependent. It is noted that the control force can be adjusted to a desired magnitude by varing the input weighting parameter in the performance index.

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