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SOME INTEGRAL TRANSFORMATIONS INVOLVING THE H-FUNCTION OF SEVERAL VARIABLES

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1. Introduction

Following the notations explained fairly fully by Srivastava and Panda [15] and [16], the H-function of several complex variables $z_1, \cdots, z_r$ is defined as below by means of the multiple contour integral:

$$H^{O,\ \lambda:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\left[\begin{matrix}[(a):\ \theta',\ \cdots,\ \theta^{(r)}]:\ [(b'):\ \phi'];\ \cdots;\ [(b^{(r)}):\ \phi^{(r)}];\ z_1\\ [(c):\ \varepsilon',\ \cdots,\ \varepsilon^{(r)}:\ [(d'):\ \delta'];\ \cdots;\ [(d^{(r)}:\ \delta^{(r)}];\ z_r\end{matrix}\right]$$
$$=\frac{1}{(2\pi w)^r}\int_{L_1}\cdots\int_{L_r} R_1(s_1)\cdots R_r(s_r)T(s_1,\ \cdots,\ s_r)z_1^{s_1}\cdots z_r^{s_r}ds_1\cdots ds_r,\quad w=\sqrt{-1} \tag{1.1}$$

where

$$R_i(s_i)=\frac{\prod_{j=1}^{u^{(i)}}\Gamma[d_j^{(i)}-\delta_j^{(i)}s_i]\prod_{j=1}^{v^{(i)}}\Gamma[1-b_j^{(i)}+\phi_j^{(i)}s_i]}{\prod_{j=u^{(i)}+1}^{D^{(i)}}\Gamma[1-d_j^{(i)}+\delta_j^{(i)}s_i]\prod_{j=v^{(i)}+1}^{B^{(i)}}\Gamma[b_j^{(i)}-\phi_j^{(i)}s_i]}\quad i=1,\ \cdots,\ r; \tag{1.2}$$

$$T(s_1,\ \cdots,\ s_n)=\frac{\prod_{j=1}^{\lambda}\Gamma[1-a_j+\sum_{i=1}^{k}\theta_j^{(i)}s_i]}{\prod_{j=\lambda+1}^{A}\Gamma[a_j-\sum_{i=1}^{r}\theta_j^{(i)}s_i]\prod_{j=1}^{C}\Gamma[1-c_j+\sum_{i=1}^{r}\varepsilon_j^{(i)}s_i]} \tag{1.3}$$

an empty product is interpreted as 1, the coefficients $\theta_j^{(i)}$, $j=1, \cdots, A$; $\phi_j^{(i)}$, $j=1, \cdots, B^{(i)}$; $\varepsilon_j^{(i)}$, $j=1, \cdots, C$; $\delta_j^{(i)}$, $j=1, \cdots, D^{(i)}$; and $i=1, \cdots, r$, are positive numbers, and λ, $u^{(i)}$, $v^{(i)}$, A, $B^{(i)}$, C, $D^{(i)}$ are integers such that $0\leqq\lambda\leqq A$, $0\leqq u^{(i)}\leqq D^{(i)}$, $C\geqq 0$, and $0\leqq v^{(i)}\leqq B^{(i)}$, $i=1, \cdots, r$. The contour L_i in the complex s_i-plane is of the Mellin-Barnes type which runs from $-w_\infty$ to $+w_\infty$ with indentations, if necessary, in such a manner that all the poles of $\Gamma[d_j^{(i)}-\delta_j^{(r)}s_i]$, $j=1, \cdots, u^{(i)}$, are to the right, and those of $\Gamma[1-b_j^{(i)}+\phi_j^{(i)}s_i]$, $j=1, \cdots, v^{(i)}$, and $\Gamma[1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}s_i]$, $j=1, \cdots, \lambda$, to the left, of L_i, the various parameters being so restricted that the poles are all simple and none of them coincide; and with the points $z_i=0$, $i=1, \cdots, r$, being tacitly excluded, the multiple integral in (1.1) converges absolutely if

$$|\arg(z_i)| < \frac{1}{2} T_i \pi, \quad i=1, \cdots, r \tag{1.4}$$

where

$$T_i = \sum_{j=1}^{\lambda} \theta_j^{(i)} - \sum_{j=\lambda+1}^{A} \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{C} \varepsilon_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)}$$
$$- \sum_{j=u^{(i)}+1}^{D^{(i)}} \delta_j^{(i)} > 0, \quad i=1, \cdots, r. \tag{1.5}$$

Whenever, there is no ambiguity or confusion, we shall use a contracted notation and write the first member of (1.1) in the abbreviated form

$$H_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}^{O,\ \lambda:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})} \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix} \tag{1.6}$$

Also

$$H_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}^{O,\ \lambda:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})} \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix} = \begin{cases} O(|z_1|^{\alpha_1} \cdots |z_r|^{\alpha_r}, & \max\{|z_1|, \cdots, |z_r|\} \to 0 \\ O(|z_1|^{\beta_1} \cdots |z_r|^{\beta_r}, \ \lambda \equiv 0, & \min\{|z_1|, \cdots, |z_r|\} \to \infty, \end{cases} \tag{1.7}$$

where, with $i=1, \cdots, r$,

$$\begin{cases} \alpha_i = \min\{\mathrm{Re}(d_j^{(i)}/\delta_j^{(i)}\}, \ j=1, \cdots, u^{(i)} \\ \beta_i = \max\{\mathrm{Re}(b_j^{(i)}-1)/\phi_j^{(i)}\}, \ j=1, \cdots, v^{(i)}. \end{cases} \tag{1.8}$$

Again, throughout the present paper, we employ in abbreviation (a) to denote the sequence of A parameters $a_1, \cdots, a_A$; for each $i=1, \cdots, r$, $(b^{(i)}$ abbreviates the sequence of $B^{(i)}$ parameters $b_j^{(i)}$, $j=1, \cdots, B^{(i)}$, with similar interpretations for (c), $(d^{(i)})$, etc., $i=1, \cdots, r$; it will be understood, for example, that $b^{(1)} = b'$, $b^{(2)} = b''$, and so on. Also, for the sake of brevity, we use the following contracted notations:

$$((a))_n = \prod_{j=1}^{A} (a_j)_n, \ ((b^{(i)}))_n = \prod_{j=1}^{B^{(i)}} (b_j^{(i)})_n, \quad i=1, \cdots, r, \text{ etc.}, \tag{1.9}$$

where $(a)_n$ is the Pochhamer symbol defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1, & \text{if } n=0, \\ a(a+1)\cdots(a+n-1), & \text{if } n=1, 2, 3, \cdots \end{cases} \tag{1.10}$$

2. Integral transformations

We now state our main results by the following integral transformations:

$$\int_0^1 x^{w-1}(1-x)^{\beta} F_{\iota}(a;\ b;\ d,\ e,\ 1+\alpha,\ 1+\beta;\ y,\ z,\ -xt,\ (1-x)t)$$

$$\times H^{0,\ \lambda:(u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\begin{bmatrix} z_1\ x^{h_1} \\ \vdots \\ z_r\ x^{h_r} \end{bmatrix}dx$$

$$=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n(-t)^n\Gamma(1+\beta+n)}{(1+\alpha)_n(1+\beta)_n L^n}F_4\ (a+n,\ b+n;\ d,e;\ y,z)$$

$$H^{0,\ \lambda+2:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A+2;\ C+2:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\begin{bmatrix}[1-\omega;\ h_1,\ \cdots,\ h_r], & [1+\alpha-\omega:h_1,\ \cdots,\ h_r], \\ [1-\omega+\alpha+n:h_1,\ \cdots,\ h_r], & [-\beta-\omega-n:h_1,\ \cdots,\ h_r],\end{bmatrix}$$

$$\left.\begin{matrix}[(a):\ \theta',\ \cdots,\ \theta^{(r)}];\ [(b'):\ \phi'];\ \cdots;\ [(b^{(r)}):\phi^{(r)}]: & z_1 \\ [(c):\ \varepsilon',\ \cdots,\ \varepsilon^{(r)}]:\ [(d'):\ \delta'];\ \cdots;\ [(d^{(r)}):\delta^{(r)}]: & \vdots \\ & z_r\end{matrix}\right], \tag{2.1}$$

where $\mathrm{Re}(\beta)>-1$, $\mathrm{Re}(\omega+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)})>0$, $j=1,\ \cdots,\ u^{(i)}$, $h_i>0$, $|t|<1$ and conditions corresponding appropriately to (1.4) and (1.5) are assumed to hold for the multivariable H-functions involved.

$$\int_0^t x^{-p-1}(t-x)^{p-1}e^{-q/x}\ H^{0,\ 0:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A,\ C:\ [B'D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\begin{bmatrix} z_1\left(\frac{t-x}{2}\right)^{h_1} \\ \vdots \\ z_r\left(\frac{t-x}{2}\right)^{h_r}\end{bmatrix}dx$$

$$=t^{p-1}\ \exp\left(\frac{-q}{t}\right)q^{-p}\ H^{0,\ 1:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A+1,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}$$

$$\begin{bmatrix}[1-p:\ h_1,\ \cdots,\ h_r],\ [(a):\ \theta',\ \cdots,\ \theta^{(r)}]:\ [(b'):\ \phi'];\ \cdots;\ [(b^{(r)}):\ \phi^{(r)}]; & z_1(t/q)^{h_1} \\ [(c):\ \varepsilon',\ \cdots,\ \varepsilon^{(r)}]:\ [(d'):\ \delta'];\ \cdots;\ [(d^{(r)}:\ \delta^{(r)}]; & \vdots \\ & z_r(t/q)^{h_r}\end{bmatrix}, \tag{2.2}$$

where $\mathrm{Re}(p)>0$, $\mathrm{Re}\left[p+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}\right]>0$, $h_i>0$, $t>0$, $i=1,\ \cdots,$ and conditions corresponding appropriately to (1.4) and (1.5) are assumed to hold for the multivariable H-functions involved.

$$\int_0^{\infty} e^{-\omega t}\ t^{p-1}\ E(\alpha,\ \beta;\ \omega t)\ H^{0,\ 0:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\begin{bmatrix} z_1 t^{h_1} \\ \vdots \\ z_r t^{h_r}\end{bmatrix}dt$$

$$=\frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta)}{(\omega)^p}\ H^{0,\ 1;\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A+1,\ C+1:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}$$

$$\times\begin{bmatrix}[p+\beta:\ h_1,\ \cdots,\ h_r],\ [(a):\ \theta',\ \cdots,\ \theta^{(r)}]:\ [(b'):\ \phi'];\ \cdots;\ [(b^{(r)}:\ \phi^{(r)}]; & z_1/w^{h_1} \\ [p+\alpha+\beta:\ h_1,\ \cdots h_r],\ [(c):\varepsilon',\ \cdots,\ \varepsilon^{(r)}]:\ [(d'):\ \delta'];\ \cdots;\ [(d^{(r)}):\ \delta^{(r)}]; & \vdots \\ & z_r/w^{h_r}\end{bmatrix}, \tag{2.3}$$

where $\mathrm{Re}[p+\alpha+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}]>0$, $\mathrm{Re}[p+\beta+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}]>0$, $h_i>0$,

$\mathrm{Re}(\omega)>0$,

$i=1, \cdots, r,$ and the conditions mentioned in (1.4) and (1.5) are satisfied.

$$\int_0^\infty x^{\omega-\frac{1}{2}}(x+p)^{-\omega}(x+q)^{-\omega}\, H^{0,\ \lambda:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\left[\begin{matrix} z_1\{x/(x+p)(x+q)\}^{h_1} \\ \vdots \\ z_r\{x/(x+p)(x+q)\}^{h_r}\end{matrix}\right] dx$$

$$=\sqrt{\pi}(\sqrt{p}+\sqrt{q})^{1-2\omega}\, H^{0;\ \lambda+1:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A+1,\ C+1:\ [B',\ D'];\ \cdots;\ [B^{(r)},D^{(r)}]}\left[\begin{matrix}[\frac{3}{2}-\omega:\ h_1,\ \cdots,\ h_r], \\ [1-\omega;\ h_1,\ \cdots,\ h_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\ \theta',\ \cdots,\ \theta^{(r)}]:\ [(b'):\ \phi'];\ \cdots;\ [(b^{(r)}):\ \phi^{(r)}]: \\ [(c):\ \varepsilon',\ \cdots,\ \varepsilon^{(r)}]:\ [(d'):\ \delta'];\ \cdots;\ [(d^{(r)}):\ \delta^{(r)}]:\end{matrix}\ \begin{matrix} z_1(\sqrt{p}+\sqrt{q})^{-2h_1} \\ \vdots \\ z_r(\sqrt{p}+\sqrt{q})^{-2h_r}\end{matrix}\right], \tag{2.3}$$

where $\mathrm{Re}(\omega)>0$, $\mathrm{Re}\left(\omega+\sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}-\frac{1}{2}\right)>0$, $h_i>0$, p, q are positive, $i=1, \cdots, r,$ and the conditions mentioned in (1.4) and (1.5) are satisfied.

Proofs of the integral formulas (2.1) to (2.4)

To establish the integral formula (2.1), we start with the following result [13]:

$$F_C(a,\ b;\ d;\ e,\ 1+\alpha,\ 1+\beta;\ y,\ z,\ -xt,\ (1-x)t) = \sum_{n=0}^{\infty}\frac{(a)_n(b)_n(t)^n}{(1+\alpha)_n(1+\beta)_n}$$

$$F_4(a+n,\ b+n;\ d, e; y,\ z)\ P_n^{(\alpha,\ \beta)}(1-2x),\quad 0<x<1,\quad |t|<1 \tag{2.5}$$

Now, multiplying both the sides of (2.5) by $f(x)$, integrate it with respect to x between the limits 0 to 1, and then interchange the order of integration and summation (which is permissible) of the result thus obtained on its right handside, we find that

$$\int_0^1 F_c\,(a,\ b:\ d,\ e,\ 1+\alpha,\ 1+\beta;\ y,\ z,\ -xt,\ (1-x)\,t)f(x)\ dx$$

$$=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n t^n}{(1+\alpha)_n(1+\beta)_n}\ F_4(a+n,\ b+n;\ d,\ e;\ y,\ z)$$

$$\int_0^1 P_n^{(\alpha,\beta)}\ (1-2x)\ f(x)\ dx. \tag{2.6}$$

If, in the integral relationship (2.6), we set

$$f(x)=x^{\omega-1}\ (1-x)^{\beta}\ H^{0,\ \lambda:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A,\ C:\ [B',\ D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}\left[\begin{matrix} z_1\ x^{h_1} \\ \vdots \\ z_r\ x^{h_r}\end{matrix}\right], \tag{2.7}$$

evaluate the x-integral by means of a familiar result [16, p.131 (2.2)], we shall be led fairly easily to our integral formula (2.1) under the various (sufficient) conditions stated already.

In order to derive the integral formula (2.2) to (2.4), we apply the well known results [5, p.286, 3. 197(7); p.399, 3.471(3)] and [7] respectively, and interpret the resulting integral as an H-function of r variables, we shall arrive at the desired integral formula (2.2), (2.3) and (2.4) under the conditions stated already. The various steps that mentioned are essentially similar to those in our derivation of (2.1), and we omit the details.

3. Applications

At the outset we should remark that the integral formulas (2.1) through (2.4) of the present paper are quite general in character: indeed these and their various specialised forms considered in the proceeding sections can be suitably applied to derive several intergral formulas involving a remarkably wide variety of useful functions or product of several such functions; which are expressible in terms of the E, F, G or H functions of one or more variables. Thus the various results presented in this paper and of course, in our earlier works [9] through [11] can be first reduced fairly easily to hold for the product of several H-functions and hence also G-functions, E-functions or Wright's generalized hypergeometric functions and so on, of different arguments. In passing we should recall the fact that a great many of the special functions that occur in problems of applied mathematics and mathematical analysis can be expressed in terms of the G-function, and hence also as an H-function. Evidently, therefore, our results should apply not only to the simple special functions of mathematical physics such as the Legendre functions, the Bessel functions $J_{\nu}(x)$ or $I_{\nu}(x)$, the Whittaker function $M_{u,v}(x)$, the classical orthogonal polynomials of Hermite, Jacobi, Laguerre, etc., which are all particular cases of the hypergeometric function ${}_pF_q$, but also to the relatively more involved Bessel functions $K_{\nu}(x)$ or $Y_{\nu}(x)$, the Whittaker function $W_{u,v}(x)$, their various combinations and other related functions.

Now, we turn to the special cases of our results contained in (2.1) to (2.4). Obviously, for $r=2$, our integral formulas (2.1) to (2.4) would provide us with interesting formulas involving the H-function of two variables already given in [2], [4], [1] and [13].

If, we use the known relationship of the multivariate H-function with the generalized Lauricella function of several complex variables [15, p.272, eq. (4.7)] in our main results, we get the corresponding integral formulas for the generalized Lauricella function of several complex variables. Here it is remarkable to note that if we take $r=2$, θ's, ϕ's, δ's, ε's, h_1, h_2 to$=1$ in the results obtained by (2.4) and make use of the formulas [6, p, 231(3)], we get a result obtained by Gupta and Srivastava [3, p. 496(22)].

Again, if we take $y=z=0$ in (2.1), it gives

$$\int_0^1 x^{\omega-1}(1-x)^{\beta} F_4(a,b;\ 1+\alpha,\ 1+\beta; -xt,\ (1-x)t)$$

$$H^{0,\ \lambda:\ (u',\ v');\ \cdots;\ u^{(r)},\ (v^{(r)})}_{A,\ C:\ [B',\ D'];\ \cdots;\ [(B^{(r)},\ D^{(r)}]}\begin{bmatrix} z_1\ x^{h_1} \\ \vdots \\ z_r\ x^{h_1}\end{bmatrix} dx=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n(-t)^n\ \Gamma(1+\beta+n)}{(1+\alpha)_n\ (1+\beta)_n\ L_n}$$

$$H^{0,\ \lambda+2:\ (u',\ v');\ \cdots;\ (u^{(r)},\ v^{(r)})}_{A+2,\ C+2:\ [B', D'];\ \cdots;\ [B^{(r)},\ D^{(r)}]}$$

$$\begin{bmatrix}[1-\omega:h_1,\ \cdots,\ h_r],\ [1+\alpha-\omega:\ h_1,\ \cdots,\ h_r],\ [(a):\ \theta',\ \cdots,\ \theta^{(r)}]: \\ [1-\omega+\alpha+n:\ h_1,\ \cdots,\ h_r],\ [-\beta-\omega-n:\ h_1,\ \cdots,\ h_r],\ [(c):\ \varepsilon',\ \cdots,\ \varepsilon^{(r)}]:\end{bmatrix}$$

$$\begin{matrix}[(b');\ \phi'];\ \cdots;\ [(b^{(r)}):\ \phi^{(r)}];\ \\ [(d'):\ \delta];\ \cdots;\ [(d^{(r)}):\ \delta^{(r)}];\ \end{matrix}\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\Bigg], \tag{3.1}$$

where $\mathrm{Re}(\beta)>1$, $\mathrm{Re}\left(\omega+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}\right)>0$, $h_i>0$, $|t|<1$, $0<x<1$, $i=1,\ \cdots,\ r$, and the conditions corresponding appropriately to (1.4) and (1.5) are assumed to hold for the multivariate H-functions involved.

The above integral formula given in (3.1) appears to be new and for $r=2$ it yields a well known integral formula already obtained by Mittal [8].

In passing, we should remark here also that for $r=1$ (2.1) reduces to an another interesting integral formula given by Chaurasia [2]. Here, it is worthy to note that for $r=1$ (3.1) gives us a result recently obtained by Mittal [8].

Also, in view of the reduction formula [16], the integral formula (2.3) reduces to a result obtained earlier by Chaurasia [1, p. 129, eq. (4.2)].

In the last, if we set $r=2$, θ's, ϕ's, ε's, δ's, h_1, h_2 equal to unity, in (2.1), we get another interesting integral formula given by Sharma [13].

We conclude our present paper by the remark that in the preceding papers we have obtained multiple integral transformation involving the H-function of several complex variables with a product of two H-functions.

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