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—Research Paper—

A General Decision-Theoretic Model for a Couple's Family Building Process

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Abstract

During the course of history, more and more reliable birth control methods have become available. Hence, to a certain degree, the possibility of avoiding any or additional children, and of spacing the family building process has arisen. The advancement of sex predetermination technology, whereby couples can influence the sex of their children, gives couples another decision variable. Assuming a rational acting couple, we present a general decision-theoretic model which describes the family building process and its optimization through maximizing the expected utility concerning the spacing, ordering, sex, and number of their children.

Introduction

In a vast literature on the economics of fertility, costs and values of children are regarded as the determining force of human fertility behavior. In this paper, we concentrate on the decision-theoretic perspective of family building processes at a micro-demographic level.

Terhune and Kaufman [10] point out that popular fertility survey questions focus on the *most* preferred family size, while actually a *spectrum* of family size preferences should be taken into account. To indicate strength of preference, they introduce the concept of utility as the difference between rewards (gratifications) and costs (penalties). Several types of family size utility functions are discussed,and an operational method for determining family size utility functions is developed. However,in order to formulate a couple's family building process, we have to incorporate the utilities of spacing, ordering, and sex of children, too.

Techniques that give preference scales for number and sex of children, are proposed, among others, in [1] and [5]. We avoid these measurement problems by stating that our rational acting couple-like the *homo economicus* in economics has a utility function which numerically represents its preferences related to spacing, ordering, sex, and number of their children.

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Obviously,number and spacing of children are affected by the couple's fertility, frequency of intercourse, and by the use of contraceptive methods. O'Hara[8]gives the optimal timing for a birthcontrol applying couple which wants to minimize its expected number of children under the restriction that with probability 95% or more, at least one son will be alive when the father reaches age 65. With the advent of sex-selection techniques, i.e. methods that determine the probabilities that a subsequent child will be a son or a daughter, the sex proportion and moreover the offspring's ordering can be influenced. McDonald [6], Mason and Bennett [4], and Keyfitz [3, pp. 338~344] derive strategies which optimize specific goals. Probability distributions of i boys(girls) and 0-2 girls (boys) are given by Smith [9] for some common family building strategies.

In the following, we present a wider framework for the individual family building process and show that it is a generalization of the cited work.

Modelling the Family Building Decision Process

We assume that the couple agree son its reproductive goals completely, i. e. the couple acts as one decision-maker, though the bargaining dynamics behind this question may be part of the decision process.

At every point in time, the couple has to choose among several actions. First, they may use one of m available birth control methods $c_1 \ldots, c_m$ (pill, IUD, sterilization etc.,or combined actions like the pill and a low frequency of intercourse, etc.). Second, we assume that n sex-selection techniques $s_1, \ldots, s_n$ are available. This sounds somewhat futuristic, but Mason and Bennett [4] already report on one of these methods, namely a sperm-separation method increasing the probability of bearing a son to as much as 0.9. Third, the couple may apply neither contraceptional method nor sex selection technique; they choose the 'natural technique' denoted by c_0. Thus, the set A of possible actions is

$$A = \{c_0, c_1, \cdots, c_m, s_1, \cdots, s_n\}. \qquad (1)$$

Each action is followed by one of the following consequences: a boy (B), a girl (C), or no child (N) will be conceived according to probabilities explained later. For purposes of simplicity, we do not consider multiple births and fetal wastage, stillbirths, etc. at the moment. Hence, the set Ω of possible outcomes is given by

$$\Omega = \{B, G, N\}. \qquad (2)$$

Furthermore we assume that a new born child will survive the end of the couple's fertility.

The time reference is discrete. Since the couple will not choose an action $a \in A$ on a day-to-day basis it is reasonable and practical to have the menses or one month as time unit. The points in time (stages) will be denoted by t, $t=1, \cdots, T$, where $t=1$ is the couple's first period of intercourse, and $t=T$ the end of their fecundity.

Let $H_t = A \times \Omega \times A \times \Omega \times \cdots \times A \times \Omega = (A \times \Omega)^t$, $1 \leq t \leq T$, the t-fold Cartesian product of $A \times \Omega$. Then, H_t is the set of all family histories, i.e. the set of all possible sequences of actions and corresponding outcomes from stage 1 up to stage t.

Now the outcome of each birth is a random variable, depending on the previous family history and current action. Formalizing this idea, for each $a \in A$ we have a discrete transition probability on Ω for the initial stage, denoted by $p_1(a,.)$. For purposes of illustration, a fecund young couple, using the 'natural method' c_0 may have the distribution

$$p_1(c_0, w) = \begin{cases} 0.46 & \text{for } w=B \\ 0.44 & \text{for } w=G \\ 0.10 & \text{for } w=N \end{cases} \tag{3}$$

Using a nearly perfect birth control method c_i, yields a distribution like

$$p_1(c_1, w) = \begin{cases} 0.01 & \text{for } w=B \\ 0.01 & \text{for } w=G \\ 0.98 & \text{for } w=N \end{cases} \tag{4}$$

Or using the sperm separating technique s_j, the distribution may be

$$p_1(s_j, w) = \begin{cases} 0.81 & \text{for } w=B \\ 0.09 & \text{for } w=G \\ 0.10 & \text{for } w=N \end{cases} \tag{5}$$

Analogously, for each stage t, $2 \leq t \leq T$, each $h_{t-1} \in H_{t-1}$, and each $a \in A$, the outcomes vary stochastically according to the transition probabilities $p_t(h_{t-1}, a, .)$. For example, consider sterilization, action c_e. If c_e is a coordinate of h_{t-1}, or if $a = c_e$, we have

$$p_t(h_{t-1}, a, w) = \begin{cases} 0.0 & \text{for } w=B \\ 0.0 & \text{for } w=G \\ 1.0 & \text{for } w=N. \end{cases} \tag{6}$$

As a last example, if the outcome at stage $t-1$ was B or G, i.e. a conception took place at stage $t-1$, the previous distribution holds for all $a \in A$, too, and remains valid to stage $t+k-1$, where $k=9$ months (for pregnancy) $+x$ months (for post-partum period). All these probabilities apply only to the couple under consideration. They may vary from couple to couple and must be known at the start of the decision process.

A family building plan f (or briefly a plan) is a sequence $\{f_t\}$ of maps $f_t : H_{t-1} \rightarrow A$, $2 \leq t \leq T$, and $f_1 \in A$. That is the couple may choose its current action depending on the family history that happened so far. Associated with each plan f is a probability distribution P_f on $\Omega \times \cdots \times \Omega = \Omega^T$ which is the product of appropriate transition probabilities,

$$P_f(w_1, w_2, \cdots, w_T) = p_1(f_1, w_1) \cdot p_2(h_1, f_2(h_1), w_2) \cdots\cdots p_T(h_{T-1}, f_T(h_{T-1}), w_T), \tag{7}$$

where $h_1 = (f_1, w_1)$, $h_2 = (f_1, w_1, f_2(h_1), w_2)$, etc.

The final cornerstone of our model is the couple's utility function. Formally, we define as its utility function u a map from H_T to the set of real numbers. By this definition, not only the offspring's ordering (and thus number and sex) and spacing is allowed for but also the rewards and costs of the actions are taken into account, as well as the date of birth. For example, traditional Catholic parents, who think they should not have more children, are much more comfortable (i.e. have greater utility) reaching this goal not by means of the pill but by a birth control method permitted by the church. For a couple which strongly desires a son, the utility of getting a son when using the 'natural technique' is greater than when using a new sex-selection technique because of some possible health hazards.

On account of the stochastic nature of the decision process, we look at expected utility instead of utility itself. For a plan f, the expected utility $U(f)$ is given by

$$U(f)=\sum_{\Omega_T} u(f_1, w_1, f_2(h_1), w_2, \cdots, f_T(h_{T-1}), w_T)\cdot P_f(w_1, w_2, \cdots, w_T), \tag{8}$$

or compactly $U(f)=\int_{\Omega_T} u\ dP_f\cdot$ (P_f is a counting measure. $U(f)$ exists and is finite, if for example the utility function does not attain the values $\pm\infty$.)

Finally, a family building plan f^* is said to be optimal. if.

$$U(f^*)=\max_f U(f)=\max_f \int_{\Omega_T} u\ dP_f. \tag{9}$$

Since the set of all plans is finite, the maximum exists, but there may be more than one plan with maximum expected utility. This model seems to be applicable to any couple's decision process. Even the simplifications we have met can be released. For example, multiple births can be incorporated by enlarging the set of outcomes Ω. The assumptions that each conception leads to a live birth and each child survives stage T can be removed by considering the according probabilities and denoting its further existence or loss (respectively its survival or death), in the set of family histories H_t. Moreover, there is no need for exact knowledge of the transition probabilities p_t or the time stability of the utility function u. As long as probability distributions for both can be found, we just have to integrate once more to get $U(f)$.

Optimization of the Family Decision Process

Finding an optimal plan can be very time-consuming, even impossible, because of the combinatorial magnitude of the problem. However, the dimensionality can be drastically reduced, if the quite general utility function $u: H_T \rightarrow \boldsymbol{R}$ is decomposible in the following sense. Let a system of *marginal* utility functions $u_t: H_t \rightarrow \boldsymbol{R}$, $1 \leq t \leq T$, be given such that

$$u_T(a_1, w_1, a_2, w_2, \cdots a_{T-1}, w_{T-1}, a_T, w_T)=u_1(a_1, w_1)+u_2(a_1, w_1, a_2, w_2)+\cdots$$
$$+u_T(a_1, w_1, a_2, w_2, \cdots a_{T-1}, w_{T-1}, a_T, w_T). \tag{10}$$

Herein $u_1(a_1, w_1)$ is the utility of action a_1 if prompted by outcome w_1, and, in general, $u_t(a_1, w_1, \cdots, a_{t-1}, w_{t-1}, a_t, w_t)$ is the additional utility of (a_t, w_t) at stage t, given the family history $h_{t-1}=(a_1, w_1, \cdots, a_{t-1}, w_{t-1})$.

Now, the theory of dynamic programming supplies an elegant and programmable algorithm to determine an optimal plan. Its 'Principle of Optimality' is explained in standard books. like [2], or [7]. In our terminology, the dynamic programming algorithm says:

First, for all $h_{T-1} \in H_{T-1}$, find

$$V_{T-1}(h_{T-1})=\max_{a\in A} \int_\Omega u_T(h_{T-1}, a, w)\ dp_T(h_{T-1}, a, w), \tag{11}$$

and set $f_T^*(h_{T-1})=a_T^*$ for some $a_T^* \in A$ with

$$V_{T-1}(h_{T-1})=\int_\Omega u_T(h_{T-1}, a_T^*, w)\ dp_T(h_{T-1}, a_T^*, w). \tag{12}$$

Then, starting with $t=T-1$ and ending at $t=1$, for all $h_{t-1} \in H_{t-1}$ (h_0 is the empty tupel), recursively compute

$$V_{t-1}(h_{t-1})=\max_{a\in A} \int_\Omega (u_t(h_{t-1}, a, w)+V_t(h_{t-1}, a, w))\ dp_t(h_{t-1}, a, w) \tag{13}$$

and set $f_t^*(h_{t-1})=a_t^*$ for some $a_t^* \in A$ with

$$V_{t-1}(h_{t-1})=\int_\Omega (u_t(h_{t-1}, a_t^*, w)+V_t(h_{t-1}, a_t^*, w))\ dp_t(h_{t-1}, a_t^*, w). \tag{14}$$

Then, max $U(f)=U(f^*)$, and max $U(f)=V_0$, i. e. both an optimal plan f^* and the maximum expected utility is determined.

Relation to Previous Research

As mentioned above, McDonald [6], Keyfitz [3, p. 338~344], and Mason and Bennett [4] give operational advice for constructing optimal family building plans. None of them considers ordering and spacing of children, but they refer to number and sex only. By ignoring spacing, they consequently preclude all birth control methods from action space A. They assume that the couple's fertility is constant (high) over time. (In our model, varying levels of fertility depending on age and previous fertility through h_t, are suggested by the system of transition probabilities $\{p_t\}$.) They neglect the financial, moral, psychological, or health-related costs of the various actions by assuming that the costs of all techniques are essentially the same. Hence, there is room for three actions only, namely the sex-selection techniques with maximum probability of producing a son, or a daughter respectively, and the 'natural' technique as a kind of dummy action.

Mason and Bennett[4] derive an optimal family building plan for a couple wanting exactly x boys and y girls without exceeding family size $x+y$. A plan which maximizes the probability of this event, is optimal. Transferred to our framework, we get $T=x+y, \Omega=\{B, G\}$, $A=\{s_1, s_2\}$, where s_1 is the son-producing technique with probability q_1 for a son, and s_2 the daughter-producing technique with probability q_2 for a daughter. The transition laws become stationary;

$$p_t(h_{t-1}, s_1, B)=q_1 \quad p_t(h_{t-1}, s_1, G)=1-q_1 \tag{15}$$
$$p_t(h_{t-1}, s_2, B)=1-q_2 \quad p_t(h_{t-1}, s_2, G)=q_2,$$

for all $h_{t-1}\in H_{t-1}$, and all t, $1\leq t\leq T$. Finally, by choosing the marginal utility functions u_t as

$$u_t(h_{t-1}, a, w)\equiv 0 \text{ for } t,\ 1\leq t\leq T-1, \text{ and} \tag{16}$$

$$u_T(h_{T-1}, a, w)=\begin{cases} 1 & \text{if } h_{T-1} \text{ contains } x-1 \text{ boys, } y \text{ girls and } w=B, \text{ or if } h_{T-1} \\ & \text{contains } x \text{ boys, } y-1 \text{ girls and } w=G, \\ 0 & \text{else,} \end{cases} \tag{17}$$

this approach is seen as a special case of our model.

McDonald [6] and Keyfitz [3] present another special case. With A, Ω, and $\{p_t\}$ as above, their couple desires at least x boys and y girls under the restriction to minimize the expected family size. Now there is no finite time horizon T, since, as Keyfitz [3, p. 344] puts it, 'they must be willing and able to have an infinite number of children if necessary'. Clearly this is not realistic, but it can be set in our framework. With $T=\infty$, the Principle of Optimality is no longer simple. By means of stopping sets $\Phi_t=\{h_t\in H_t;\ h_t$ contains at least x boys and y girls$\}$ (see [2, p57]), with B_t the complement of Φ_t relative to H_t, and marginal utility functions $u_t(h_{t-1}, a, w)\equiv -1$ for all t, h_{t-1}, a, w, we get the recursive scheme of optimality equations.

$$V_{t-1}(h_{t-1})=\max_{a\in A}\int_{\Omega}(u_t(h_{t-1}, a, w)+V_t(h_{t-1}, a, w)). \tag{18}$$
$$.\, 1_{B_t}\ (h_{t-1}, a, w)\, dp_t(h_{t-1}, a, w)$$

Where 1_{B_t} is the indicator function of set B_t.

Hereafter it is obvious, how the model of O'Hara [8] can be fitted in our framework. Since its formulation is lengthy and of technical interest only, we waive the elaboration.

Conclusion

In this article, the perspective of the family building decision process is a micro-demographic and operational one, and not a description of these processes within a population. Technical advice is provided to any couple which wants to maximize its expected utility concerning spacing, ordering, number, and sex of its intended children. It is assumed that the couple is rational in the sense that it knows all birth control and sex selection methods available to them. Rationality also means that the couple is able and willing to state its preferences in form of a utility function, and the stochastic dynamics in forms of transition probabilities. Our model becomes manageable only by reducing its dimensionality. This can be achieved by suggesting the overall utility as sum of marginal utilities and thus permitting the application of the well-known 'Principle of Optimality'. Previous research can be seen as special, simple examples, underlining the richness of our model.

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