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ON TRANSPORTS DRIVEN BY TIME-VARYING WINDS IN HORIZONTALLY UNBOUNDED SHALLOW SEAS

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ABSTRACT

We present theoretical models for the unsteady transports driven by the time-varying wind stress in horizontally unbounded shallow seas of an uniform depth. We derive linearized transport equations that include the acceleration, the Coriolis force, the wind stress and the bottom friction. The steady transport in a shallow sea is different from the classical Ekman transport because of a presence of non-negligible bottom friction. The transient transport in a shallow sea after an onset of a wind stress is decribed by a superposition of the Ekman transport and an inertial oscillation of which magnitude decreases with time. A sinusoidal wind stress generates a transport ellipse of which frequency of rotation is the same as the frequency of the wind-stress forcing. The transport associated with a wind stress of which direction changes linearly with time is decribed by a superposition of a free inertial oscillation with a period of one inertial day and a forced oscillation of which period is the same as the rotation period of the wind stress. The theoretical models of the transports are useful in understanding the time-varying currents and the transports of nutrients in shallow seas.

1. INTRODUCTION

In the classical Ekman theory of wind-driven currents one usually assumes a steady state balance between the wind stress and the Coriolis force. An analytic solution for the steady Ekman problem can be easily obtained in a homogenous ocean of an infinite depth with a simplifying assumption that the austausch coefficient or the eddy viscosity is constant (see for example, von Schwind, 1980, 128~134). The unsteady Ekman problem after an onset of a steady and uniform wind stress was investigated by Ekman (1905) for an infinitely deep ocean and by Hidaka (1933) for the case of a finite depth (see Krauss, 1973, 249~253).

In the shallow seas, such as the Yellow Sea or the East China Sea, the depths of the sea of 100m or less are only of the same order or shallower than the Ekman depth, and therefore one needs to consider the bottom friction as well as the wind stress. Most previous studies on the wind-driven currents in shallow seas, except numerical models (e.g., Choi, 1982), do not include the effect of bottom friction. In theoretical studies on the Ekman transport in a shallow sea one usually assumes a constant austausch coefficient and applies a no-slip condition at the bottom (e.g., Krauss, 1973, 245f). However, both of these assumptions are quite remote from the reality.

In this paper we present theoretical models of the transports in horizontally unbounded shallow seas driven by various types of time-varying wind stress forcing. After we derive governing equations for the transports (Section 2) we obtain a steady solution in Section 3. Then we present the solution for the transient state transport after an onset of a steady and uniform wind stress (Section 4). In Sections 5 and 6 we study transports associated with the wind stress of which magnitude or direction changes with time. The applicability and limitations of the theoretical models are discussed in Section 7. The mathematical symbols used in this paper are summerized in the Appendix.

2. GOVERNING EQUATIONS

The horizontal momentum equations for a homogeneous ocean in an f-plane are (cf. von Schwind, 1980, 149)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv$$
$$=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(A_h\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_h\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_v\frac{\partial u}{\partial z}\right)\right]$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}$$
$$+\frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(A_h\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A_h\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(A_z\frac{\partial v}{\partial z}\right)\right], \quad (2.1)$$

where u, v and w are velocities in the x(eastward), y(northward) and z(upward) direction, respectively, $f=2\Omega\sin\varphi$ the Coriolis parameter, Ω the angular velocity of the earth's rotation, φ the latitude, ρ the density of a homogeneous sea water, and A_h and A_v are horizontal and vertical austausch coefficients, respectively.

For a mathematical simplicity we assume the followings:

1) The ocean is horizontally unbounded.
2) There is no horizontal pressure gradients, i.e., $\frac{\partial p}{\partial x}=\frac{\partial p}{\partial y}=0$.
3) There is no lateral variation in the velocity field, i.e., $\nabla u=\nabla v=0$, where $\nabla=\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$.
4) The vertical velocity is negligibly small, i.e., $w=0$.

With these simplifying assumptions, (2.1) becomes

$$\rho\left(\frac{\partial u}{\partial t}-fv\right)=\frac{\partial}{\partial z}\left(A_v\frac{\partial u}{\partial z}\right)$$
$$\rho\left(\frac{\partial v}{\partial t}+fu\right)=\frac{\partial}{\partial z}\left(A_v\frac{\partial v}{\partial z}\right). \quad (2.2)$$

We integrate (2.2) from the bottom($z=-H$) to the free surface ($z=0$) and get

$$\frac{\partial U}{\partial t}-fV=\left(A_v\frac{\partial u}{\partial z}\right)_{z=0}-\left(A_v\frac{\partial u}{\partial z}\right)_{z=-H}$$
$$\frac{\partial V}{\partial t}+fU=\left(A_v\frac{\partial v}{\partial z}\right)_{z=0}-\left(A_v\frac{\partial v}{\partial z}\right)_{z=-H}, \quad (2.3)$$

where $\mathbf{V}=(U, V)$ is the horizontal mass transport vector, i.e.,

$$U(t)=\rho\int_{-H}^{0}u(z,t)dz, \quad V(t)=\rho\int_{-H}^{0}v(z,t)dz. \quad (2.4)$$

The surface and bottom boundary conditions are, from the continuity of the stresses,

$$\left(A_v\frac{\partial u}{\partial z}\right)_{z=0}=T_x, \quad \left(A_v\frac{\partial v}{\partial z}\right)_{z=0}=T_y,$$
$$\left(A_v\frac{\partial u}{\partial z}\right)_{z=-H}=B_x, \quad \left(A_v\frac{\partial v}{\partial z}\right)_{z=-H}=B_y, \quad (2.5)$$

where $\mathbf{T}=(T_x, T_y)$ is the wind stress vector at the sea surface and $\mathbf{B}=(B_x, B_y)$ is the stress due to bottom friction. The wind stress $\mathbf{T}$ can be computed by

$$\mathbf{T}=\rho_a C_d|\mathbf{W}|\mathbf{W}, \quad (2.6)$$

where ρ_a is the density of the air, C_d is a dimensionless drag coefficient, and $\mathbf{W}$ is the wind stress vector. The bottom stress $\mathbf{B}$ can be parameterized as a linear function of horizontal mass transport $\mathbf{V}$ by (cf. von Schwind, 1980, 194)

$$\mathbf{B}=-r\mathbf{V}, \quad (2.7)$$

where r is a friction coefficient in units of sec^{-1}.

By using (2.5) and (2.7) into (2.3) we get a set of equations for the transport in a shallow sea:

$$\frac{\partial U}{\partial t}-fV=T_x-rU$$
$$\frac{\partial V}{\partial t}+fU=T_y-rV. \qquad (2.8)$$

The equations (2.8) will be the basis of our theoretical investigations in the subsquent sections. The equations (2.8) are different from the governing equations of the classical Ekman theory in the following aspects:

1) Unknowns are expressed in terms of transports instead of a velocity field
2) Equations (2.8) do not include any austausch coefficient.
3) The bottom stress is explicitly included.
4) Temporal changes of the transport are explicitly included.

3. TRANSPORTS IN A STEADY STATE

Let's suppose that a uniform wind has blown for a sufficiently long time and a steady state is maintained. Then (2.8) becomes

$$-fV=T_x-rU$$
$$fU=T_y-rV, \qquad (3.1)$$

and the solution of these equations is

$$U=\frac{rT_x+fT_y}{r^2+f^2},\quad V=\frac{rT_y-fT_x}{r^2+f^2}. \qquad (3.2)$$

The dot product of the wind stress vector $\mathbf{T}=(T_x, T_y)$ and the transport vector $\mathbf{V}=(U, V)$ yields

$$\mathbf{T}\cdot\mathbf{V}=\frac{r}{r^2+f^2}(T_x{}^2+T_y{}^2)=\frac{r}{r^2+f^2}|\mathbf{T}|^2. \qquad (3.3)$$

This shows that the wind stress and the transport are not orthogonal to each other, except for a special case of a vanishing bottom friction ($r=0$). In other words, due to an existence of the bottom friction, the mass transport is not to the right 90° of the wind stress in the northern hemisphere.

When the wind is blowing northward ($T_x=0$, $T_y>0$) the transport is decribed by

$$U=\frac{fT_y}{r^2+f^2},\quad V=\frac{rT_y}{r^2+f^2}. \qquad (3.4)$$

Fig. 1 shows the steady transport described by (3.4). This figure shows that the Ekman transport in a shallow sea associated with a northward wind is not eastward, but it lies between northward and eastward. The deviation from the classical Ekman transport is given by $\tan^{-1}(r/f)$. The magnitude of transport is

$$\sqrt{U^2+V^2}=\frac{T_y}{\sqrt{f^2+r^2}}=\frac{T_y}{f\sqrt{1+r^2/f^2}}. \qquad (3.5)$$

From this we see that the bottom friction reduces the magnitude of transport by a factor of $(1+r^2/f^2)^{-\frac{1}{2}}$. As an example, let's assume that $r=f/4$. In this case the bottom friction reduces the magnitude of the transport by a factor of 0.97, and it rotates the direction of transport 14° northward from the eastward.

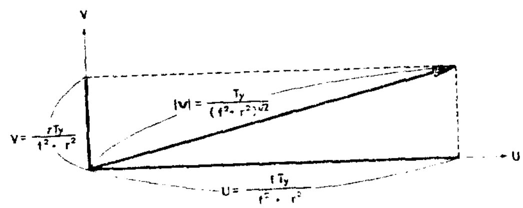

Fig. 1. Steady state transport in a shallow sea driven by a uniform northward wind stress.

4. TRANSIENT TRANSPORT AFTER AN ONSET OF A STEADY WIND

In the previous section we considered a steady state solution. In this section we consider the mass transport in a transient state after an onset of a steady and uniform wind stress. We assume that a constant wind stress is applied from a certain time, say, $t=0$. That is,

$$\mathbf{T}=0, \quad t<0$$
$$\mathbf{T}=\text{constant}, \quad t\geq 0 \tag{4.1}$$

The transient transport is described by a solution of (2.8) with initial conditions

$$U=V=0 \text{ at } t=0. \tag{4.2}$$

We introduce the complex-valued transport V^* and the complex-valued wind stress T^* defined by

$$V^*=U+iV$$
$$T^*=T_x+iT_y, \tag{4.3}$$

where i is a unit imaginary number. An introduction of these complex-valued quantities is very useful, because we can combine two equations in (2.8) into a single equation in a complex-plane. By multiplying a unit imaginary number i to the second equation of (2.8) and by adding this to the first equation of (2.8) we get

$$\frac{\partial V^*}{\partial t}+ifV^*+rV^*=T^*. \tag{4.4}$$

The solution of this equation with an initial condition $V^*(0)=0$ is

$$V^*=\frac{T^*}{if+r}(1-e^{-rt}e^{-ift}). \tag{4.5}$$

That is,

$$U+iV=\frac{(T_x+iT_y)(r-if)}{f^2+r^2}\times$$
$$[(1-e^{-rt}\cos ft)+ie^{-rt}\sin ft]. \tag{4.6}$$

The x and y components of the mass transport can be found simply by equating the real and the imaginary parts of this equation.

When the wind blows northward ($T_x=0$, $T_y>0$), the transient transport is given by

$$U(t)=\frac{T_y}{f^2+r^2}[f(1-e^{-rt}\cos ft)-re^{-rt}\sin ft]$$
$$V(t)=\frac{T_y}{f^2+r^2}[fe^{-rt}\sin ft+r(1-e^{-rt}\cos ft)]. \tag{4.7}$$

The transport vector (4.7) describes a spiral. The period of the "damped oscillation" of the transport is one inertial day, $12\text{hr}/\sin\varphi$. As the time increases the magnitude of the oscillatory component of the transport decreases exponentially with time, and the rate of the decrease depends on the friction coefficient r. After a sufficiently long time (4.7) becomes identical to the steady solution (3.4), i.e.,

$$U(t\to\infty)=\frac{fT_y}{f^2+r^2}, \quad V(t\to\infty)=\frac{rT_y}{f^2+r^2}. \tag{4.8}$$

Fig. 2 shows the temporal evolution of the vertically-averaged velocity, after an onset of a constant wind stress, obtained by dividing the transport of (4.7) by ρH. Numeral values used in this figure are: $T_y=1$ dyne/cm^2, $f(35°\text{N})=8.4\times10^{-5}\text{sec}^{-1}$, $r=0.25f$, and $H=5\times10^3$cm.

The trajectory $X(t)$ of a water particle averaged over the water column is

$$X(t)=\frac{1}{\rho H}\int_0^t U(t)dt$$
$$=\frac{1}{\rho H}\frac{T_y}{f^2+r^2}\left[ft+\frac{r^2-f^2}{r^2+f^2}e^{-rt}\sin ft+\frac{2rf}{r^2+f^2}e^{-rt}\cos ft-\frac{2rf}{r^2+f^2}\right] \tag{4.9}$$

$$Y(t)=\frac{1}{\rho H}\int_0^t V(t)dt$$
$$=\frac{1}{\rho H}\frac{T_y}{f^2+r^2}\left[rt+\frac{r^2-f^2}{r^2+f^2}e^{-rt}\cos ft\right.$$

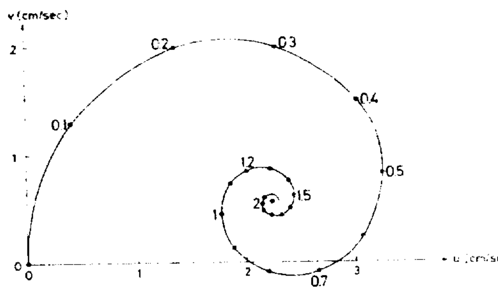

Fig. 2. Vertically-averaged transient velocity (cm/sec) of water particles in a shallow sea of 50m after an onset of a uniform northward wind stress of 1 dyne/cm². The numbers show the time in units of inertial days after an onset of the wind stress. The Coriolis parameter is evaluated at 35°N, and the friction coefficient $r=f/4$.

$$-\frac{2rf}{r^2+f^2}e^{-rt}\sin ft-\frac{r^2-f^2}{r^2+f^2}\Big]$$

If we put $X'=X+\frac{T_y}{\rho H}\frac{2rf}{(r^2+f^2)^2}$ and $Y'=Y+\frac{T_y}{\rho H}\frac{r^2-f^2}{(r^2+f^2)^2}$, then from (4.9) we can easily obtain an equation:

$$\left(X'-\frac{f}{\rho H}\frac{T_y}{r^2+f^2}t\right)^2+\left(Y'-\frac{r}{\rho H}\frac{T_y}{r^2+f^2}t\right)^2 = e^{-2rt}\left(\frac{1}{\rho H}\frac{T_y}{r^2+f^2}\right)^2. \qquad (4.10)$$

This equation shows that the trajectory of the water particle is decribed by a superposition of a translational motion with a velocity $\left(\frac{f}{\rho H}\frac{T_y}{r^2+f^2},\ \frac{r}{\rho H}\frac{T_y}{r^2+f^2}\right)$ and a circular motion with a radius of $e^{-rt}\left(\frac{1}{\rho H}\frac{T_y}{r^2+f^2}\right)$. The period of the circular motion is one inertial day, $12\text{hr}/\sin\varphi$. After a sufficiently long time the circular motion is diminished and the trajectory becomes a straight line. Fig. 3 shows the trajectory calculated by (4.9) using the values the same as in Fig. 2.

It is interesting to consider the transport for the case of a vanishing friction damping, i.e., $r=0$. In this case (4.7) becomes

$$U(t)=\frac{T_y}{f}(1-\cos ft)$$
$$V(t)=\frac{T_y}{f}\sin ft. \qquad (4.11)$$

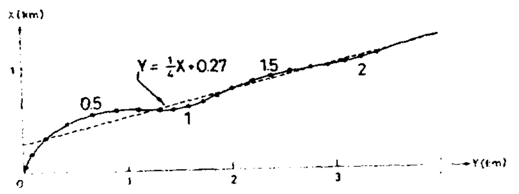

Fig. 3. The trajectory (km) of a water particle, averaged vertically in a shallow sea of 50m, after an onset of a uniform northward wind stress of 1dyne/cm². The numbers show the time in inertial days after an onset of the wind stress forcing. The Coriolis parameter is evaluated at 35°N, and the friction coefficient $r=f/4$.

The transport vector of (4.11) describes a circle centered at $(T_y/f, 0)$ in the (U, V)-plane with a radius of T_y/f. The time average of the transport over an inertial day is the same as the classical Ekman transport, namely, an eastward transport of T_y/f. The trajectory $\mathbf{X}(t)$ of a water particle averaged vertically over the water column for the case of $r=0$ is a cycloid described by

$$X(t)=\frac{1}{\rho H}\int_0^t U(t)dt=\frac{T_y}{H\rho f}\left(t-\frac{1}{f}\sin ft\right)$$
$$Y(t)=\frac{1}{\rho H}\int_0^t V(t)dt=\frac{T_y}{H\rho f^2}(1-\cos ft). \qquad (4.12)$$

5. TRANSPORT ASSOCIATED WITH A SINUSOIDAL WIND STRESS

So far we have considered the steady and transient transports associated with a constant wind stress. The direction and magnitude of the wind stress in the ocean vary randomly with time. The random wind stress can be decomposed into the time-averaged mean field and the fluctuations. The transports associated with the mean field were already discussed in Sections 3 and 4. The fluctuation part of the x and y components of the wind stress can be represented by the Fourier series expansion. In this section we will investigate the transports associated with a single Fourier component of the wind stress which changes its magnitude but does not change its direction. The transport associated with a wind stress of which direction varies linearly with time will be discussed in Section 6.

For a mathematical simplicity we assume a vanishing bottom friction (i.e., $r=0$). We consider a transport associated with a sinusoidally varying wind stress in the meridional direction, that is,

$$T_x(t)=0,\quad T_y(t)=\widetilde{T}_y\cos\omega t, \tag{5.1}$$

vhere $\widetilde{T}_y$ is the amplitude of the meridional vind stress and ω is the associated frequency. n this case (2.8) becomes

$$\frac{\partial U}{\partial t}-fV=0$$
$$\frac{\partial V}{\partial t}+fU=\widetilde{T}_y\cos\omega t, \tag{5.2}$$

ınd yields a solution

$$U(t)=\frac{f\widetilde{T}_y}{f^2-\omega^2}\cos\omega t,$$
$$V(t)=-\frac{\omega\widetilde{T}_y}{f^2-\omega^2}\sin\omega t. \tag{5.3}$$

The temporal characteristics of the transport ·an be seen more clearly by representing (5.3) ıs

$$\frac{U^2(t)}{\left(\frac{f\widetilde{T}_y}{f^2-\omega^2}\right)^2}+\frac{V^2(t)}{\left(\frac{\omega\widetilde{T}_y}{f^2-\omega^2}\right)^2}=1. \tag{5.4}$$

This equation shows that the transport vector, ıssociated with a sinusoidal wind-stress for·ing, describes a "transport ellipse", the freıuency of which is the same as that of the ·orcing. When the frequency of the meridional wind stress is smaller than f, that is, when the period of the wind stress is larger than an inertial day, the major axis of the transport ellipse is orthogonal to the direction of the wind. In other words, if $\omega \ll f$, then the oceanic transport associated with a sinusoidally varying meridional wind stress is almost zonal, and if $\omega \gg f$, then the oceanic transport is almost meridional. When the period of the wind stress forcing is equal to one inertial day (i.e., $\omega=f$), the oceanic transport in an ocean without friction, described by (5.3) or (5.4), will be infinitely large.

From (5.3) we see that the transport, associated with a sinusoidal wind stress, averaged over one cycle of the forcing is zero. The sense of rotation of the transport ellipse is clockwise for the case of $\omega<f$ and is anticlockwise for $\omega>f$ in the northern hemisphere. Fig. 4 shows the temporal evolution of the wind stress (5.1) and the associated vertically averaged velocity, which is obtained by multiplying $1/\rho H$ to the transport (5.3). Numeral

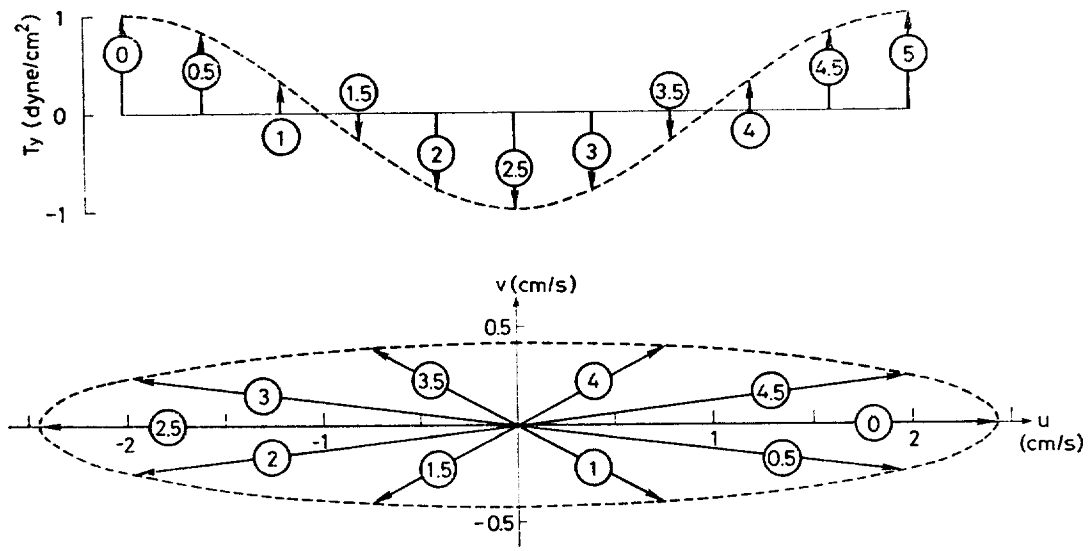

Fig. 4. The upper figure: the meridional sinusoidal wind stress with an amplitude of 1 dyne/cm² and a period of 5 inertial days. The lower figure: the horizontal velocity(cm/sec) averaged vertically in a shallow sea of 50m. The numbers inside of circles in dicate the time in units of inertial days. The Coriolis parameter is evaluated at 35°N, and the bottom friction is neglected.

values used in Fig .4 are: $\tilde{T}_y=1$dyne/cm^2 (the magnitude of wind speed is about 7 m/s), f (35°N) $=8.4\times10^{-5}$sec^{-1} (inertial period of 21 hours), $\omega=1.45\times10^{-5}$sec^{-1}(forcing period of 5 days), and $H=50$m.

6. TRANSPORT ASSOCIATED WITH A ROTATING WIND STRESS

In this section we investigate the transport associated with a wind stress of which direction varies linearly with time. We assume that a wind stress of a constant magnitude rotates with a constant angular velocity and the bottom friction is negligible. With these simplifying assumptions, (2.8) becomes

$$\frac{\partial U}{\partial t}-fV=\tilde{T}\cos\theta t$$
$$\frac{\partial V}{\partial t}+fU=\tilde{T}\sin\theta t, \qquad (6.1)$$

where $\tilde{T}$ is the magnitude of the wind stress and θ is a constant angular velocity of the rotation of the wind stress. (6.1) can be written in terms of the complex-valued transport, $V^*=U+iV$, as

$$\frac{\partial V^*}{\partial t}+ifV^*=\tilde{T}e^{i\theta t}, \qquad (6.2)$$

and this has a solution

$$V^*(t)=\tilde{a}e^{-ift}-\frac{i\tilde{T}}{f+\theta}e^{i\theta t}$$
$$=\tilde{a}(\cos ft-i\sin ft)+$$
$$\frac{\tilde{T}}{f+\theta}(\sin\theta t-i\cos\theta t), \qquad (6.3)$$

where $\tilde{a}$ is an arbitrary constant associated with the homogeneous solution of (6.2). The real and imaginary parts of (6.3) yield the desired solution of (6.1):

$$U(t)=\tilde{a}\cos ft+\frac{\tilde{T}}{f+\theta}\sin\theta t$$
$$V(t)=-\tilde{a}\sin ft-\frac{\tilde{T}}{f+\theta}\cos\theta t. \qquad (6.4)$$

The solution (6.4) shows that the transport consists of a free inertial oscillation with an undetermined amplitude $\tilde{a}$ and a forced oscillation with an amplitude of $\tilde{T}/(f+\theta)$. Note that the free inertial oscillation does not require any external forcing, and therefore the amplitude $\tilde{a}$ is undetermined. The forced oscillation rotates with an angular velocity θ which is the same as that of the wind stress forcing. For a special case of $\tilde{a}=0$, that is, when there is no free inertial oscillation, we can easily show that the transport vector is always to the right 90° of the wind stress vector.

It is instructive to plot the temporal evolution of the transport described by (6.4). The vertically-averaged velocity can be obtained by multiplying $1/\rho H$ to the transport of (6.4). If, as an example, we take numeral values of $f=8.4\times10^{-5}$ sec^{-1}, $\theta=0.5f$ (rotation period of the wind stress is two inertial days), $\tilde{T}=$ 1 dyne/cm^2, $\tilde{a}=\tilde{T}/(f+\theta)$ (amplitudes of the free and the forced oscillations are the same), and $H=50$m, then the vertically averaged velocity $(\bar{u},\bar{v})$ becomes

$$\bar{u}(t)=\frac{\tilde{a}}{\rho H}(\cos ft+\sin\theta t)$$
$$=1.6(\cos ft+\sin\theta t)\,(\text{cm/sec})$$
$$\bar{v}(t)=-\frac{\tilde{a}}{\rho H}(\sin ft+\cos\theta t)$$
$$=-1.6(\sin ft+\cos\theta t)\,(\text{cm/sec}). \qquad (6.5)$$

Fig. 5 shows the temporal evolution of velocity computed by (6.5). Note that the three-leaved rose curve of the velocity shown in Fig. 5 is not necessarily a general curve of (6.4) but is only one specific example of the velocity computed by (6.5).

7. DISCUSSION AND CONCLUSIONS

The theoretical studies in this paper provide us insights on the qualitative features of the transports in a shallow sea driven by time-

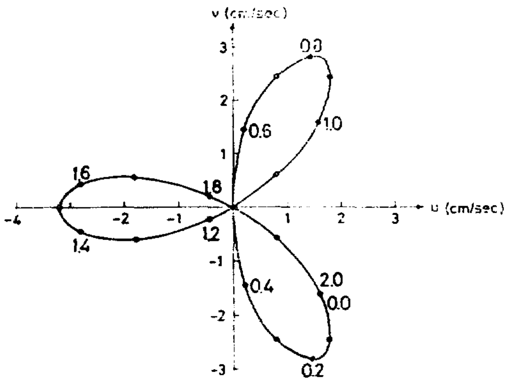

Fig. 5. Temporal evolution of the velocity(cm/sec), vertically averaged in a shallow sea of 50m, driven by a rotating wind stress with a magnitude of 1 dyne/cm^2 and a rotation period of two inertial days. The amplitude of the free inertial oscillation and that of the forced oscillation are assumed to be the same. The numbers indicate the time in units of inertial days. The Coriolis parameter is evaluated at 35°N, and the bottom friction is neglected.

varying wind stress forcing. In particular, we have shown the influences of the bottom friction on the transports in a shallow sea. It is noteworthy that our transport model does not include any austausch coefficient. This implies that the analytic models of transports for various types of wind-stress forcing do not depend on the vertical distribution of the austausch coefficient.

We should, however, be aware of the limitations of the theoretical results of this paper. We assumed that the sea is horizontally unounded and homogeneous and the wind stress is horizontally uniform. We further assumed that the velocity field is horizontally homogeneous. These assumptions require the depth of the sea to be constant. Hence the transport models discussed in this paper should work only in shallow seas of a uniform depth located far away from the coastlines. Our transport models can give us quantitative values of transport, but they do not provide us any information on the vertical distribution of the velocity field.

The theoretical results of this paper can be useful in understanding the horizontal transports of water masses and nutrients associted with time-varying wind stress in a shallow sea. Also, although our main concern in this paper was concentrated to the transports in a shallow sea, the theoretical results of this paper can, in principle, be applied to the time-varying transports in the surface Ekman layer of the open ocean provided the friction coefficient is adjusted accordingly.

ACKNOWLEDGEMENTS

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APPENDIX: NOTATIONS

Mathematical symbols used in this paper are summerized below with their c.g.s. units in the parenthesis.

a Amplitude of the transport associated with a free inertial oscillation (gm cm^{-1} sec^{-1})

A_h, A_v Horizontal and vertical austausch coefficients or eddy viscosity(gm cm^{-1} sec^{-1})

$\mathbf{B}=(B_x, B_y)$ Bottom friction stress(gm cm^{-1} sec^{-2})

C_d Drag coefficient ($C_d=1.2\times10^{-3}$)

$f=2\Omega\sin\varphi$ Coriolis parameter(sec^{-1})

H Depth of the sea(cm)

$i=\sqrt{-1}$ A unit imaginary number (dimensionless)

p Pressure(gm cm^{-1} sec^{-2}, or dyne cm^{-2})

r Coefficient of bottom friction(sec^{-1})

$\mathbf{T}=(T_x, T_y)$ Wind stress vector(gm cm^{-1}sec^{-2}, or dyne cm^{-2})

$\tilde{T}$ Amplitude of the time-varying wind stress(gm cm^{-1} sec^{-2})

$T^*=T_x+iT_y$ Complex-valued wind stress(gm cm^{-1} sec^{-2})

u, v, w Velocity components of water particles in the x, y and z directions, respectively(cm sec^{-1})

$\mathbf{V}=(U, V)$ Horizontal mass transport vector(gm cm^{-1} sec^{-1})

$V^*=U+iV$ Complex-valued horizontal mass transport(gm cm^{-1} sec^{-1})

$\mathbf{W}$ Wind velocity(cm sec^{-1})

x, y, z Local cartesian coordinate; x=eastward, y=northward, and z=upward (cm)

$X=(X, Y)$ Trajectory of water particles (cm)

θ Angular velocity of a rotating wind stress (sec^{-1})

ρ Density of the sea water($\rho=1$ gm cm^{-3})

ρ_a Density of the air($\rho_a=1.225\times10^{-3}$ gm cm^{-3})

φ Latitude(degrees)

Ω Angular velocity of the earth's rotation($\Omega=7.27\times10^{-5}$sec^{-1})

ω Angular frequency of a sinusoidal wind stress (sec^{-1})

시간변화적 바람에 따른 넓은 천해에서의 해수유량

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요약 : 수평적으로 무한히 넓은 천해에 시간에 따라 크기와 방향이 바뀌는 바람이 불 때의 해수의 유량(transports)에 대한 이론적인 계산을 할 수 있는 모델을 만들었다. 이 모델에서 사용하는 선형화된 이론식은 가속력, 편향력(Coriolis force), 바람의 응력(wind stress) 및 해저면 마찰력을 포함하고 있으며, 이 모델로부터 해수 유량에 대한 다음과 같은 결과를 얻었다. 일정한 바람이 계속 불 때의 해수 유량은 해저면 마찰로 인하여 바람 오른쪽 방향 90°보다 작다. 바람이 불기 시작한 후의 천이상태(transient state)의 유량은 정상류적(steady)인 에크만류(Ekman current)에다가 시간이 지남에 따라서 크기가 감소하는 관성진동(inertial oscillation)과의 합성으로 나타난다. 바람의 크기가 정현적(sinusoidal)으로 바뀌는 경우 유량 벡터는 바람변화의 주기와 같은 시간동안 일주회전을 하는 유량 타원(transport ellipse)을 그린다. 바람의 방향이 바뀔 경우 해수유량은 자유관성진동(free inertial oscillation)과 강제진동의 합으로 나타나는데, 이 경우 강제진동의 주기는 바람 방향의 회전주기(rotation period)와 같다. 이 이론적인 해수유량모델은 천해에서 시간에 따라 바람이 바뀔 경우 해수와 영양염의 이동에 대한 해석을 하는데 유용하다.