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A NOTE ON GENERIC SUBMANIFOLDS OF AN ODD-DIMENSIONAL SPHERE

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0. Introduction

Recently many papers on generic submanifolds of a Riemannian manifold have been submitted by several authors who found out a general notion of an intrinsic character of hypersurfaces by using various methods (see [3], [4], [5], [6], [7] and [8]).

But the present authors [6] studied a generic submanifold M of an odd-dimensional sphere $S^{2m+1}(1)$ under the condition that the structure tensor f induced on M and the second fundamental tensor h commute.

The purpose of the present paper is to explore the generic submanifolds of an odd-dimensional sphere tangent to the Sasakian structure vector when the structure tensor f and the second fundamental tensor h anticommute.

In 1, we recall fundamental properties and structure equations for generic submanifolds immersed in a Sasakian manifold and define the structure tensor induced on M is antinormal.

In 2, we look into the fundamental properties of submanifolds of $S^{2m+1}(1)$ and introduce some theorems for later use.

In the last 3, we characterize the generic submanifold of $S^{2m+1}(1)$ tangent to the Sasakian structure vector field.

1. Generic submanifolds of a Sasakian manifold

Let M^{2m+1} be a $(2m+1)$-dimensional Sasakian manifold covered by a system of coordinate neighborhoods $\{U ; z^h\}$ and $(F_j^{\ h}, g_{ji}, \xi^h)$ the set of structure tensors of M^{2m+1}, where here and in the sequel the indices h, i, j and k run over the range $\{1, 2, \cdots, 2m+1\}$. We then have

$$(1.1)\qquad F_j^{\ h}F_i^{\ j}=-\delta_i^h+\xi_i\xi^h,\ \ \xi_jF_i^{\ j}=0,\ \ F_j^{\ h}\xi^j=0,$$
$$\xi_j\xi^j=1,\ \ \xi_j=g_{ji}\xi^i,\ \ F_j^{\ h}F_i^{\ k}g_{hk}=g_{ji}-\xi_j\xi_i$$

and

(1.2) $$\nabla_j \xi^h = F_j^{\ h}, \quad \nabla_j F_i^{\ h} = -g_{ji}\xi^h + \delta_j^h \xi_i,$$

where ∇_j denotes the operator of covariant differentiation with respect to g_{ji}.

Let M be an n-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V ; x^a\}$, which is isometrically immersed in M^{2m+1} by the immersion $i : M \longrightarrow M^{2m+1}$, and identify $i(M)$ with M itself and represent the immersion i by $y^h = y^h(x^a)$ (throughout this paper the indices a, b, c, d and e run over the range $\{1, 2, \cdots, n\}$). If we put $B_b^{\ h} = \partial_b y^h$, $\partial_b = \partial/\partial x^b$, then $B_b^{\ h}$ are n linearly independent vectors of M^{2m+1} tangent to M. Denoting by g_{cb} the fundamental metric tensor of M, we then have

(1.3) $$g_{cb} = B_c^{\ j} B_b^{\ i} g_{ji}$$

because the immersion is isometric.

We now denote by $C_x^{\ h}$ $2m+1-n$ mutually orthogonal unit normals of M (the indices u, v, w, x, y and z run over the range $\{n+1, \cdots, 2m+1\}$). Thus, denoting ∇_c by the operator of van der Waerden-Bortolotti covariant differentiation with respect to the Christoffel symbols $\{{}_c{}^a{}_b\}$ formed with g_{cb}, we obtain equations of Gauss and Weingarten

(1.4) $$\nabla_c B_b^{\ h} = h_{cb}^{\ \ x} C_x^{\ h},$$

(1.5) $$\nabla_c C_x^{\ h} = -h_c^{\ a}{}_x B_a^{\ h}$$

respectively, where $h_{cx}^{\ \ b}$ are the second fundamental forms with respect to the normals $C_x^{\ h}$ and $h_c^{\ a}{}_x = h_{cb}^{\ \ y} g^{ab} g_{xy}$, g_{xy} being the metric tensor of the normal bundle of M given by $g_{xy} = g_{ji} C_x^{\ j} C_y^{\ i}$, and $(g^{cb}) = (g_{cb})^{-1}$.

A submanifold M of a Sasakian manifold M^{2m+1} is called a *generic* (an *anti-holomorphic*) submanifold if the normal space $N_p(M)$ of M at any point $P \in M$ is always mapped into the tangent space $T_p(M)$ by the action of the structure tensor F of the ambient manifold M^{2m+1}, that is, $FN_p(M) \subset T_p(M)$ for all $P \in M$ (see [6], [7]).

From now on, we consider throughout this paper generic submanifolds immersed in a Sasakian manifold M^{2m+1}. Then we can put in each coordinate neighborhood

(1.6) $$F_j^{\ h} B_c^{\ j} = f_c^{\ a} B_a^{\ h} - f_c^{\ x} C_x^{\ h},$$

(1.7) $$F_j^{\ h} C_x^{\ j} = f_x^{\ a} B_a^{\ h},$$

(1.8) $$\xi^h = \eta^a B_a^{\ h} + \xi^x C_x^{\ h},$$

where $f_c^{\ a}$ is a tensor field of type (1,1) defined on M, $f_c^{\ x}$ a local 1-form for each fixed index x, η^a a vector field and ξ^x a function for each fixed index x, and $f_x^{\ a} = f_c^{\ y} g^{ac} g_{yx}$.

Applying F to (1.6) and (1.7) respectively and using (1.1) and these equations, we easily find ([3], [7], [8])

(1.9) $$\begin{cases} f_c^{\ e} f_e^{\ a} = -\delta_c^{\ a} + f_c^{\ x} f_x^{\ a} + \eta_c \eta^a, \\ f_c^{\ e} f_e^{\ x} = -\eta_c \xi^x, \\ f_x^{\ e} f_e^{\ y} = \delta_x^{\ y} - \xi_x \xi^y, \\ \eta^e f_e^{\ a} = -\xi^x f_x^{\ a}, \quad \eta^e f_e^{\ x} = 0, \\ g_{de} f_c^{\ d} f_b^{\ e} = g_{cb} - f_c^{\ x} f_{xb} - \eta_c \eta_b, \\ \eta_a \eta^a + \xi_x \xi^x = 1, \end{cases}$$

where $\eta_a = g_{ea} \eta^e$. But, the last relationship follows from (1.3), (1.8) and the fact that $\xi_j \xi^j = 1$.

Putting $f_{cb} = f_c^{\ a} g_{ba}$ and $f_{cx} = f_c^{\ y} g_{yx}$, then we easily verify from (1.9) that $f_{cb} = -f_{bc}$ and $f_{cx} = f_{xc}$.

When the submanifold M is a hypersurface of M^{2m+1}, (1.9) becomes the so-called (f, g, u, v, λ)-structure ([1], [2]), where we have put $f_c^{\ x} = u_c$, $\eta^a = v^a$, $\xi^x = \xi_x = \lambda$.

The aggregate $(f_c^{\ a}, g_{cb}, f_c^{\ x}, \eta^a, \xi^x)$ satisfying (1.9) is said to be *antinormal* ([3], [7]) if

(1.10) $$h_c^{\ e}{}_x f_e^{\ a} + f_c^{\ e} h_e^{\ a}{}_x = 0$$

holds, or equivalently

(1.11) $$h_{ce}^{\ \ x} f_b^{\ e} = h_{be}^{\ \ x} f_c^{\ e}.$$

Transvecting (1.11) with $f_a^{\ b}$ and using the first relation of (1.9), we find

$$h_{ce}^{\ \ x}(-\delta_a^{\ e} + f_a^{\ z} f_z^{\ e} + \eta_a \eta^e) = h_{be}^{\ \ x} f_c^{\ e} f_a^{\ b},$$

from which, taking the skew-symmetric part with respect to the indices c and a,

$$(1.12)\qquad (h_{ce}{}^{x}f_{z}^{e})f_{a}^{z}-(h_{ae}{}^{x}f_{z}^{e})f_{c}^{z}+(h_{ce}{}^{x}\eta^{e})\eta_{a}-(h_{ae}{}^{x}\eta^{e})\eta_{c}=0.$$

Differentiating (1.6)~(1.8) covariantly along M and using (1.1)~(1.5), we find respectively (see [3], [6], [7])

$$(1.13)\qquad \nabla_{c}f_{b}^{a}=-g_{cb}\eta_{a}+\delta_{c}^{a}\eta_{b}+h_{cb}{}^{x}f_{x}^{a}-h_{cax}f_{b}^{x},$$

$$(1.14)\qquad \nabla_{c}f_{b}^{x}=g_{cb}\xi^{x}+h_{ce}{}^{x}f_{b}^{e},$$

$$(1.15)\qquad h_{ecx}f_{e}^{y}=h_{c}^{ey}f_{ex},$$

$$(1.16)\qquad \nabla_{c}\eta_{b}=f_{cb}+h_{cb}{}^{x}\xi_{x},$$

$$(1.17)\qquad \nabla_{c}\xi^{x}=-f_{c}^{x}-h_{ce}{}^{x}\eta^{e}$$

with the aid of (1.6)~(1.8).

2. Intrinsic properties of submanifolds of $S^{2m+1}(1)$

Let M be an n-dimensional submanifold of an odd-dimensional unit sphere $S^{2m+1}(1)$, then the equations of Gauss, Codazzi and Ricci for M are respectively given by

$$(2.1)\qquad K_{dcb}{}^{a}=\delta_{d}^{a}g_{cb}-\delta_{c}^{a}g_{db}+h_{d}^{a}{}_{x}h_{cb}{}^{x}-h_{c}^{a}{}_{x}h_{db}{}^{x},$$

$$(2.2)\qquad \nabla_{d}h_{cb}{}^{x}-\nabla_{c}h_{db}{}^{x}=0,$$

$$(2.3)\qquad K_{dcy}{}^{x}=h_{de}{}^{x}h_{c}^{e}{}_{y}-h_{ce}{}^{x}h_{d}^{e}{}_{y},$$

$K_{dcb}{}^{a}$ and $K_{dcy}{}^{x}$ being the curvature tensor of M and that of the connection induced in the normal bundle respectively.

From the Ricci identity

$$\nabla_{d}\nabla_{c}h_{ba}{}^{x}-\nabla_{c}\nabla_{d}h_{ba}{}^{x}=-K_{dcb}{}^{e}h_{ae}{}^{x}-K_{dca}{}^{e}h_{be}{}^{x},$$

we have

$$(2.4)\qquad (g^{da}\nabla_{d}\nabla_{a}h_{cb}{}^{x})h^{cb}{}_{x}-(\nabla_{c}\nabla_{b}h^{x})h^{cb}{}_{x}=K_{ce}h_{be}{}^{y}h^{cb}{}_{y}-K_{dcba}h^{day}h^{cb}{}_{y}$$

because of (2.2), where we have put $h^{x}=g^{cb}h_{cb}{}^{x}$, $K_{dcba}=K_{dcb}{}^{e}g_{ae}$, $K_{cb}=g^{da}K_{dcba}$.

We have from (2.1)

$$(2.5)\qquad K_{cb}=(n-1)g_{cb}+h^{x}h_{cbx}-h_{c}^{e}{}_{x}h_{be}{}^{x},$$

which implies

$$(2.6)\qquad K=n(n-1)+h^{x}h_{x}-h_{cb}{}^{x}h^{cb}{}_{x},$$

K being the scalar curvature of M.

We now suppose that the connection induced in the normal bundle of M is flat, that is, $K_{dcy}{}^{x}=0$. Then we have from (2.3)

$$h_{de}{}^{x} h^{e}_{c\ y}=h_{ce}{}^{x} h^{e}_{d\ y}. \tag{2.7}$$

Substituting (2.1) and (2.5) into (2.4) and taking account of the identity

$$\frac{1}{2}\Delta(h_{cb}{}^{x} h^{cb}{}_{x})=(g^{da}\nabla_d\nabla_a h_{cb}{}^{x})h^{cb}{}_{x}+\|\nabla_d h_{cb}{}^{x}\|^2,$$

we have

$$\frac{1}{2}\Delta(h_{cb}{}^{x} h^{cb}{}_{x})=nh_{cb}{}^{x} h^{cb}{}_{x}-h_x h^x+h^x h_{cex} h_b{}^{ey} h^{cb}{}_{y}-(h_{cb}{}^{x} h^{cby})(h_{dax} h^{da}{}_{y}) \tag{2.8}$$
$$+(\nabla_c\nabla_b h^x)h^{cb}{}_{x}+\|\nabla_d h_{cb}{}^{x}\|^2$$

with the help of (2.7), where Δ is the Laplacian given by $\Delta=g^{da}\nabla_d\nabla_a$.

For a submanifold of an m-dimensional sphere S^m, K. Yano and M. Kon [10] proved the following theorem:

THEOREM A. *Let M be a complete n-dimensional submanifold of S^m with flat normal connection. If the second fundamental form of M is parallel, then M is a small sphere, a great sphere or a pythagorean product of a certain number of spheres. Moreover, if M is of essential codimension $m-n$, then M is a pythagorean product of the form*

$$S^{p_1}(r_1)\times\cdots\times S^{p_N}(r_N),\quad r_1^2+\cdots+r_N^2=1,\ N=m-n+1,$$

or a pythagorean product of the form

$$S^{p_1}(r_1)\times\cdots\times S^{p_{N'}}(r_{N'})\subset S^{m-1}(r)\subset S^m,\ r^2+\cdots+r^2=r^2<1,\ N'=m-n.$$

On the other hand, by K. Yano and M. Kon [10] and K. Yano and S. Ishihara [11], we can easily obtain the following theorem:

THEOREM B. *Let M be a complete minimal submanifold of dimension n immersed in S^m with parallel second fundamental form and flat normal connection. If the length of second fundamental form is constant, then M is a great sphere of S^m or a pythagorean product of the form*

$$S^{p_1}(r_1)\times\cdots\times S^{p_N}(r_N),\quad r_t=\sqrt{p_t/n}\ \ (t=1,\ \cdots,\ N),$$

and with essential codimension $N-1$, where $p_1, \cdots, p_N\geqq 1$, $p_1+\cdots+p_N=n$.

3. Generic submanifolds of an odd-dimensional sphere $S^{2m+1}(1)$ tangent to the Sasakian structure vector field

In this section we assume that the Sasakian structure vector field defined on S^{2m+1} (1) is tangent to the submanifold M, that is, $\xi^x=0$ identically, the normal connection of M is flat and (1.11) is satisfied. Then (1.9) reduces to

$$(3.1)\qquad \begin{cases} f_c^{\ e} f_e^{\ a} = -\delta_c^{\ a} + f_c^{\ x} f_x^{\ a} + \eta_c \eta^a, \\ f_c^{\ e} f_e^{\ x} = 0, \ \ \eta^e f_e^{\ a} = 0, \ \ \eta^e f_e^{\ x} = 0, \\ f_x^{\ e} f_e^{\ y} = \delta_x^{\ y}, \\ g_{de} f_c^{\ d} f_b^{\ e} = g_c^{\ b} - f_c^{\ x} f_{xb} - \eta_c \eta_b, \\ \quad \eta_e \eta^e = 1 \end{cases}$$

and (1.14)~(1.17) to

$$(3.2)\qquad \nabla_c f_b^{\ x} = h_{ce}^{\ \ x} f_b^{\ e},$$

$$(3.3)\qquad h_{ce}^{\ \ x} f_e^{\ y} = h_c^{\ ey} f_{ex},$$

$$(3.4)\qquad \nabla_c \eta_b = f_{cb},$$

$$(3.5)\qquad h_{ce}^{\ \ x} \eta^e = -f_c^{\ x}.$$

LEMMA 3.1. *Let M be an n-dimensional generic submanifold with flat normal connection of an odd-dimensional unit sphere S^{2m+1} (1). If the structure induced on M is antinormal and the Sasakian structure vector on S^{2m+1} (1) is tangent to M, then we have*

$$(3.6)\quad \frac{1}{2}\Delta(h_{cb}^{\ \ x} h^{cb}_{\ \ x}) = 2(n-m-1)\{2h_{cb}^{\ \ x} h^{cb}_{\ \ x} + h_x h^x\} + (\nabla_c \nabla_b h^x) h^{cb}_{\ \ x} + \|\nabla_d h_{cb}^{\ \ x}\|^2.$$

PROOF. Transvecting (1.11) with $f_y^{\ b} f_d^{\ c}$ and taking account of (3.1),

$$-h_{bd}^{\ \ x} f_y^{\ b} + (h_{be}^{\ \ x} \eta^e) f_y^{\ b} \eta_d + (h_{be}^{\ \ x} f_y^{\ b} f_z^{\ e}) f_d^{\ z} = 0.$$

from which, using (3.5),

$$(3.7)\qquad h_{ce}^{\ \ x} f_y^{\ e} = P_{yz}^{\ \ x} f_c^{\ z} - \delta_y^{\ x} \eta_c,$$

where we have put

$$P_{yz}^{\ \ x} = h_{cb}^{\ \ x} f_y^{\ c} f_z^{\ b}.$$

We put $P_{yzx} = P_{yz}^{\ \ w} g_{wx}$, then we see from (3.3) that P_{yzx} is symmetric for all indices.

If we transvect (1.11) with f^{cb} and make use of (3.1), then we get

$$h^x = h_{ce}{}^x f_z{}^e + h_{ce}{}^x \eta^c \eta^e,$$

or, use (3.5) and (3.7),

$$h^x = P^x, \tag{3.8}$$

where we have put $P^x = g^{yz} P_y{}^x{}_z$.

Since the normal connection of the submanifold M is flat, by transvecting (2.7) with $f_z{}^b$ and taking account of (3.5) and (3.7), we get

$$P_{yz}{}^w (P_{wv}{}^x f_c{}^v - \delta_w{}^x \eta_c) + g_{yz} f_c{}^x = P_{wz}{}^x (P_{vy}{}^w f_c{}^v - \delta_y{}^w \eta_c) + \delta_z{}^x f_{cy},$$

from which, transvecting $f_u{}^c$ and using (3.1),

$$P_{yz}{}^w P_{wu}{}^x + g_{yz} \delta_u{}^x = P_{wz}{}^x P_{uy}{}^w + \delta_z{}^x g_{yu}. \tag{3.9}$$

Contraction with respect to the indices x and z yields

$$P_{yzx} P_u{}^{zx} = P_z P_{yu}{}^z + (p-1) g_{yu}, \tag{3.10}$$

where $p = 2m+1-n$, and consequently

$$P_{xyz} p^{xyz} = h_x h^x + p(p-1) \tag{3.11}$$

with the aid of (3.8).

Differentiating (3.7) covariantly and substituting (3.2) and (3.4), we find

$$(\nabla_d h_{ce}{}^x) f_y{}^e + h_c{}^{ex} h_{day} f_e{}^a = (\nabla_a P_{yz}{}^x) f_c{}^z + P_{yz}{}^x h_{de}{}^z f_c{}^e - \delta_y{}^x f_{dc},$$

from which, taking the skew-symmetric part with respect to d and c, and using (1.11) and (2.2),

$$2 h_c{}^{ex} h_{eay} f_d{}^a = (\nabla_d P_{yz}{}^x) f_c{}^z - (\nabla_c P_{yz}{}^x) f_d{}^z - 2\delta_y{}^x f_{dc}. \tag{3.12}$$

If we transvect (3.12) with $f_w{}^d$ and use (3.1), then we obtain

$$\nabla_d P_{yz}{}^x = (f_z{}^e \nabla_e P_{yw}{}^x) f_c{}^w.$$

Substituting this into (3.12) and using $P_{yz}{}^x = P_{zy}{}^x$, we have

$$h_{cex} h_a{}^e{}_y f_d{}^a = g_{yx} f_{cd}.$$

Transvection $f_b{}^d$ gives

$$h_{cex} h_a{}^{ey} (-\delta_b{}^a + f_b{}^z f_z{}^a + \eta_b \eta^a) = g_{yx} (g_{cb} - f_c{}^z f_{bz} - \eta_c \eta_b),$$

from which, using (3.5) and (3.7),

$$(3.13)\quad h_{cex}h_b{}^e{}_y=P_{yz}{}^wP_{wvx}f_c^{\ v}f_b^{\ z}-P_{yzx}(f_b^{\ z}\eta_c+f_c^{\ z}\eta_b)+2g_{yx}\eta_c\eta_b+f_{cx}f_{by}$$
$$-g_{yx}(g_{cb}-f_c^{\ z}f_{bz}).$$

Transvecting (3.13) with g^{cb} and taking account of (3.1) and (3.10), we get

$$h_{cbx}h^{cb}{}_y=P^zP_{zyx}+(2p+2-n)g_{yz},$$

from which,

$$(3.14)\qquad h_{cb}{}^xh^{cb}{}_x=h_xh^x+p(2p+2-n)$$

and

$$(3.15)\quad (h_{cb}{}^xh^{cby})(h_{dax}h^{da}{}_y)=P_{yzx}h^yh^zh^x+(p-1)h_xh^x+2(2p+2-n)h_xh^x$$
$$+p(2p+2-n)^2$$

with the help of (3.8) and (3.10).

Since we have from (3.10) and (3.13)

$$h_{ce}{}^xh_b{}^e{}_x=P^xP_{xyz}f_c^{\ y}f_b^{\ z}+2p(f_b^{\ x}f_{cx}+\eta_c\eta_b)-P^x(f_{bx}\eta_c+f_{cx}\eta_b)-pg_{cb},$$

it follows that

$$(3.16)\qquad h^xh_{bax}h_c{}^a{}_yh^{cby}=P_{yzx}h^yh^zh^x+(2p+1)h_xh^x$$

because of (3.7)~(3.10).

Substituting (3.14) and (3.15) into (2.8), we find

$$\frac{1}{2}\Delta(h_{cb}{}^xh^{cb}{}_x)=(n-p-1)\{3h_xh^x+2p(2p+2-n)\}+(\nabla_c\nabla_bh^x)h^{cb}{}_x+\|\nabla_dh_{cb}{}^x\|^2,$$

from which, using (3.14) and the fact that $p=2m+1-n$,

$$\frac{1}{2}\Delta(h_{cb}{}^xh^{cb}{}_x)=2(n-m-1)\{2h_{cb}{}^xh^{cb}{}_x+h_xh^x\}+(\nabla_c\nabla_bh^x)h^{cb}{}_x+\|\nabla_dh_{cb}{}^x\|^2.$$

Thus, Lemma 3.1 is completely proved.

LEMMA 3.2. *Let M be an n-dimensional generic submanifold with flat normal connection of an odd-dimensional sphere $S^{2m+1}(1)$ whose Sasakian structure vector is tangent to M. If the structure tensor induced on M is antinormal and the mean curvature vector is parallel in the normal bundle, then M is of dimension $m+1$ with parallel second fundamental form.*

PROOF. Since the mean curvature vector is parallel in the normal bundle, it is shown from (3.14) that $h_{cb}{}^xh^{cb}{}_x$ is constant. Thus (3.6) impiles

$$(3.17)\qquad (n-m-1)\{2h_{cb}{}^xh^{cb}{}_x+h_xh^x\}=0$$

and

$$\nabla_d h_{cb}{}^x = 0. \tag{3.18}$$

But, we can see from the first relationship of (3.1) that $n \geqq m+1$. If $2h_{cb}{}^x h^{cb}{}_x + h_x h^x = 0$ and hence $h_{cb}{}^x = 0$, then (3.5) becomes $P_{yz}{}^x f_c{}^z - \delta_y{}^x \eta_c = 0$. Transvection with η^c gives $p = 2m+1-n$ because of (3.1). It contradicts the fact that the codimension $p \geqq 1$. And consequently $n = m+1$. Therefore, our lemma is proved.

On the other hand, the submanifold M does not admit any umbilical section because of (3.7) and hence M is of essential codimension m. Thus, combining Lemma 3.1, 3.2 and this fact with Theorem A and B in 2, we have

THEOREM 3.3. *Let M be an n-dimensional complete generic submanifold with flat normal connection of an odd-dimensional unit sphere $S^{2m+1}(1)$. Suppose that the mean curvature vector is parallel in the normal bundle and the induced structure on M is antinormal. If the Sasakian structure vector defined on $S^{2m+1}(1)$ is tangent to the submanifold, then M is a pythagorean product of the form*

$$S^1(r_1) \times \cdots \times S^1(r_{m+1}), \quad r_1^2 + \cdots + r_{m+1}^2 = 1, \quad r_t \neq \sqrt{1/m+1}, \quad (t = 1, 2, \cdots, m+1).$$

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REFERENCES

[1] Blair, D.E., G.D. Ludden and K. Yano, ***Hypersurfaces of an odd-dimensional sphere***, J. Diff. Geo., 5(1971), 479—486.

[2] Ishihara, S. and U-H. Ki, *Complete Riemannian manifolds with* (f, g, u, v, λ)*-structure*, J. Diff. Geo., 8(1873), 541—554.

[3] Ki, U-H., *On generic submanifolds with antinormal structure of an odd-dimensional sphere*, to appear in Kyungpook Math. J.

[4] ______ and J.S. Pak, *Generic submanifolds of an even-dimensional Euclidean space*, to J. Diff. Geo., 16(1981), 293—303.

[5] ____________________ and Y.H. Kim, *Generic submanifolds of a complex projective space with parallel mean curvature vector*, Kodai Math J., 4(1981), 137—151.

[6] ______, and _______, *Generic submanifolds of an odd-dimensional sphere*, to appear.

[7] ____________________, *Generic submanifolds with parallel mean curvature vector of*

an odd-dimensional sphere, Kodai Math. J., 4(1981), 353—370.

[8] Okumura, M., *Submanifolds of real codimension of a complex projective space*, Atti della Accademia Nazionale dei Lincei, 4(1975), 544—555.

[9] Pak, J.S., *Note on anti-holomorphic submanifolds of real codimension of a complex projective space*, Kyungpook Math. J., 20(1980), 59—76.

[10] Yano, K. and M. Kon, *Generic submanifolds*, Annali di Mat., 123(1980), 59—92.

[11] ______, and S. Ishihara, *Submanifolds with parallel mean curvature vector*, J. Diff. Geo., 6(1971), 95—118.