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Hypersurfaces with quasi-integrable (f, g, u, v, λ)-structure of an odd-dimensional sphere

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§ 0. Introduction

It is well known that a submanifold of codimension 2 of an almost Hermitian manifold or a hypersurface of an almost contact metric manifold induces the so-called (f, g, u, v, λ)-structure ([13], [14]). Thus, hypersurfaces of an odd-dimensional sphere S^{2n+1} admit an (f, g, u, v, λ)-structure. In terms of this structure, many authors ([1], [5], [6], [8], [12] etc.) studied hypersurfaces of S^{2n+1} under the condition that the structure tensor f induced on M and the second fundamental tensor H anticommute.

In 1971, D. E. Blair, G. D. Ludden and K. Yano proved the following:

Theorem A ([1]). Let M be a complete hypersurface of S^{2n+1} such that the structure tensor f induced on M and the second fundamental tensor H anticommute. If the scalar curvature of M is constant and the function λ is not locally constant, then M is a great sphere $S^{2n}(1)$ or a product of two spheres $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

On the other hand, H. Suzuki ([10]) investigated the integrability conditions of an almost complex structure F constructed from (f, g, u, v, λ)-structure.

In the present paper, we improve Theorem A by using one of the integrablity conditions of F above and characterize a compact hypersurface of S^{2n+1} under certain conditions.

In § 1, we recall fundamental properties and structure equations for hypersurfaces immersed in S^{2n+1}, and define the (f, g, u, v, λ)-structure induced on the hypersurface to be quasi-integrable.

§ 2 is devoted to study a hypersurface M with quasi-integrable (f, g, u, v, λ)-structure of S^{2n+1} such that the scalar curvature of M is constant.

In the last § 3, we examine compact hypersurfaces with the same kind of this structure of S^{2n+1} over which the sectional curvature $\gamma(u, v)$ of the section spanned by u and v is constant.

§1. Preliminaries

Let N be a $(2n+1)$-dimensional Sasakian manifold with structure tensors (ϕ, ξ, g) covered by a system of coordinate neighborhoods $\{\overline{W}; y^h\}$, where here and in this section the indices h, i, j and k run over the range $\{\overline{1}, \overline{2}, \cdots, \overline{2n+1}\}$. Then the structure tensors of N satisfy ([9]).

$$\phi_j^h \phi_i^j = -\delta_i^h + \xi_i \xi^h \quad \xi_j \phi_i^j = 0, \quad \phi_j^h \xi^j = 0, \tag{1.1}$$
$$\phi_j^k \phi_i^h g_{kh} = g_{ji} - \xi_j \xi_i, \quad \xi_j \xi^j = 1, \quad \xi_j = g_{ji} \xi^i$$

and

$$\nabla_j \phi_i^h = -g_{ji} \xi^h + \delta_j^h \xi_i, \quad \nabla_j \xi^h = \phi_j^h, \tag{1.2}$$

where ∇_j denotes the operator of the covariant differentiation with respect to the metric tensor g_{ji}.

Let M be a $2n$-dimensional orientable hypersurface in N covered by a system of coordinate neighborhoods $\{W; x^c\}$, where here and in the sequel, the indices a, b, c, d and e run over the range $\{1, 2, \cdots, 2n\}$. Then the local parametric expression of M is represented by $y^h = y^h(x^c)$. If we put $B_b^h = \partial_b y^h$, $\partial_b = \partial / \partial x^b$, then B_b^h are $2n$ linearly independent vectors tangent to M. The induced metric tensor of M is given by $g_{cb} = B_c^j B_b^i g_{ji}$ becouse the immersion is isometric. Since M is orientable, we can choose a unit normal C^h to M globally along M. Thus, the transforms $\phi_i^h B_c^i$ of B_c^i by ϕ_i^h can be expressed as linear combinations of B_a^h and C^h, that is,

$$\phi_i^h B_c^i = f_c^a B_a^h - u_c C^h, \tag{1.3}$$

where f_c^a is a tensor field of type $(1, 1)$ and u_c a 1-form on M. And the transform $\phi_i^h C^i$ of C^i by ϕ_i^h, being orthogonal to C^h, can be written as

$$\phi_i^h C^i = u^a B_a^h, \tag{1.4}$$

where $u^a = u_b g^{ba}$ is a vector field on M, $(g^{cb}) = (g_{cb})^{-1}$.

Similarly we can put

$$\xi^h = v^a B_a^h + \lambda C^h, \tag{1.5}$$

where v^a is a vector field and λ a function on M.

Applying ϕ to the both sides of (1.3)~(1.5) respectively and taking account of (1.1) and these equations, we find (*cf.* [1], [8]).

$$
(1.6)\qquad \begin{cases} f_c^e f_e^a = -\delta_c^a + u_c u^a + v_c v^a, \\ u_e f_c^e = -\lambda v_c, \quad v_e f_c^e = \lambda u_c, \\ u_e u^e = v_e v^e = 1 - \lambda^2, \quad u_e v^e = 0, \\ g_{de} f_c^d f_b^e = g_{cb} - u_c u_b - v_c v_b. \end{cases}
$$

Thus, M admits the (f, g, u, v, λ)-structure ([5], [14]).

If we put $f_{cb} = f_c^e g_{eb}$, then we easily verify that $f_{cb} = -f_{bc}$.

Denoting by ∇_c the operator of van der Waerden-Bortolotti covariant differentiation along the hypersurface M, we obtain the equations of Gauss and Weingarten:

$$
(1.7)\qquad \nabla_c B_b^h = h_{cb} C^h, \quad \nabla_c C^h = -h_c^a B_a^h
$$

respectively, where h_{cb} is the second fundamental tensor of M and $h_c^a = h_{cb} g^{ab}$.

Differentiating (1.3)~(1.5) covariantly along M and using (1.2), (1.7) and these equations, we find (*cf.* [1], [8]).

$$
(1.8)\qquad \nabla_c f_b^a = -g_{cb} v^a + \delta_c^a v_b + h_{cb} u^a - h_c^a u_b,
$$

$$
(1.9)\qquad \nabla_c u_b = \lambda g_{cb} + h_{ce} f_b^e,
$$

$$
(1.10)\qquad \nabla_c v_b = f_{cb} + \lambda h_{cb},
$$

$$
(1.11)\qquad \nabla_c \lambda = -u_c - h_{ce} v^e.
$$

We now define a tensor field T of type (0, 2) by

$$
(1.12)\qquad T_{cb} = f_b^e \nabla_e v_c - f_c^e \nabla_e v_b + (\nabla_c f_b^e - \nabla_b f_c^e) v_e - \lambda(\nabla_c u_b - \nabla_b u_c).
$$

If T_{cb} vanishes identically, it is said that the (f, g, u, v, λ)-structure is quasi-integrable (*cf.* [10]).

Substituting (1.8)~(1.10) into (1.12), we obtain

$$
(1.13)\qquad T_{cb} = (h_{ce} v^e) u_b - (h_{be} v^e) u_c.
$$

Assuming the ambient manifold N being unit sphere S^{2n+1} of dimension $2n+1$, equations of Gauss and Codazzi are given respectively by

$$
(1.14)\qquad K_{dcb}{}^a = \delta_d^a g_{cb} - \delta_c^a g_{db} + h_d^a h_{cb} - h_c^a h_{db},
$$

$$
(1.15)\qquad \nabla_d h_{cb} - \nabla_c h_{db} = 0,
$$

where $K_{dcb}{}^a$ is the Riemann-Christoffel curvature tensor of M.

Contraction (1.14) yields

$$
(1.16)\qquad K_{cb} = (2n-1) g_{cb} + h h_{cb} - h_{ce} h_b^e,
$$

where K_{cb} is the Ricci tensor of M and $h = g^{cb} h_{cb}$. Thus the scalar curvature K of M is written in the form

$$
(1.17)\qquad K = 2n(2n-1) + h^2 - h_{cb} h^{cb}.
$$

We note from (1.9) that the function $1 - \lambda^2$ does not vanish almost everywhere on M because h_{cb} is symmetric and f_{cb} is skew-symmetric.

§ 2. Hypersurfaces of S^{2n+1} with quasi-integrable structure

Let M be a hypersurface of an odd-dimensional sphere S^{2n+1} with equasiintegrable (f, g, u, v, λ)-structure.

Then we have from (1.13)

$$h_{be}v^e=\beta u_b \tag{2.1}$$

because the function $1-\lambda^2$ is not zero almost everywhere, where we have put $h(u, v)=h_{cb}u^c v^b=\beta(1-\lambda^2)$. Thus (1.11) reduces to

$$\nabla_b\lambda=-(\beta+1)u_b. \tag{2.2}$$

Differentiating this covariantly along M and taking account of (1.9), we find

$$-\nabla_c\nabla_b\lambda=(\nabla_c\beta)u_b+(\beta+1)(\lambda g_{cb}+h_{ce}f^e_b),$$

from which, taking the skew-symmetric part,

$$(\nabla_c\beta)u_b-(\nabla_b\beta)u_c+(\beta+1)(h_{ce}f^e_b-h_{be}f^e_c)=0. \tag{2.3}$$

If we transvect (2.3) with v^c and use (1.6) and (2.1), then we obtain

$$\lambda(\beta+1)h_{be}u^e=\lambda(\beta+1)\beta v_b-(v^e\nabla_e\beta)u_b. \tag{2.4}$$

Also, transvecting (2.3) with u^b and making use of (1.6), (2.1) and (2.4), we get

$$\lambda(1-\lambda^2)\nabla_c\beta=\lambda(u^e\nabla_e\beta)u_c+\lambda(v^e\nabla_e\beta)v_c. \tag{2.5}$$

Substituting (2.5) into (2.3), we have

$$\lambda(\beta+1)(h_{ce}f^e_b-h_{be}f^e_c)+\frac{\lambda}{1-\lambda^2}(v^e\nabla_e\beta)(v_c u_b-v_b u_c)=0. \tag{2.6}$$

If we transvect this with f^{cb} and take account of (1.6), (2.1) and (2.4), then we find

$$\lambda(\beta+1)h+v^e\nabla_e\beta=0. \tag{2.7}$$

Thus, (2.6) reduces to

$$\lambda(\beta+1)(h_{ce}f^e_b-h_{be}f^e_c)=\frac{\lambda^2(\beta+1)h}{1-\lambda^2}(v_c u_b-v_b u_c). \tag{2.8}$$

We now suppose that the function λ is not locally constant. Then we see from (2, 2) that $\lambda(\beta+1)$ does not vanish almost everywhere on M. Therefore, (2.4) and (2.8) becomes respectively

$$h_{be}u^e=hu_b+\beta v_b, \tag{2.9}$$

$$h_{ce}f^e_b-h_{be}f^e_c=\frac{\lambda h}{1-\lambda^2}(v_c u_b-v_b u_c) \tag{2.10}$$

because of (2.7).

Differentiating (2.9) covariantly and substituting (1.9) and (1.10), we find

$$(\nabla_c h_{be})u^e + h_b^e(\lambda g_{ce} + h_{ca} f_e^a) = (\nabla_c h) u_b + h(\lambda g_{cb} + h_{ce} f_b^e) + (\nabla_c \beta) v_b + \beta(f_{cb} + \lambda h_{cb}),$$

from which, taking the skew-symmetric part and using (1.15) and (2.10),

$$(2.11) \qquad 2 h_{be} h_{ca} f^{ea} = (\nabla_c h) u_b - (\nabla_b h) u_c + \frac{\lambda h^2}{1-\lambda^2}(v_c u_b - v_b u_c) + (\nabla_c \beta) v_b - (\nabla_b \beta) v_c + 2\beta f_{cb}.$$

If we transvect this with u^b and make use of (1.6), (2.1) and (2.9), then we obtain

$$(2.12) \qquad (1-\lambda^2)\nabla_c h = (u^e \nabla_e h) u_c + \lambda(2\beta - 2\beta^2 - h^2 + u^e \nabla_e \beta) v_c.$$

Substituting (2.12) into (2.11) and using (2.5), we find

$$h_{be} h_{ca} f^{ea} = \frac{\lambda}{1-\lambda^2}(1-\beta)\beta(v_c u_b - v_b u_c) + \beta f_{cb},$$

from which, taking account of (2.1), (2.9) and (2.10),

$$(1-\lambda^2)(h_{be} h_a^e f_c^a - \beta f_{cb}) = \lambda\beta(1-\beta)(v_c u_b - v_b u_c) - \lambda h^2 v_c u_b - \lambda h \beta(v_c v_b - u_c u_b).$$

Transvecting this with f_d^c and taking account of (1.6), we get

$$(1-\lambda^2)\{h_{be} h_a^e(-\delta_d^a + u_d u^a + v_d v^a) - \beta(-g_{db} + u_d u_b + v_d v_b)\}$$
$$= \beta\lambda^2(1-\beta)(u_d u_b + v_d v_b) - \lambda^2 h^2 u_d u_b - \lambda^2 h \beta(u_d v_b + v_d u_b),$$

from which, using (2.1) and (2.9),

$$(2.13) \qquad (1-\lambda^2)(h_{ce} h_b^e - \beta g_{cb}) = (h^2 + \beta^2 - \beta) u_c u_b + \beta(\beta - 1) v_c v_b + \beta h(u_c v_b + u_b v_c).$$

Transvection g^{cb} gives

$$(2.14) \qquad h_{cb} h^{cb} - h^2 = 2n\beta + 2\beta(\beta - 1).$$

Hence, (1.7) reduces to

$$(2.15) \qquad K = 2n(2n-1) - 2n\beta + 2\beta(\beta - 1).$$

We now suppose that the scalar curvature K of M is constant. Then it follows that β is constant on M and hence the hypersurface M is minimal because of (2.7) and the fact that λ is not locally constant. Consequently we see from (2.12) that $\beta(\beta - 1) = 0$ on M. Thus, (2.10) and (2.13) reduce respectively to

$$h_{ce} f_b^e - h_{be} f_c^e = 0 \qquad \text{and} \qquad h_{ce} h_b^e = \beta g_{cb}.$$

According to Theorem A, M is a great sphere $S^{2n}(1)$ if $\beta = 0$ or $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ if $\beta = 1$ (*cf.* [1], [11]).

Thus we have

Theorem 1. Let M be a complete orientable hypersurface of an odd-dimensional sphere S^{2n+1} with quasi-integrable (f, g, u, v, λ)-structure. If the scalar curvature of M is constant and the function λ is not locally constant, then M is a great sphere $S^{2n}(1)$ or a product of two spheres $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

We now denote by $\gamma(u, v)$ the sectional curvature of the section spanned by the vectors u^a and v^a. Then $\gamma(u, v)$ is given by

$$\gamma(u, v) = -\frac{K_{dcba}u^d v^c u^b v^a}{(g_{da}g_{cb}-g_{ca}g_{db})u^d v^c u^b v^a}.$$

If we take account of (1.6), (1.14) and (2.1), then we have

$$\gamma(u, v) = \beta^2 - 1 \tag{2.16}$$

because the function $1-\lambda^2$ is nonzero almost everywhere. Thus, if $\gamma(u, v)$ is constant on M, then we see from (2.15) that the scalar curvature K of M is constant.

Therefore, we have

Corollary 2. In Theorem 1, replacing the constancy of the scalar curvature k by $\gamma(u, v)=$const., M is $S^{2n}(1)$ or $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$.

§ 3. Compact hypersurfaces of S^{2n+1} with quasi-integrable structure

Let M be a hypersurface of S^{2n+1} with quasi-integrable (f, g, u, v, λ)-structure such that the sectional curvature $\gamma(u, v)$ of the section spanned by u^a and v^a is constant on M. Then we see from (2.16) that β is constant on M. Thus (2.3) reduces to

$$(\beta+1)(h_{ce}f_b^e - h_{be}f_c^e) = 0$$

and consequently $\beta=-1$ or $h_{ce}f_b^e - h_{be}f_c^e=0$ on M. In latter case, the complete hypersurface M is $S^{2n}(1)$ or $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ because of Theorem 1.

Secondly, if $\beta=-1$, then it follows from (2.2) that λ is constant on M.

Assuming the hypersurface M is compact, we see from (1.9) that λ vanishes identically.

From the Ricci identity for u^a, that is,

$$\nabla_d\nabla_c u_b - \nabla_c\nabla_d u_b = -K_{dcb}{}^a u_a,$$

we have

$$u^c(\nabla^b\nabla_c u_b) = K_{cb}u^c u^b \tag{3.1}$$

with the aid of (1.9) and the fact that $\lambda=0$

If we take account of (1.6) and (1.16), then (3.1) becomes

(3.2) $$u^c(\nabla^b\nabla_c u_b)=2n-1+h(u,\ v)h-h_{ce}h^e_b u^c u^b.$$

On the other hand, we have from (1.9) with $\lambda=0$

(3.3) $$\|\nabla_c u_b\|^2=h_{cb}h^{cb}-h_{ce}h^e_b u^c u^b-1$$

with the help of (2.1) with $\beta=-1$.

Substituting (3.2) and (3.3) into the identity:

$$\nabla^b(u^c\nabla_c u_b)=\frac{1}{2}\|\nabla_c u_b+\nabla_b u_c\|^2-\|\nabla_c u_b\|^2+u^c\nabla^b\nabla_c u_b,$$

we find

$$\nabla^b(u^c\nabla_c u_b)=\frac{1}{2}\|\nabla_c u_b+\nabla_b u_c\|^2+h(u,\ u)h+2n-h_{cb}h^{cb}.$$

Since M is compact, by Green's theorem, we have

$$\int_M\left(\frac{1}{2}\|\nabla_c u_b+\nabla_b u_c\|^2+h(u,\ u)h+2n-h_{cb}h^{cb}\right)d\sigma=0,$$

where $d\sigma$ is the volume element of M.

Now, suppose that the second fundamental tensor h^a_c is positive semidefinite and satisfies $h_{cb}h^{cb}\leqq 2n$ on M, we have

(3.4) $$h_{cb}h^{cb}=2n,$$

(3.5) $$h(u,\ u)h=0,$$

(3.6) $$\nabla_c u_b+\nabla_b u_c=0,\quad i.e.,\quad h_{ce}f^e_b+h_{be}f^e_c=0$$

because of (1.9) with $\lambda=0$.

Transvecting (3.6) with $u^b f^c_d$ and taking account of (1.6) with $\lambda=0$ and (2.1) with $\beta=-1$, we find

(3.7) $$h_{be}u^e=-v_b+\alpha u_b,$$

where we have put $\alpha=h(u,\ u)$.

Differentiating (3.7) covariantly and substituting (1.9) and (1.10) with $\lambda=0$, we get

$$(\nabla_c h_{be})u^e+h_{be}h_{ca}f^{ea}=-f_{cb}+(\nabla_c\alpha)u_b+\alpha h_{ce}f^e_b,$$

From which, taking the skew-symmetric part and using (1.15) and (3.6),

(3.8) $$h_{be}h^e_a f^a_c=f_{cb}+\alpha h_{be}f^e_c+\frac{1}{2}\{(\nabla_b\alpha)u_c-(\nabla_c\alpha)u_b\}.$$

If we transvect this with u^b and make use of (1.6) with $\lambda=0$, (2.1) with $\beta=-1$ and (3.7), then we find

(3.9) $$\nabla_c\alpha=\mu u_c$$

where we have put $\mu=u^e\nabla_e\alpha$. Thus (3.8) becomes

$$h_{be}h^e_a f^a_c=f_{cb}+\alpha h_{be}f^e_c$$

Transvection f_a^b gives

(3.10) $$h_{ce} h_b^e = g_{cb} + \alpha h_{cb}$$

with the aid of (1.6) with $\lambda=0$, (2.1) with $\beta=-1$ and (3.7).

Differentiating (3.9) covariantly and using (1.9) with $\lambda=0$, we find

$$\nabla_c \nabla_b \alpha = (\nabla_c \mu) u_b + \mu h_{ce} f_b^e,$$

from which, taking the skew-symmetric part and making use of (3.6),

$$(\nabla_c \mu) u_b - (\nabla_b \mu) u_c + 2\mu h_{ce} f_b^e = 0,$$

which implies that $\mu h_{ce} f_b^e = 0$ with the aid of (1.6) with $\lambda=0$.

Now we put a set $S = \{ P \varepsilon M \mid \mu(P) \neq 0 \}$. Then S is open in M. Thus, at each point of S we have

(3.11) $$h_{ce} f_b^e = 0, \quad i.\ e., \quad \nabla_c u_b = 0$$

because of (1.9) with $\lambda=0$. Transvection f_a^b gives

$$h_{ca} = \alpha u_c u_a - (v_c u_a + v_a u_c)$$

with the aid of (1.6) and (2.1) with $\beta=-1$ and (3.7).

Differentiating this covariantly and taking account of (3.9) and (3.11), we find on S

$$\nabla_d h_{ca} = \mu u_d u_c u_a - f_{dc} u_a - f_{da} u_c,$$

from which, taking the skew-symmetric part with respect to the indices d and c and using (1.15), $2 f_{dc} u_a - f_{da} u_c + f_{ca} u_d = 0$. Thus, it follows that $f_{dc}=0$, which contradicts the fact that $n>1$. There by the set S is empty and so α is constant because of (3.9). Therefore we have $\alpha=0$ or $h=0$ on M because of (3.5).

In the first place, if $h=0$, that is, M is minimal, then owing to Chern-do Carmo and Kobayashi's result ([2]), M is $S^n \times S^n$ since we have (3.4).

In the next place, if $\alpha=0$ on M, then we see from (3.10) that $h_{ce} h_b^e = g_{cb}$. From this and (1.15) we easily see that $\nabla_d h_{cb} = 0$.

On the other hand, we can easily see that the second fundamental tensor h_b^a has two principal curvatures 1 and -1. Moreover, using (2.1) with $\beta=-1$, (3.6) and (3.7) with $\alpha=0$, we can easily verify that the mutiplicities of ± 1 are odd. Thus, M is a product of two odd-dimensional spheres (*cf*, [4], [7]).

Developed above we conclude

Theorem 3. Let M be a complete hypersurface of an odd-dimensional sphere S^{2n+1}, $(n>1)$ with quasi-integrable (f, g, u, v, λ)-structure such that the sectional curvaturg $\gamma(u, v)$ of the section spanned by u and v is constant on M. If the second fundamental tensor H is positive semi-definite and satisfies trace ${}^tHH \leqq 2n$, then M is a great sphere $S^{2n}(1)$ of radius I or a product of two spheres $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ or $S^p \times S^{2n-p}$, where p is odd.

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Abstract

Hypersurfaces with quasi-integrable (f, g, u, v, λ)-structure of an odd-dimensional sphere

Let M be a complete and orientable hypersurface of an odd-dimensional sphere S^{2n+1} with quasi-integrable (f, g, u, v, λ)-structure.

The purpose of the present paper is to prove the following two theorems.

(I) If the scalar curvature of M is constant and the function λ is not locally constant, then M is a great sphere $S^{2n}(1)$ or a product of two spheres with the same dimension $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$.

(II) Suppose that the sectional curvature of the section $\gamma(u, v)$ spanned by u and v is constant on M and M is compact. If the second fundamental tensor H of M is positive semi-definite and satisfies trace ${}^tHH \leq 2n$, then M is a great sphere $S^{2n}(1)$ or a product of two spheres $S^n \times S^n$ or $S^p \times S^{2n-p}$, p being odd.