Transactions of the Korean Society of Mechanical Engineers Vol. 7, No.1, pp. 83~95 1983.

<Original>

Application of Multivariable Optimal Control Theory to Active Plate Vibration Control

S.W. Hong* and H.S. Cho**

(Received October 6, 1982)

최적제어 이론의 圓板진동 감소에의 응용

홍 석 우 · 조 형 석

초 록

원판의 진동은 위치 좌표에 따라 진동변위 및 속도가 달라질 뿐 아니라, 또한 여러 진동모우드를 포함하고 있다. 이러한 특성을 갖는 시스템의 진동을 감소시키기 위해서는 진동 제어력을 발생시켜 주는 구동장치와 진동변수를 측정하는 측정장치를 어느 위치에 두느냐가 무엇보다도 중요하다. 더구나 여러 진동 모우드를 동시에 최적으로 제어하기 위해서는 진동 모우드 갯수 만큼의 많은 측정장치가 필요하게 되는데 이러한 난점을 해결하기 위해서 본 논문에서는 측정장치의 역활을 대신해 주는 관측기(obsevrer)를 부착하여 측정장치의 필요 갯수를 줄였다. 최적제어 이론을 바탕으로 구동장치와 측정장치의 위치를 최적으로 결정하였고, 제어기의 제어상수를 최적으로 설계하였다. 이와 같이 설계된 최적 제어 시스템의 제어효과를 평가해 보기 위해서 원판의 진동을 예를 들어 시뮬레이션해 보았는데 그 결과를 자세히 기술하였다.

1. Introduction

The use of active control to reduce vibrations of mechanically flexibe systems has received considerablle attention in recent years. Several studies discussing this class of control problem includies modal control of plate bending vibration(1), feedback control of circular saw vibration(2), active control of a spinning flexible spacecraft(3,4), control of civil engineering structures(5,6), and active control of a simply supported beam(7). Active control of these flexible systems requires feedback control of infinite dimensional vibration modes, but to design an implementable control system a finite dimensional controller should be considered. This produces several aspects of controller design problems which complicate the design decisions. These include truncation and the resulting spillover problem(7), uncertainty in dynamic modelling and modelling error(8,9,10), and controller implementation(3,11). Detailed surverys on these problems have been given in the references(12,13,14).

Truncation method which includes a finite

* Department of Mechanical Engineering, Choongnam University

** Department of Production Engineering, Korea Advanced Institute of Science and Technology, Member, KSME

number of terms for the dynamic modelling has been extensively used as the only feasible alternative in previous works. A basis for this approach is that higher frequency modes (residual modes) are rarely encountered in practice and can not be easily excited. Furthermore, the bandwidths of control devices (actuators and sensors) cannot respond to these modes. However, when the actuators excite the actuators excite the residual modes, the control and observation spillover have been shown to lead instabilites in the closed-loop system and thus to seriously degrade the system performance[7]. In a series of papers Balas[7,15,16] proposed some excellent remedies for the spillovers to remove the instability mechanism. Also, Skelton and Likins[17] proposed an orthogonal filtering as compensation for the residual effects. These works provide a significant contribution to the solution of control and observation spillovers problem.

Implementation of the feedback controller is also important from a design consideration. The implementation often necessitates measurement of all state variables such as amplitude and velocity of the individual vibration mode. In actual practice, measurement of all these variables is costly or even impossible when a large number of vibration mode are to be controlled. One solution is to use a linear dynamic observer which estimates the state variables which are inaccessible to physical measurement. In this case the observer feedback gains greatly influence the overall control system performance. Previously the observer theory has been used for this estimation[3,7], but a large error occurred during the early transient period of estimation due to preselection of the observer feedback gains.

Another important consideration is to achieve better control effectiveness with fewer actuators and sensors needed to implement a governing control law. A rational approach to achieve this end is to optimize design parameters such as the number and location of actuators and sensors. In previous studies the locations of actuators and sensors have been prespecified arbitrary and consequently the design problem to determine their optimal locations has not been fully investigated

In the design method proposed in this paper such criticisms are avoided. The method determines optimal locations of actuators and sensors and uses an optimal observer for feedback implementation to make the estimation error to be minimized at an early period of the observation. The vibratory system to be studied here is a centrally clamped circular plate which has important applications in turbine disc, circular saw, and computer memory disc. The plate dynamics are formulated by a finite dimensional state variable model, incorporating the control force due to the actuators. This finite dimensional modelling seems to be feasible in the case of a centrally-clamped circular plate, since higher frequency modes (usually nodal circle modes) are not usually observed from a spectral analysis. For some practical reasons, if any significant higher modes can not be included for the controller design, the spillover compensation methods proposed by Balas and others may be used to suppress the unwanted residual modes effects. Therefore, in this paper the residual mode effects will be assumed negligible from a stability view point.

The optimal controller gains and actuator locations are obtained based upon minimization of a prespecified performance index which represents the vibrating plate energy. This method is similar to that of the reference[18] described for a general class of distibuted parameter control systems. The optimal observer gains and sensor locations are derived from the minimization of a performance index taken as a measure of deviation of the estimation states from true responses. Response characteristics of the controlled plate system such as modal damping ratios and natural frequencies are obtained and compared for various design parameters. These parameters include the number and locations of actuators and sensors, and the weighting factors used in the performance index. The effects of these parameters on the controlled response characteristics are discussed in some detail. Also, it is shown that an optimally designed observer drastically decreases the control period required for effective vibration suppression, as compared with the non-optimally-designed one.

2. Analytical Control System Model

A pictorial scheme for the active control system of a stationary circular plate is shown in Fig. 1. The plate is clamped at inner radius, $r=a$, and is free at outer radius, $r=b$. Actuators which provide control forces are located at points (r_i^a, θ_i^a), $i=1, 2, \cdots, N$, on the plate. The N point forces control the plate motion, obeying the actuator command signal,

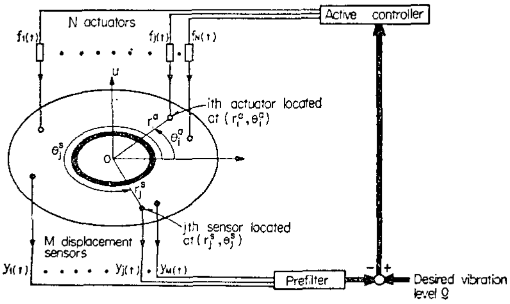


Fig. 1 Active control of a circular plate vibration.

$f_i(t)$ generated by a feedback controller. M point sensors measure the instantaneous displacement of the plate and are located at points (r_j^s, θ_j^s), $j=1, 2, \dots M$. The equation of motion governing small amplitude, transverse motion of a thin plate subjected to N point actuating forces is described in a dimensionless form:

$$\frac{Eh^2}{3\rho(1-\nu^2)b^4\omega_{00}^2}\nabla^4\bar{u}+\frac{\partial^2\bar{u}}{\partial\tau^2}=\bar{A}\sum_{i=1}^{N}\delta(\theta-\theta_i^a)$$
$$\frac{\delta(\bar{r}-\bar{r}_i^a)}{\bar{r}_i^a}\bar{f}_i(\tau) \qquad (1)$$

where $\bar{u}$ is the plate displacement, h is the half thickness, ρ is the density of the plate, E is the elastic modulus, ν is poission ratio, $\delta(\cdot)$ denotes the dirac delta function, and the right hand side of the equation denotes is force per unit area applied to circular plate by the actuators. In the above nondimensional quantities are defined as

$$\bar{\nabla}^4=b^4\nabla^4,\ \bar{u}=\frac{u}{b},\ \bar{r}=\frac{r}{b},\ \bar{y}=y/b,\ \bar{\omega}_{mn}=\frac{\omega_{mn}}{\omega_{00}}$$
$$\tau=\omega_{00}t,\ \bar{A}=A/b^2,\ \bar{f}_1=\frac{f_i}{\pi\rho h(b^2-a^2)\omega_{00}^2}$$

where the super bar denotes dimensionless quantities, the subscripts, m and n, refer to integer number of the nodal dimeters and nodal circle modes, respectively, ω_{mn} are natural frequency, and A is the plate surface area, f_i is the time dpendent force magnitude, and a and b are the clamped inner and outer radius, respectively. The bounday conditions at the clamped inner radius, $\bar{r}=a$ are

$$\bar{u}=0$$
$$\frac{\partial\bar{u}}{\partial\bar{r}}=0 \qquad (2)$$

and the free edge boundary conditions on the outer radius, $\bar{r}=b$ are

$$\frac{\partial\bar{\nabla}^2\bar{u}}{\partial\bar{r}}+\frac{1-\nu}{\bar{r}^2}\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial\bar{u}}{\partial\bar{r}}-\frac{\bar{u}}{\bar{r}}\right)=0$$
$$\frac{\partial^2\bar{u}}{\partial\bar{r}^2}+\nu\left(\frac{1}{\bar{r}}\frac{\partial\bar{u}}{\partial\bar{r}}+\frac{1}{\bar{r}^2}\frac{\partial^2\bar{u}}{\partial\theta^2}\right)=0 \qquad (3)$$

The dimensionless displacements of the circular plate to be measured by M point sensors are given by

$$\bar{y}_j(t)=\bar{u}(\bar{r}_j^s, \theta_j^s, t), \quad j=1, 2, ...M. \qquad (4)$$

The solution of the equation (1) may be expressed as an eigenfunction expension of the free circular plate,

$$\bar{u}(\bar{r}, \theta, \tau)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} q_{mn}(\tau)R_{mn}(\bar{r})\cos m\theta \qquad (5)$$

The radial mode shape function is

$$R_{mn}(\bar{r})=C_1 J_m(\lambda_{mn}\bar{r})+C_2 Y_m(\lambda_{mn}\bar{r}) +C_3 I_m(\lambda_{mn}\bar{r})+C_4 K_m(\lambda_{mn}\bar{r})$$

where the eigenvalue λ_{mn} are related to the natural frequencies of the plate

$$\lambda_{mn}^4=\frac{3\rho(1-\nu^2)b^4\omega_{mn}^2}{Eh^2}$$

The functions J_m, Y_m, I_m and K_m are Bessel functions of order m, and the coefficients, C_i and the values of λ_{mn} can be determined from the boundary conditions (4) and (5). They are tablated for clamping ratio, $\frac{a}{b}=0.5$.[19]

Control System Model

As can be seen from equation(5). circular plate is an infinite dimensional system, consisting nodal diameters and nodal circle modes. If the high frequency modes effects on control system performance is assumed negligible, a truncation modal expansion may be used to restrict the control system to a few significant lower modes which can be selected for the system performance requirement such as dynamic stability and vibration tolerance. Let p be the number of vibration modes to be actively controlled, consisting 0 to r nodal diameter modes and 0 to s nodal circle modes. Then the dimensionless displacement, $\bar{u}$ may be written as

$$\bar{u}(\bar{r}, \theta, \tau)=\sum_{m=0}^{r}\sum_{n=0}^{s} q_{mn}(\tau)R_{mn}(\bar{r})\cos m\theta \qquad (6)$$

where $p=r\times s$.

Substituting equation (6) into equation (1), multiplying the resulting equation by $\bar{r}R_{lq}(\bar{r})\cos l\theta$, integrating both sides over the domain $(\theta\varepsilon\{0, 2\pi\}\bar{r}\varepsilon\{0.5, 1\})$ and using the orthognality relationship of the eigenfunctions lead to the following time-dependent modal equation.

$$\ddot{q}_{mn}(\tau)+\bar{\omega}_{mn}^2 q_{mn}(\tau)=\bar{A}\sum_{i=1}^{N} R_{mn}(r_i^a, \theta_i^a) \cos m\theta_i^a \bar{f}_i(\tau) : m=0, 1, 2, ...r, \quad n=0, 1, 2, ...s \qquad (7)$$

Let the amplitudes $q_{mn}(\tau)$ and velocities $\dot{q}_{mn}(\tau)$ form the state of the controlled system. The state variables is defined as,

$$\underset{\sim}{x}(\tau)=\begin{bmatrix} \underset{\sim}{q}(\tau) \\ \hdashline \dot{\underset{\sim}{q}}(\tau) \end{bmatrix}$$

where $x(\tau)$ is $2p\times 1$ vector, $q(\tau)=\{q_{00}, q_{01}, q_{rs}\}^T$ is $p\times 1$ vector, $\dot{q}(\tau)=\{\dot{q}_{00}, \dot{q}_{01}, ...\dot{q}_{rs}\}^T$ is $p\times 1$ vector, and T denotes the transpose. Then the equation (7) can be put into the following state equation :

$$\dot{\underset{\sim}{x}}(\tau)=A\underset{\sim}{x}(\tau)+B\underset{\sim}{u}(\tau) \qquad (8)$$

where control input vector $u(\tau)$ is

$$\underset{\sim}{u}(\tau)=\{\bar{f}_1(\tau), \bar{f}_2(\tau), ... \bar{f}_N(\tau)\}^T$$

and the system matrix A and input matrix, B are

$$A=\begin{bmatrix} 0 & I \\ \Lambda & 0 \end{bmatrix}, \qquad B=\begin{bmatrix} 0 \\ B^* \end{bmatrix}$$

The Λ is a $p\times p$ diagonal matrix whose diagonal entries are the squares of the controlled mode frequencies,

$$\Lambda=\mathrm{diag}\{\bar{\omega}_{00}^2, \bar{\omega}_{10}^2, \bar{\omega}_{rs}^2\} \qquad (9)$$

and the B^* is $P\times N$ matrix whose rows are the controlled mode shape functions $\phi_{mn}(\bar{r}, \theta)$ evaluated at the actuator locations,

$$B^*=\bar{A}\begin{bmatrix} \phi_{00}(\bar{r}_1^a, \theta_1^a)......\phi_{00}(\bar{r}_N^a, \theta_N^a) \\ \phi_{rs}(\bar{r}_1^a, \theta_1^a)......\phi_{rs}(\bar{r}_N^a, \theta_N^a) \end{bmatrix} \qquad (10)$$

where $\phi_{mn}(\bar{r}, \theta)=R_{mn}(\bar{r})\cos m\theta$.

The sensor outputs in equation (4) may be written in vector form,

$\bar{y}(\tau)=\{\bar{y}_1(\tau),\ \bar{y}_2(\tau),\ldots\ldots\bar{y}_M(\tau)\}^T$

and satisfies

$$\bar{y}(\tau)=C\underset{\sim}{x}(\tau). \tag{11}$$

Since the sensors measure only the displacement at each location and contain only the P controlled mode signals, the output matrix C can be represented as ;

$$C=[C^*\ \vdots\ 0]. \tag{12}$$

In the above the C^* matrix is a $M\times P$ matrix,

$$C^*=\begin{bmatrix}\phi_{00}(\bar{r}_1^s,\ \theta_1^s)\ \ldots\ldots\ \phi_{rs}(\bar{r}_1^s,\ \theta_1^s)\\ \phi_{00}(\bar{r}_M^s,\ \theta_M^s)\ldots\ldots\phi_{rs}(\bar{r}_M^s,\ \theta_M^s)\end{bmatrix} \tag{13}$$

It is noted that the C^* matrix is dependent entirely upon the sensor locations and also that B^* in equation (10) is a function of the actuator locations. Thus the locations of actuators and sensors will be optimized to give the best vibration suppression performance.

3. Synthesis of Optimal Feedback Control System

Minimization of system energy provides a natural goal for this vibration problem. The dimensionless total system energy, E_T for the p controlled modes is given by

$$E_T=E_P+E_K=\sum_{m=0}^{r}\sum_{n=0}^{s}\{\bar{\omega}_{mn}^2q_{mn}^2(\tau)+\dot{q}_{mn}^2(\tau)\}$$

where E_P and E_K are dimensionless potential energy and kinetic energy, respectively. The above equation may rewritten in terms of state variables x

$$E_T=\frac{1}{2}\underset{\sim}{x}^T(\tau)D\underset{\sim}{x}(\tau)$$

where $D=\left[\begin{array}{c:c}\Lambda & 0\\ \hdashline 0 & I\end{array}\right]$; $2p\times 2p$ matrix.

Also, it is necessary to constrain the control force, $\underset{\sim}{u}(\tau)$ in minimizing the vibration energy. Then a quadratic periormance index to be minimized is chosen as

$$J=\frac{1}{2}\int_0^\infty(\underset{\sim}{x}^TQ\underset{\sim}{x}+\underset{\sim}{u}^TR\underset{\sim}{u})d_\tau \tag{14}$$

where weighting matrices Q and R are nonnegative and positive definite, respectively and selected to have following forms:

$Q=W^TDW$

$R=\text{diag}\{\beta,\ \beta,\ldots\ldots\beta\}$

where $W=\text{diag}\ \{1,\ \alpha,\ \alpha^2\ldots\alpha^{P-1},\ 1,\ \alpha,\ \alpha^2\ldots\alpha^{P-1}\}$ permits the modes to be weighted relative to each other.

Provided that the control system in equation (8) is completely controllable for the given actuator set, i.e.,

$\text{Rank}\{B,AB,\ A^{2P-1}B\}=2p.$

then the optimal control law minimizing the performance index can be put in the form.[20]

$$\underset{\sim}{u}(\tau)=F\underset{\sim}{x}(\tau),\ \ F=R^{-1}B^TK \tag{15}$$

where the matrix K is a symmetric positive definite solution of Riccati equation

$$KA+A^TK-\text{KBR}^{-1}B^TK+Q=0. \tag{16}$$

Since the actuator positions determine the B matrix, the optimal gain matrix, K depends upon the locations of actuators.

The control forces to be generated by actuators, as given in equation (15) are analoguous to those generated from a passive damper: The force component proportional to velocity, $\dot{q}(\tau)$ play the same role as viscous damping, while the component proportional to displacement, $q(\tau)$ play the same role as a spring force.

Optimal Actuator Location

The optimal performance index J^0 is given by Levine and Athans[21]

$$J^0=\frac{1}{2}\underset{\sim}{x}^T(0)K\underset{\sim}{x}(0) \tag{17}$$

where $\underset{\sim}{x}(0)$ is initial values of the state variables and K satisfies Riccati equation(16). Since the actuator locations $(\bar{r}_1^a,\theta_i^a)$, $i=1,2,$ N, affect the matrix K through the input matrix B, the J^0 also depends upon the actuator locations. The J^0 is proportional to trace (K), as pointed out by Levine and Athans,[21] and thus the problem may be reformulated to be

$$\text{minimize} \quad \text{trace}(K) \qquad (18)$$
$$(\bar{r}_i^a,\ \theta_i^a,\ i=1,\ 2, \ldots\ldots N)$$

The minimization of $T_r(K)$ over the actuator location may be carried out by using Rosenbrock algorithm,[22] once the initial positions of the actuator and sensor are established. At each stage of Rosenbrock algorith, Riccati equation[16] must be solved by iterative technique proposed by Kleiman.[23] Initial positions of actuators and initial positions of actuators and initial feedback gain matrix L_0 are so chosen that the matrix $A+BL_0$ all have arbitrary specified negative real parts.

4. Active Controller Implementation

Implementation of the optimal control law as given by equation (15) necessitates measurement of all feedback variables such as modal amplitudes, $q(\tau)$, and velocities $\dot{q}(\tau)$ of the P vibration modes. To measure all these variables, $2p$ number of displacement sensors and velocity sensors must be available. However, it is costly or even impossible to use such a large number of sensors. Since only $M(<2p)$ displacement sensors are assumed to be used, the feedback variables are not all available directly from the physical measurements. One solution to overcome this difficulty is to use a dynamic observer which produces estimate value, $\hat{x}(\tau)$ of the true state $x(\tau)$, provided that the control system in Eqs. (8) & (11) is completely observable for the given set of sensors. If a full state observer is used for illustration of the feedback system design, then, the state equations describing the observer dynamics have the form:

$$\dot{\hat{x}}=A\hat{x}+Bu+G(y-\hat{y}),\quad \hat{x}(0)=0 \qquad (19)$$
$$\hat{y}=C\hat{x}$$

where initial state of observer $\hat{x}(\tau)$ is taken as zero conveniently and the observer gain, G is a $2p\times M$ matrix. Defining error variable $e=x-\hat{x}$, the error equation is obtained by

$$\dot{e}(\tau)=(A-GC)e(\tau),\quad e(0)=x(0) \qquad (20)$$

Thus, the observer gains G may be chosen to make the estimation error decay at a prescribed rate, so that $\hat{x}(\tau)$ may be driven as close to $x(\tau)$ as possible. Note that the sensor locations also affect the error dynamics through C matrix. The optimal feedback control, $u(\tau)$ in equation (15) may be rewritten, if the estimated state is substituted for the true state.

$$u(\tau)=F\hat{x}(\tau),\quad F=-R^{-1}B^T K \qquad (21)$$

This completes the implementation of active controller, but it remains to determine the gain G and the sensor locations. The complete optimal feedback control system with the state estimator is shown in Fig. 2.

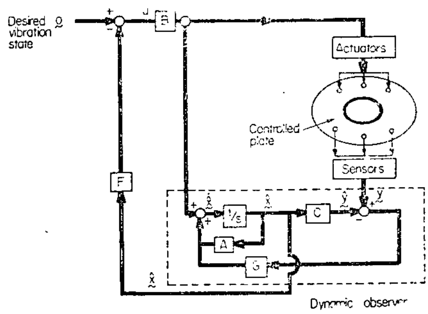


Fig. 2 Optimal feedback controller with a state estimator

Optimal Observer Design and Sensor Location

One method of observer design which makes the estimate value close to the actual value of true state is to minimize a performance measure of the deviation of the estimate value from the actual state. The performance measure may be chosen as an increment of the performance index given by equation (14) due to using an estimate value of $\hat{x}(\tau)$, instead of the actual state $x(\tau)$ for the feedback control $u(\tau)$. The increment, J is given by

$$\Delta J=\frac{1}{2}\int_0^\infty e^T F^T R\ Fe\ d\tau \qquad (22)$$

From Eqs. 20 & 22 the ΔJ can be rewritten by

$$\Delta J=\frac{1}{2}x^T(0)Lx(0) \qquad (23)$$

where the symmetric positive definite matrix Lyapunov equation,

$$(A-GC)^TL+L(A-GC)=-F^TRF \qquad (24)$$

As clearly shown in the Lyapunov equation, the matrix L depends on the sensor locations $(\bar{r}_j^s,\ \theta_j^s)$ through the output matrix C, and observer gain G. Therefore, minimization of ΔJ subjected to equation (24) leads to an optimal design of the dynamic observer as well as sensor locations. Since the $^1/_2\ x^T(0)\ Lx(0)$ in equation (23) is proportional to trace $[L]$, the optimal design problem determining sensor locations $(\bar{r}_j^s,\ \theta_j^s)$ and observer gain G may be reformulated as follows:

$$\text{minimize } T_r[L]$$
$$(\bar{r}_j^s, \theta_j^s;\ j=1, 2, \ldots\ldots M) \qquad (25)$$

where the symmetric positive definite matrix L satisfies the Lyapunov equation.

$$(A-GC)^TL+L(A-GC)=-F^TRF \qquad (26)$$

and the characteristicequation of observer matrix must satisfy

$$|sI-(A-GC)|=(s-s_1)(s-s_2)\ldots(s-s_{2P})\ldots\ldots \qquad (27)$$

where $s_1, s_2, \ldots s_{2p}$ are the desired locations of the eigenvalues in the complex plane. If each column of G is designated as a column vector, $\{g_i\}$ and assumed to be identical for simplicity, then

$$\{g_i\}=\{g_j\}$$

where $\{g_i\}$ is $2p\times 1$ vector.

The minimization of $T_r[L]$ is carried out by using Rosenbrock algorithm, while satisfying the equations (26) and (27) at each stage.

5. Numerical Results

For the present simulation the circular plate is assumed to predominantly vibrate in the lowest four modes, namely from zero to three nodal diameter modes with zero nodal circle. Then, the displacement of the controlled modes in equation (6) are represented by

$$\bar{u}(\bar{r},\ \theta,\ \tau)=\sum_{m=0}^{3} q_{m0}(\tau)\ R_{m0}(\bar{r})\cos m\theta \qquad (28)$$

The performance index to be minimized, then, contains the modified energy term associated with the first four modes. The dimensionless natural frequencies of those modes are given by Mote:[19]

number of nodal diameters, m :	0	1	2	3
dimensioness natural frequency, $\bar{\omega}_{m0}(\omega_{m0}/\omega_{00})$:	1	1.0204	1.1290	1.4252

5.1. Optimal Actuator Position

The number of force actuators was considered from one to three while up to two sensors considered. Since maximum amplitude of the plate vibration accurs at the plate periphery $(\bar{r}=1, 0)$, this radial position will be the most effective to control the plate motion. Hence, all actuators and sensors were assumed to be located at $\bar{r}=1.0$. The weighting factors chosen for th oatimization procedure were $\alpha=1.0$ for the modal weighting and $10^{-4}\ \beta=1.0$ for the control weighting.

Numerical calculations to obtain the optimal feedback gain F and actuator position, θ_i^a were carried out for $N=1$, $N=2$, $N=3$, using equation (14), (15), (16) and (18). For a single actuator case, the actuator was located at a fixed position $\theta_1^a=0°$, $\bar{r}_1^a=1.0$, and this position was also used for the first actuator location for the cases of multiple actuators.

Table 1 Opimal actuator position and controller gatn matrix ($\alpha=1.0$, $10^{-4}\ \beta=1.0$).

Numbir of actuator N	Optimal actuator position $\theta_i{}^a$	Controller gain matrix F ($\times 10^{-2}$)							
1	0^0	−0.9570	0.5257	0.5654	−0.4518	1.0470	1.3221	−1.3222	−1.378
2	0^0	−0.0880	−0.2260	−0.4150	−0.2341	0.8371	1.1292	−1.1881	−0.9880
	119.0^0	0.0431	0.1993	0.1661	−0.1120	1.1361	−0.0802	0.6581	−0.9960
3	0^0	−0.0131	−0.0916	−0.1724	−0.1535	0.6576	0.9577	−1.1101	−0.7860
	125.6^0	0.0351	0.1068	0.1892	−0.0320	0.9858	−0.9212	0.3408	−0.8187
	63.0^0	−0.0106	0.1643	−0.0708	0.0431	0.7708	0.4350	0.7737	0.8360

Table 2 Modal damping and natural frequency ($\alpha=1.0$, $10^{-4}\beta=1.0$).

Number of actuator N	1st mode		2nd mode		3rd mode		4th mode	
	ξ_{10}	$\bar{\omega}_{10}$	ξ_{20}	$\bar{\omega}_{20}$	ξ_{30}	$\bar{\omega}_{30}$	ξ_{40}	$\bar{\omega}_{40}$
0	0.0	1.0	0.0	1.0204	0.0	1.1290	0.0	1.4252
1	0.0121	1.0083	0.0320	1.0159	0.0230	1.1304	0.0051	1.4314
2	0.0383	1.0026	0.0280	1.0214	0.0261	1.1308	0.0073	1.4247
3	0.0477	1.0042	0.0301	1.0200	0.0283	1.1272	0.0096	1.4138

The optimal results for the feedback gains and locations are shown in Table 1. It is noted that the optimal position of the second actuator is little changed.

In Table 2, modal dampings ξ_{m0} and natural frequencies $\bar{\omega}_{m0}$ of the controlled plate are listed. Since the uncontrolled case is a free undamped vibration of the plate, the modal damping ratios are zero. It is clearly shown that the ζ_{m0} increases with the number of actuators for all modes except the second mode. The second mode shows a biased damping with a single actuator and slightly decreases with multiple actuators. A remarkable increase in the modal damping for the fundamental mode is achieved with three actuators, the increase of which is approximately 400% times that obtained for a single actuator. It can be concluded from this table that the use of more actuators results in better control effectiveness, and that controlled modes frequencies, $\bar{\omega}_{m0}$ are little changed from those of the original uncontrolled system.

5.2. Optimal Observer Gain and Sensor Position

The optimal observer gain and sensor position were calculated, using equations (25), (26) and (27). In this computation the actuators were assumed to be positioned at their optimal locations and the corresponding optimal gains F were used, as given in Table 1. The eigenvalues of the observer matrix $(A-GC)$ were prespecified such that the largest transient time of the estimation error would be less than about 20% of the smallest transient time of exact state variable $\underset{\sim}{x}(\tau)$. Thus, comparing eigenvalues of $A+BF$, the desired eigenvalues of observer matrix A-GC, were arbitrary specified in the left complex plane.

$$-0.51667\pm j1.6186,\quad -0.3942\pm j1.3443\cdot$$

$-0.2753\pm j1.2543,\ -0.2226\pm j0.9822$

The 8×4 gain matrix G and sensor location matrix C were computed for the following six cases:

case 1; a single actuator-a single sensor ($N=1,\ M=1$)

case 2; a single actuator-two sensors ($N=1,\ M=2$)

case 3; two actuators-a single sensor ($N=2,\ M=1$)

case 4; two actuators-two sensors ($N=2,\ M=2$)

case 5; three actuators-a single sensor ($N=3,\ M=1$)

case 6; three actuators-two sensors ($N=3,\ M=2$)

The optimization results are shown in Table 3. The use of a single sensor yields an optimal location $\theta_1{}^s=16.11^0$ when a single actuator is used. This optimal value shows a slightly increasing trend with the addition of actuators. When two sensors are used, the optimal locations vary only slightly regardless of the number of actuators used.

Tabl 3 Optimal sensor locations.

Number of actuators / Number of sensors	One		Two		Three	
	1	2	1	2	1	2
Sensor locations $\theta_i{}^s$	16.11^0	22.92^0 49.21^0	39.4^0	24.58^0 53.66^0	38.0^0	24.23^0 51.74^0

5.3. System Response

Since the optimal actuator and sensor locations, and controller design parameters such as controller feedback gain and observer gain were obtained, it is interesting to investigate the performance of this control system. Although the above design parameters all affects the system performance, the discussion is concentrated only on the effect of sensor position and observer design. For this purpose the response character of the controlled plate is simulated for the following two cases:

1) Two optimally-positioneds ensors with an optimal observer (optimal system)
2) Two sensors located at nonoptimal positions with a non-optimal observer(nonoptimal system)

For both cases three actuators were used and assumed to be optimally located as given in Table 1, and the following initial conditions were used:

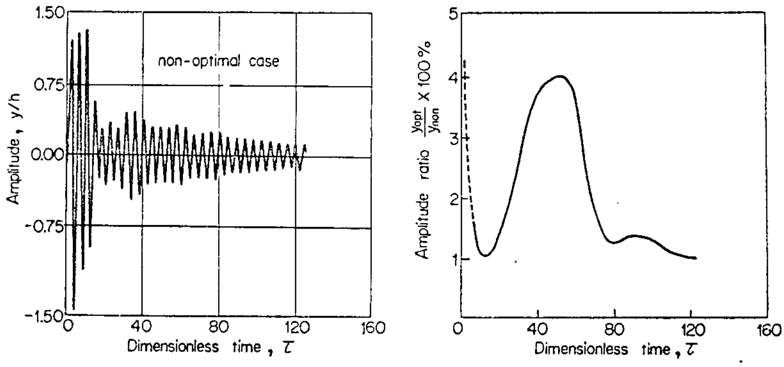
$$\underset{\sim}{x}(0)=\{0.01,\ 0.005,\ 0.003,\ 0.002,\ 0,\ 0,\ 0,\ 0\}^T$$

Sensor locations for the nonoptimal observer gain was chosen to be very near to those of the optimal one:

$$\{g_i\}=\{7.620,\ -18.136,\ -17.214,\ -1237.6,\ 24.421,\ -34.726,\ -12.960,\ -1081.9\}$$

The dynamic responses of the controlled plate are illustrated for the nonoptimal and optimal systems in Fig. 3a and Fig.2b, respectively. In Fig. 3b amplitude ratio of the optimal case to the nonoptimal case is shown. These responses were computed at a location $\bar{r}=1.0,\ \theta=0^0$. In Fig.4 the estimation errors, namely, $(y-\hat{y})/h$ are shown. The left side figure is presented for the optimal system while the right side figure for the nonoptimal system. Note that vertical scales of the estimation error for the two cases are different.

Since both optimal and nonoptimal cases have the same specified eigenvalue of observer



(a) (b)

Fig. 3 Responses of the controlled plate.
(a) Response of the nonoptimal system.
(b) Response of the optimal system: y_{opt}; amplitude of optimal response y_{non}; amplitude of nonoptimal response

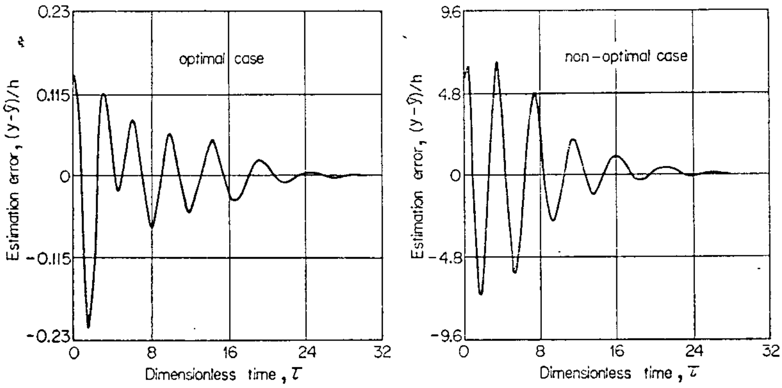


Fig. 4 Estimation error v.s. time.

matrix, $A-GC$, the decay rates of the estimation errors for both cases are identical. However, in an early stage of estimation, error for the nonoptimal case is much larger than that obtained for the optimal case, as can be seen from Fig.4. Due to this large estimation error, the response of nonoptimal case shows a slight instability in the early stage as shown in Fig. 3a. Comparison of the two response figures (Figs. 3a and 3b) shows that the optimal observer and optimally positioned sensors drastically decrease the control period required for effective vibration suppression. Qualitative implication of this result is that observer design as well as sensor location are very important in designing an active vibration control system.

5.4. Effect of Weighting Factor

The modal weighting parameter α and control weighting parameter β used in the performance index affect the optimal actuator and sensor locations and modal dampings through equations (16), (18) and (24). The choice of these values generally depends upon the relative significance of each vibration mode for the system performance requirement and also the force-generating capacity of the actuator. Within the parameter range specified an optimal choice of these value depends entirely upon the designer. However, it is usually very difficult to specify in terms of α and β such response characteristics as decay rate and decaying time. Therefore these parameters were arbitrary varied to investigate their effects. In this investigation, two actuators were used and their optimal values and the eigenvalues of the matrix $A+BF$ were computed.

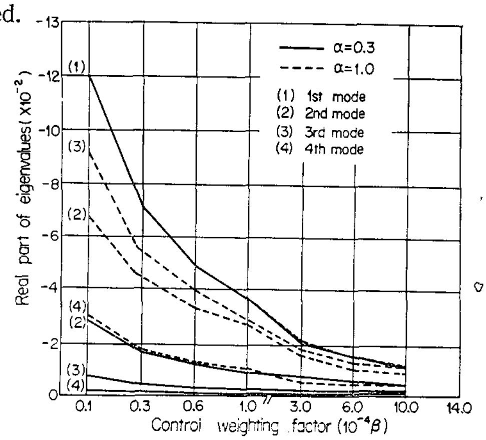


Fig. 5 Effect of weighting factor on modal damping.

In Fig.5 the effect of α and β on the negative real value of the eigenvalues which is indicative of modal damping is illustrated. The dotted lines are displayed for $\alpha=1.0$ and the solid lines for $\alpha=0.3$. The modal damping of each mode all except the first tends to increase with increasing α value. This is expected because increasing the α value means heavier weighting on those modes, as can be seen from the weighting matrix Q, thus resulting in much larger values of the modal damping. Decreasing β value increases the modal damping of all modes. Within the range $\beta=0.1\sim1.0$ the increase is quite remarkable. This is realizable, since smaller value of β indicates relatively heavier weighting on the vibration than on the control energy to be exerted by the actuator. Therefore, by appropriate choice of modal weighting α and control weighting β, desired magnitude and distributiou of modal damping over the controlled modes can be obtained. If any particular mode is excited by external disturbance, extra weighting may be necessary for that mode.

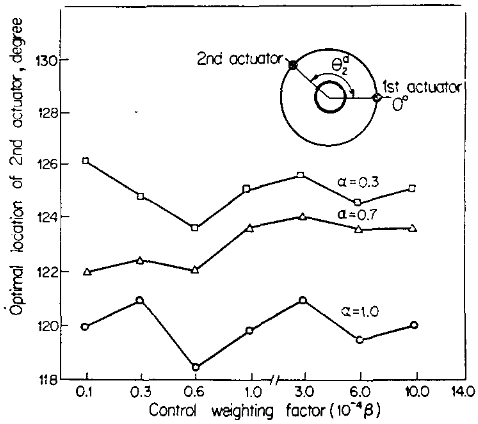


Fig. 6 Variation of optimal actuator location with weighting factor.

Fig.6 shows how the optimal position of the second actuator varies with the α and β parameters, when the first actuator is assumed to be positioned at a reference location $\theta_1^a=0^0$. The optimal location changes little with variation of the control weighting β. The maximum angular position difference is only less than 2^0, when keeping the α value constant. The location, however, appears to have a slightly greater dependency on the modal weighting parameter α. This indicates that the optimal actuator position is more sensitive to modal distribution of the vibration over the plate than to the actuator force magnitude. With α increasing, the location becomes closer to the reference angle $\theta_1^a=0^0$ where the first actuator is positioned. The maximum angular position difference is approximately 6^0.

6. Conclusions

A design procedure based upon optimal regulator problem has been applied to an active control of a vibrating circular plate. The method determines optimal actuator and sensor loctions and utilizes an optimal observer for the feedback implementation. The optimization was made for a truncated mode approximation, based upon the assumption that the compensation techniques as proposed by Balas, Skehton and Linkins, and Canavin can eliminate the neglected modes effect, if control and observation spillovers occur. Simulation of a controlled plate response was done for a four-vibration mode approximation to evaluate performance of the optimally-designed control system. The major results of this simulation show:

1. The modal damping increases as the number of actuator increases. Therefore, the use of more actuators positioned at their optimal locations reduces the vibration level more effectively.

2. An optimally-designed observer drastically decreases the control period required for effective vibration suppression. A nonoptimal observer, however, may result in dynamic instability in an early transient stage due to large estimation error, thus requiring much longer control period.

3. The choice of weighting parameters used in the performance index greatly affects the modal damping. For an appropriate choice of the modal weighting parameter α, the modal damping of all modes increasses with decreasing the control weighting parameter β and increases very rapidly especially within the parameter range $\beta=0.1\sim1.0$. This is expected because decreasing the control weighting parameter indicates relatively heavier weighting on the vibration in minimizing the performance index. The optimal actuator locations are found to slightly vary with the modal weighting parameter but to be insensitive to the control weighting parameter. This result indicates that the modal distributions over the plate is somewhat more important in determining the optimal locations than the actuator force magnitude.

In the design method proposed here the optimization was done for a four-vibration mode approximation. This may be unrealistic in some cases when the residual mode effects can not be neglected for the active controller design. Therefore, further study may be needed to investigate the residual mode effects on the optimization problem.

Acknowledgements

The authors wish to thank the Korea Science and Engineering Foundation for supporting this investigation. The authors also acknowledge Miss Y.O. Moon for her assistance in preparation of the manuscript.

References

1. Sato, T. and Tominan, N. "Modal control of two dimensional bending vibrations," Bulletin, JSME, Vol.19, No.127, 1976.
2. R.W. Ellis and C.D. Mote, Jr., "A feedback vibration controller for circular saws," Trans. ASME Dyn. Sys. Measurement & Controɪ vol, 101, 44~49, 1978.
3. Meirovitch, L. and Oz, H. "Observer moda control of dual-spin flexible spacecraft," J. Guidance and Control, Vol.2, No.2, 101-110, 1979.
4. Meirovitch, L., VanLandingham, H.F. and Oz, H., "Distributed control of spinning flexibl spacecraft," J. Guidance and Control, Vol.2, No.5, 407-415, 1979.
5. Sae-Ung, S. and "J.T.P. Active control of building structures," J. the Engineering Mechanics Division (13689), 335-350, 1978.
6. Yang, J.N. and Giannopoulos, F., "Active tendon control of structures," ASCE. J. Engineering Mechanics Division, Vol.104,

No. EMZ, 551-568, 1978.

7. Balas, M.J. "Active control of flexible systems," J. Optimization Theory and Applications, Vol.25, No.3, 415-436, 1978.
8. Balas, M.J. and Johnson, C.R., Jr., "Adaptive control of distributed paɹameter systems: The ultimate reduced-order problem," Proceedings of IEEE, Conference on Decision and Control, 1013-1017, 1979.
9. Benhabib, R.J. and Tung, F.C., "Large structure control: system identification versus direct adaptive control," Proceedings JACC, Vol.2, 1980.
10. Benhabib, R.J., Iwens, R.P. and Jackson, R.L., "Adaptive control for large space structures," Proc. IEEE, an Conference on Decision and Control, 214-217, 1979.
11. Skelton, R.E., "A control design algorithm for flexible structures," Proceedings of IEEE, Conference on Decision and Control, 1232-1234, 1980.
12. Balas, M.J., "Some trends in large space structure control theory," Proceedings, JCC, 42-55, 1979.
13. Meirovitch, L. and Öz, H., "An assessment of methods for the control of large space structure," Proceedings, JACC, 34-41, 1979.
14. Gran, R. and Rossi, M., "A survery of the large structures control problem," Proceedings of IEEE, Conference on Decision and Control, 1002-1007, 1979.
15. Balas, M.J., "Enhanced control of flexible structure via innovation feed through," Int. J. Control, Vol.32, No.6, 983-1003, 1980
16. Balas, M.J., "Modal control of certain flexible dynamic systems," SIAM J. Control and Optimization, Vol.16, No.3, 450-462, 1978.
17. Skelton, R.E., rsd Likins, P.W., "Orthogonal filte for model error compensation in t e control of nonrigid spacecraft," J. Guidance and Control, Vol.1, No.1, 41-49, 1978.
18. Orner, P.A. and Foster, A.M., "Adesign procedure for a class of distributed parameter control systems," Trans. ASME, J. Dynamic Systems Measurement, and Control, Vol.93, No.2, 86-93, 1971.
19. Mote, Jr., C.D., "Stability of circular plate subjected to moving loads," J. The Franklin Institute, Vol.290, No.4, 329-343, 1970.
20. Athands, M. and Falb, P.L., "Optimal control," McGraw-Hill N.Y., 1966.
21. Levine, W.S. and Athans, M., "On the determination of the optimal constant output feedback gains for linear multivariable system," Trans. Automatic Control, Vol. AC-15, No.5, 44-48, 1970.
22. Kuester, J.L. and Mize, J., "Optimization technique with Fortran," McGraw-Hill, N.Y., 1973.
23. Kleinman, D.L., "An iterative technique for Ricatti equation computation," IEEE, Trans. Automatic Control, Vol. AC-13, 114-115, 1968.