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A NOTE ON THE H-FUNCTION OF SEVERAL COMPLEX VARIABLES*

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In the present note an attempt has been made to derive a finite summation for the H-function of several complex variables introduced by Srivastava and Panda.

For H-function of several complex variables, which was introduced and studied systematically in a series of recent papers by H.M. Srivastava and R. Panda (cf. [2], [3] and [4]), the present author derive a finite summation. The main result (1) below, do not seem to follow easily from the general expansions for the multivariable H-function in finite series; indeed, on specializing the various parameters involved, this new summation formula will yield a number of known results including, for example (the earlier paper [1]).
The main result to be established is:

$$
\begin{aligned}
(1)\ & \sum_{m=0}^{m}(-1)^{n}\,{}^{n}c_m \frac{(1-\alpha-d_1'-b'_{B'})_m}{(1-\alpha-n)_m} H^{0,\lambda:(\mu',v');\cdots;(\mu^{(r)},v^{(r)})}_{A,C:[B',D'];\cdots;[B^{(r)},D^{(r)}]} \left(\begin{matrix} [(a):\theta',\cdots,\theta^{(r)}]: \\ [(c):\Psi',\cdots,\Psi^{(r)}]: \end{matrix} \right. \\
& \qquad \left. \begin{matrix} [b_1',\phi_1'],\cdots,[b'_{B'-1},\phi'_{B'-1}],[1-b'_{B'}+m,h];[(b''),\phi''];\cdots;[(b^{(r)}),\phi^{(r)}]; \\ [d_1'+m,h],[d_2',\delta'],\cdots,[d'_{D'},\delta'_{D'}];[(d''),\delta''];\cdots;[(d^{(r)}),\delta^{(r)}]; \end{matrix} Z, \cdots, Z_r \right) \\
& = \frac{(1-b'_{B'}-d_1')_n}{(\alpha)_n} H^{0,\lambda:(\mu'+1,v');\cdots;(\mu^{(r)},v^{(r)})}_{A\,C:[B',D'+1];\cdots;[B^{(r)},D^{(r)}]} \left(\begin{matrix} [(a):\theta',\cdots,\theta^{(r)}]:[b_1'\phi_1'],\cdots, \\ [(c):\Psi',\cdots,\Psi^{(r)}]:[d_1'+\alpha+ \end{matrix} \right. \\
& \qquad \left. \begin{matrix} [b'_{B'-1},\phi'_{B'-1}],[1-b'_{B'}+n,h],[\alpha+d_1',h];[(b''),\phi''];\cdots;[(b^{(r)}),\phi^{(r)}]; \\ n,h],[d_1',h],[d_2',\delta'],\cdots,[d'_{D'},\delta'_{D'}];[(d''),\delta''];\cdots;[(d^{(r)}),\delta^{(r)}]; \end{matrix} Z_1, \cdots, Z_r \right),
\end{aligned}
$$

provided n is a positive integer, $R(\alpha)>0$, $R(1-\alpha-d'_1-b'_{B'})>0$, $h>0$, $|arg(Z_i)| < \frac{1}{2}\pi\Delta_i$, $\forall_i \in \{1, \cdots, r\}$,

where

$$
\begin{aligned}
\Delta_i = & \sum_{j=1}^{\lambda}\theta_j^{(i)} - \sum_{j=\lambda+1}^{A}\theta_j^{(i)} + \sum_{j=1}^{v^{(i)}}\phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}}\phi_j^{(i)} \\
& - \sum_{j=1}^{C}\Psi_j^{(i)} + \sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)} - \sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)} > 0, \ \forall i \in \{1, \cdots, r\}.
\end{aligned}
$$

PROOF. Using the contour integral form of the H-function of several complex variables $z_1, \cdots, z_r$ by means of the multiple contour integral ([2], p.271, Eq

*The symbols used in this note have their usual meaning as assigned by previous authors.

(4.1); cf also [4], p. 121, Eq (1.10)) on L.H.S. of (1) and changing the order of summation and integration as permissible by absolute convergency for stated conditions and using the result (saalschutz's theorem):

$$(2) \qquad {}_3F_2\left[\begin{matrix}-n, & a, & b \\ c, & 1-c+a+b-n\end{matrix}; 1\right] = \frac{(c-a)_n (c-b)_n}{(c)_n (c-a-b)_n}.$$

REMARK 1. When $n=1$ and on multiplying both sides by α in (1), we get the recurrence relation.

REMARK 2. When all of the θ_s', ϕ_s', Ψ_s' and δ'_s are chosen to be 1, the main result (1) would reduce to the another results corresponding G-function of r variables.

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