論 文

QAM Error Performance in the Environment of Cochannel Interference and Impulsive Noise

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동일채널간섭 및 임펄스잡음 환경하에서의 QAM 신호의 오율특성

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要 約 동일 채널 간섭 및 임펄스 잡음 환경하에서 QAM시스템의 특성을 오율의 관점에서 연구했다. 반송파대 잡음 전력비, 반송파대 간섭파 전력비, 임펄스 지수 및 신호와 간섭파 간의 위상차의 함수로써 나타낸 L-level QAM신호의 오율식을 도출하여 16-QAM에 대한 수치계산을 하여 오율특성을 그래프로 보였다.

ABSTRACT We have studied and discussed the error rate performance of Quadrature Amplitude Modulation(QAM) in an environment of cochannel QAM interference and impulsive noise. A general equation of error probability for L-level QAM signal has been derived and the error rate of the 16-QAM signal, as an example, has been calculated as functions of carrier-to-noise power ratio(CNR), carrier-to-interferer power ratio(CIR), impulsive index, and the phase difference between signal and interferer.

1. Introduction

The increasing demands of data transmission through bandlimited satellite, terrestrial radio and telephony channels require highly efficient modulation techniques in terms of bandwidth. The M-ary Phase Shift Keying(PSK) which requires less bandwidth than FM or binary PSK(BPSK), Quadrature Amplitude Modulation(QAM), and Frequency Shift Keying(FSK) techniques are presently the most widely used digital transmission methods.[1]

Recently the man-made and natural electromagnetic interference, as well as cochannel and adjacent channel interference, cause a serious problem on the overcrowded communication channels in the radio frequency bands. And impulsive noise, which is generated by many electromechanical devices and the ignition spak of automobile, etc., has becomes a serious degradation factor to the receiving system in urban environments.[2]

The upper bound of error probability for the 16-QAM signals with cochannel interference had been presented.[3] Up to the present, there are few papers which deal with details of performance of QAM systems in an impulsive noise environment.

In this paper, we investigate and discuss the er-

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ror rate performance of L-level QAM system in an environment of cochannel QAM interference and impulsive noise. And using the derived general equation of error rate we evaluate and discuss the error performance of 16-QAM system in terms of carrier-to-noise power ratio (CNR), carrier-to-interferer power ratio (CIR), impulsive index, and the phase difference between signal and interferer.

2. Description of signal, interferer, and impulsive noise

1. QAM signal

A $2L$ bit QAM signal received during the Mth interval with signaling interval T and carrier frequency f_c can be represented as[3]

$$s(t)=A_i\cos\omega_c t+B_k\sin\omega_c t \quad MT\leq t\leq(M+1)T \tag{1}$$

where $A_i,\ B_k\in\{\pm d,\pm 3d,\cdots\pm(2L-1)d\}$

A_i, B_k ; the amplitudes of the signal along each axis of the coordinates to be statistically independent

$2d$; the distance between two adjacent signal points

L ; number of levels of QAM signal.

2. Interferer

The most important sources of interference are cochannel interference and adjacent channal interference.[2],[4] But now in this paper, especially cochannel interference by the other QAM signal, which is generated by the reuse of existing band in use, is considered. The cochannel QAM interferer $i(t)$ can be written as similar form of eq. (1)[3],[5]

$$i(t)=C_i\cos(\omega_c t+\Psi)+D_k\sin(\omega_c t+\Psi) \tag{2}$$

where $C_i,\ D_k\in\{\pm d',\pm 3d',\cdots\pm(2L-1)d'\}$

$2d'$; the distance between two adjacent interferering signal points

Ψ ; phase slip between signal and interferer.

3. Impulsive Noise

In digital transmission systems, there is impulsive noise in addition to Gaussian noise. Generally impulsive noise may be generated by two sources, one being man-made radio noise such as ignition spark of automobile, another being natural impulsive random noise such as cosmic noise.

In this paper, as an analytical model of impulsive noise, we introduce the Middleton's cannonical statistical physical models of electro-magnetic interference.[6]

Middleton classified impulsive noise into three types as follow :

1) Class A : It has narrower bandwidth in comparison with that of the receiving system.
(narrow band vis-à-vis the receiver)

2) Class B : It has wider bandwidth in comparisson with that of the receiving system.
(broad band vis-à-vis the receiver)

3) Class C ; Class A+Class B

In this paper, we consider only the type of Class A as an impulsive noise model.

The impulsive noise in time domain can be represented by

$$n_I(t)=N\cos(\omega_c t+\xi) \tag{3}$$

where N and ξ, which are independent random variables, are the envelope and the phase. Here the initial phase ξ is assumed to be uniformly distributed in the interval $[0,2\pi]$.

The probability density function (p. d. f.) of the instantaneous envelope of $n_I(t)$ is given by Middleton[6] as

$$p.d.f.(N)=\frac{e^{-A}}{W}\sum_{j=0}^{\infty}\frac{A^j}{j!}\frac{N}{\sigma_j^2}e^{-\frac{N^2}{2W\sigma_j^2}} \tag{4}$$

where

$W(=\sigma_G^2+\Omega_{2A})$: total impulsive noise power (Gaussian noise power component (σ_G^2) plus non-Gaussian noise power component (Ω_{2A}))

A ; impulsive index, which is the product of received average numbers per unit second and burst's emission duration

N ; instantaneous noise envelope

$\Gamma'(=\sigma_G^2/\Omega_{2A})$; the ratio of Gaussian noise power component (σ_G^2) to non-Gaussian noise power component (Ω_{2A}) of the impulsive noise

$$\sigma_j^2=\frac{j/A+\Gamma'}{1+\Gamma'}$$

3. Error rate performance of L-level QAM system

We show the phasor diagram of the received si-

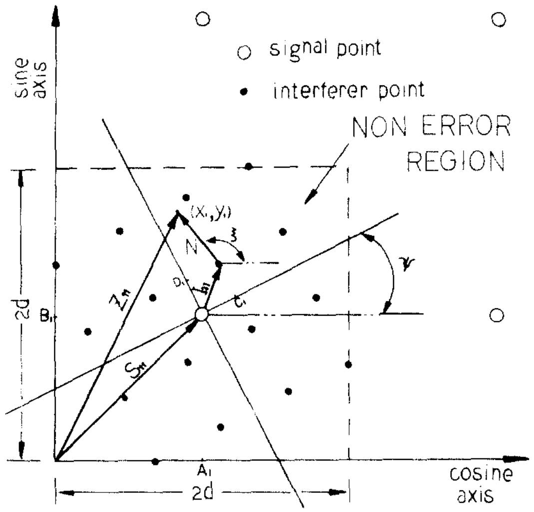

Fig. 1 The phasor diagram for the received signal point (A_1, B_1) corrupted by the interferer point (C_1, D_1) and the impulsive noise.

gnal interfered by cochannel interferer and impulsive noise for a signal point (A_1, B_1) with its non error region in Fig. 1. S_{11} and Z_{11} are the distances from orign to the signal point (A_1, B_1), and composite signal's tip (x_1, y_1) respectively. I_{11} is the distance from the signal point (A_1, B_1) to the interferer point (C_1, D_1). N and ξ are the instantaneous envelope and the phase of the noise. Ψ is the phase difference between signal's and interferer's axes. Also some signal points in the above part of the right half plane is shown for a convenience.

If the composite received signal is represented by $z(t)$, then

$$\begin{aligned} z(t) &= (A_i + C_i' + N\cos\xi)\cos\omega_c t \\ &\quad + (B_k + D_k' - N\sin\xi)\sin\omega_c t \\ &= x_i\cos\omega_c t + y_k \sin\omega_c t \end{aligned} \tag{5}$$

where, $C_i' = C_i\cos\Psi + D_k\sin\Psi$

$D_k' = D_k\cos\Psi - C_i\sin\Psi$

x_i, y_k ; the composite received signal components.

Therefore, received L-level QAM signal is considered as the sum of independently modulated sine and cosine carriers. For the case of a modulated cosine carrier, the non error region consists of rectangular area $(2d\times 2d)$ with each signal point at its center except the each outside signal point. Accordingly an error occurs under the following conditions when equal thresholds are assumed:

$$\begin{aligned} &\text{if } A_i \neq (2L-1)d \text{ and } |Z_1| > d \\ &\text{if } A_i = (2L-1)d \text{ and } Z_1 < -d, \text{ and} \\ &\quad A_i = -(2L-1)d \text{ and } Z_1 > d \end{aligned} \tag{6}$$

where, $Z_1 = x_i - A_i$

$= C_i' + N\cos\xi$

$Z_2 = y_k - B_k$

$= D_k' - N\sin\xi.$

From the symmetrical distribusion of random variables, we have that

$$\begin{aligned} \text{Prob.}_{,\Psi}\ (Z_1 > d\,|\,A_i) &= \text{Prob.}_{,\Psi}\ (Z_1 < -d\,|\,A_i) \\ &= \text{Prob.}_{,\Psi}\ (Z_1 > d). \end{aligned} \tag{7}$$

Therefore, the probability of error for the cosine carrier is given by averaging eq. (7) over A_i considering eq. (6).

$$PE_{\cos,\Psi} = \frac{2L-1}{2L}\text{Prob.}_{,\Psi}\ (|Z_1| > d). \tag{8}$$

Similarly, we can show that the error probability for the modulated sine carrier is

$$PE_{\sin,\Psi} = \frac{2L-1}{2L}\text{Prob.}_{,\Psi}\ (|Z_2| > d). \tag{9}$$

Then, the error probability PE for the L-level QAM system is the average of the error probability for the modulated sine and cosine carriers.

The error probability PE is therefore

$$\begin{aligned} PE_{,\Psi} &= \frac{1}{2}PE_{\cos,\Psi} + \frac{1}{2}PE_{\sin,\Psi} \\ &= \frac{2L-1}{4L}[\text{Prob.}_{,\Psi}\ (|Z_1| > d) \\ &\quad + \text{Prob.}_{,\Psi}\ (|Z_2| > d)]. \end{aligned} \tag{10}$$

Since sine and cosine channels are statistically independent, eq. (10) is simplified as [3]

$$PE_{,\Psi} = \frac{2L-1}{2L}[\text{Prob.}_{,\Psi}\ (|Z_1| > d)] \tag{11}$$

And eq. (11) can be rewritten using $Z_1 = C_i' + N\cos\xi$.

$$\begin{aligned} PE_{,\Psi} &= \frac{2L-1}{2L}\Big(\frac{1}{2}\text{Prob.}_{,\Psi}\ (N\cos\xi > d - C_i') \\ &\quad + \frac{1}{2}\text{Prob.}_{,\Psi}\ (N\cos\xi < -d - C_i')\Big). \end{aligned} \tag{12}$$

Averaging eq. (12) with ξ and introducing eq. (4) to integrate over the error region, we get

$$\begin{aligned} PE_{,\Psi} &= \frac{2L-1}{2L}\frac{1}{\pi}\frac{e^{-A}}{W}\sum_{j=0}^{\infty}\frac{A^j}{j!} \\ &\cdot\Big(\int_{d-C_i'}^{\infty}\frac{N^2}{\sigma_j^2}e^{-\frac{N^2}{2W\sigma_j^2}}dN - \int_{-\infty}^{-d-C_i'}\frac{N^2}{\sigma_j^2}e^{-\frac{N^2}{2W\sigma_j^2}}dN\Big), \end{aligned} \tag{13}$$

The integration in eq. (13) can be carried out by using [A. 1]. And rearranging the result by [A. 2], we obtain

$$PE_{\Psi}=\frac{2L-1}{2L}\frac{1}{\pi}\frac{e^{-A}}{W}\sum_{j=0}^{\infty}\frac{A^j}{j!}\frac{1}{\sigma_j^2}$$
$$\cdot\Big\{W\sigma_j^2(d-C_i')e^{-\frac{1}{2W\sigma_j^2}(d-C_i')^2}$$
$$-W\sigma_j^2(d+C_i')e^{-\frac{1}{2W\sigma_j^2}(d+C_i')^2}\Big\}$$
$$+\frac{\sqrt{\pi}}{\frac{2}{W\sigma_j^2}\sqrt{\frac{1}{2W\sigma_j^2}}}\Big[erfc\Big\{\sqrt{\frac{1}{2W\sigma_j^2}}(d-$$
$$C_i')\Big\}-erfc\Big\{\sqrt{\frac{1}{2W\sigma_j^2}}(d+C_i')\Big\}\Big]. \quad (14)$$

When we define the carrier-to-noise power ratio (CNR) as $\alpha=d^2/2W$, and carrier-to-interferer power ratio (CIR) as $\gamma^{-1}=A_i^2/C_i^2$ or B_k^2/D_k^2, and adopting [A. 3] and [A. 4], eq. (14) becomes

$$PE=\frac{2L-1}{2L}\frac{e^{-A}}{\pi}\sum_{j=0}^{\infty}\frac{A^j}{j!}d\Big[\Big\{1-\sqrt{\gamma}\sqrt{(2i-1)^2+(2k-1)^2}\sin\Big(\Psi+\tan^{-1}\frac{2i-1}{2k-1}\Big)\Big\}$$
$$\cdot\exp\Big(-\frac{\alpha}{\sigma_j^2}\Big\{1-\sqrt{\gamma}\sqrt{(2i-1)^2+(2k-1)^2}$$
$$\cdot\sin\Big(\Psi+\tan^{-1}\frac{2i-1}{2k-1}\Big)\Big\}^2\Big)$$
$$-\Big\{1+\sqrt{\gamma}\sqrt{(2i-1)^2+(2k-1)^2}$$
$$\cdot\sin\Big(\Psi+\tan^{-1}\frac{2i-1}{2k-1}\Big)\Big\}\exp\Big(-\frac{\alpha}{\sigma_j^2}$$
$$\cdot\Big\{1+\sqrt{\gamma}\sqrt{(2i-1)^2+(2k-1)^2}$$
$$\cdot\sin\Big(\Psi+\tan^{-1}\frac{2i-1}{2k-1}\Big)\Big\}^2\Big)\Big]. \quad (15)$$

Although PE is the function of the first order of d, no meaning can be found in the variation of d since, at constant α, the increase of d causes the increase of W to be a fixed value. Therefore, we can normalized the error rate by d. Thus,

$$PE=\frac{2L-1}{2L}\frac{e^{-A}}{\pi}\sum_{j=0}^{\infty}\frac{A^j}{j!}\Big\{(1-X)e^{-\frac{\alpha}{\sigma_j^2}(1-X)^2}$$
$$-(1+X)e^{-\frac{\alpha}{\sigma_j^2}(1+X)^2}\Big\}$$

where,

$$X=\sqrt{\gamma}\sqrt{(2i-1)^2+(2k-1)^2}$$
$$\cdot\sin\Big(\Psi+\tan^{-1}\frac{2i-1}{2k-1}\Big). \quad (16)$$

4. Numerical calculation

Using the derived general equation of symbol error rate of L-level QAM signal, we have calculated the symbol error rate for 16-QAM signal ($L=2$). The signal diagram of 16-QAM signal is shown in Fig. 2.

We have evaluated the symbol error performance of 16-QAM system with the variation of impulsive index (A), the ratio of Gaussian noise component to non-Gaussian noise power component of the impulsive noise (Γ'), carrier-to-noise power ratio (CNR), carrier-to-interferer power ratio (CIR) and the phase difference between signal and interferer (Ψ). The numerical results are shown in Fig. 3~Fig. 6.

5. Discussions and conclusions

Fig. 3 shows that in case of $A=1$ and $\Gamma'=10$ the error rate performance approachs to the Gaussian case. The reason is that the error may be occurs by mainly Gaussian noise as the impulsive index A becomes unity and Γ' increases. And we can find that the error of 16-QAM is notably dependent on CNR when CIR is above 20 dB.

As shown in Fig. 4, for a fixed value of A (= 0. 1), the larger Γ' (=Gaussine noise power/ Impulsive noise power), the more errors occur in 16

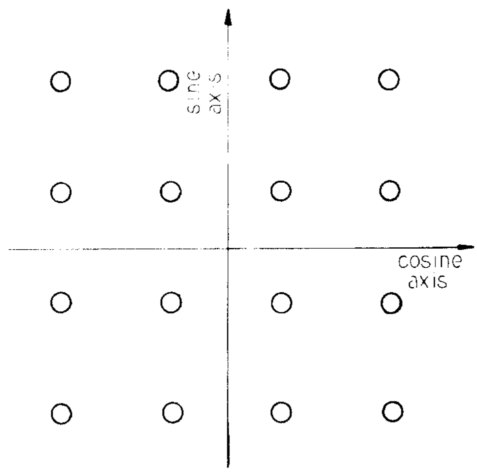

Fig. 2 The signal diagram of 16-QAM (L=2).

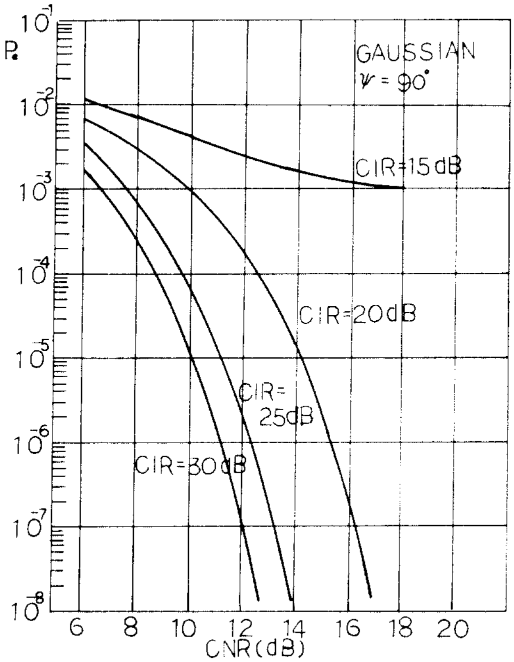

Fig. 3 Symbol error rate of QAM signal interfered by Gaussian noise and interferer.

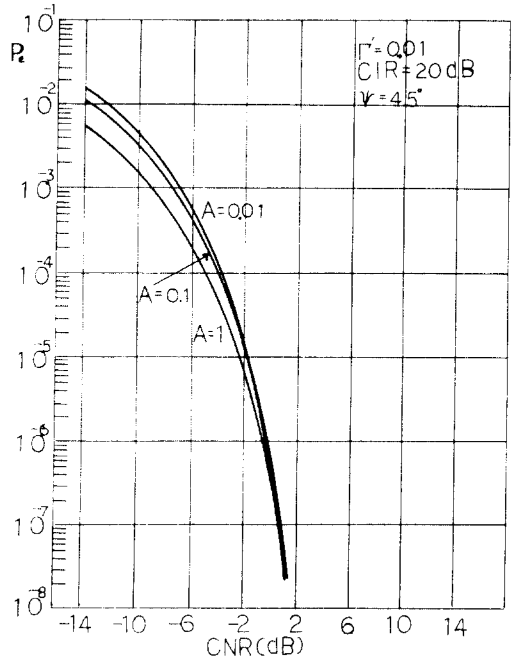

Fig. 5 Symbol error rate of QAM signal interfered by impulsive noise and one interferer (with respect to the change of A).

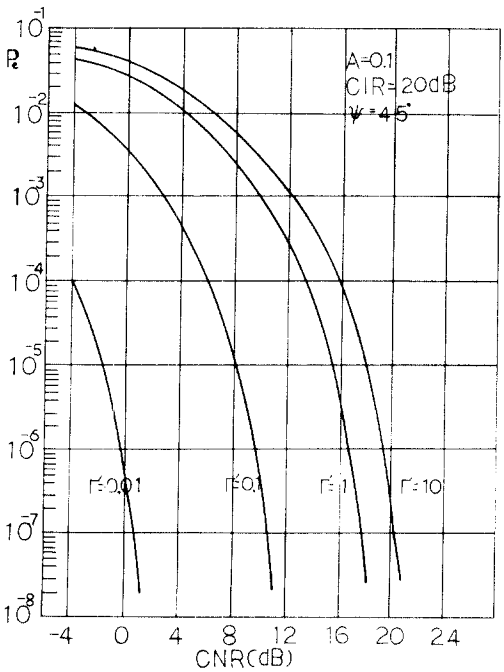

Fig. 4 Symbol error rate of QAM signal interfered by impulsive noise and one interferer (with respect to the change of Γ').

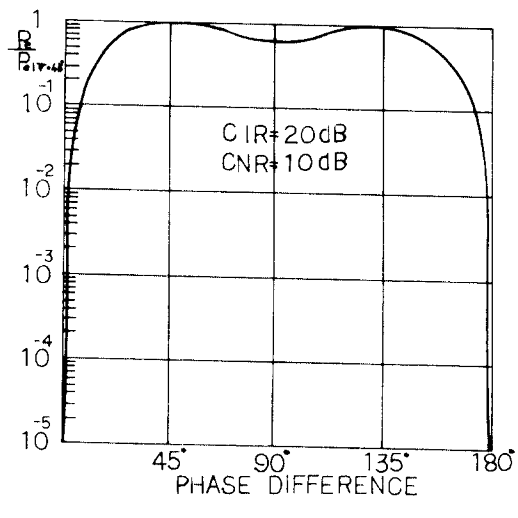

Fig. 6 Symbol error rate normalized by the worst case ($\Psi = 45°$).

-QAM system when the total noise power is constrained.

In Fig. 5, we can find that the error rate performances are nearly unrelated to A for a fixed value of Γ.

From above results we can say that the error rate performance of 16-QAM system is weakly dependent on A but sensitive to the variation of Γ. Because the error rate performances become worse for larger Γ, the QAM system may have effected more from Gaussian noise as in AM in spite of its inherent digital modulation characteristics.

Fig. 6 shows the dependence of the symbol error rates, which is normalized by the worst case (Ψ= 45°, 135°), on the phase difference. As shown in this figure, we know that the error rate performance of 16-QAM system is greatly influenced by the phase difference between signal and interferer in a cochannel interference environment. Therefore, we can consider that the QAM performance can be improved when the phase difference becomes 0° or 180°

Appendices

〔A. 1〕 $I(2)=\int_{a_1}^{\infty} X^2 \cdot e^{-bX^2} dX$

$$= -\frac{1}{2b}\left[Xe^{-bX^2}\right]_{a_1}^{\infty}+\frac{1}{2b}I(0)$$

where $I(0)=\int_{a_1}^{\infty} e^{-bX^2}\, dX$

〔A. 2〕 $erfc(x)=1-erf(x)$

$$=\frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}\, dt$$

〔A. 3〕 $erfc(x+y)=erfc(x)+\frac{2}{\sqrt{\pi}}\exp(-x^2)$

$$\sum_{i=1}^{\infty}(-1)^i H_{i-1}(x)\frac{y^i}{i!}$$

where H_K is the Hermite polynomial of order K

〔A. 4〕 $\frac{1}{2\pi}\int_0^{2\pi}\sin^n x\, dx$

$$=\begin{cases} 0 & ; n=1,3,5 \cdots\cdots \\ \dfrac{n!}{2^n\left(\frac{1}{2}n\right)!\left(\frac{1}{2}n\right)!} & ; n=0,2,4 \cdots\cdots \end{cases}$$

References

(1) C. M. Thomas, M. Y. Weidner and S. H. Durrani, "Digital Amplitude-Phase Keying with M-ary alphabets," IEEE Trans. Commu., vol. COM-22, no. 2, pp. 168-180, Feb. 1974.

(2) K. Feher, "Digital modulation techniques in an interference environment," vol. 9 of EMC Encyclopedia. Germantown, Maryland: Don White Consultants, 1977.

(3) A. Javed and R. P. Tetarenko, "Error probability upper bound for coherently detected QASK signals with cochannel interference," IEEE Trans. on Commu., vol. COM-27, no. 12, Dec. 1979.

(4) S. J. Cho, "Intersystem radio interference between analog frequency modulation system and digital phase modulation system," Ph. D. dissertation, Dept. of Commu. Eng., OSAKA Univ. Japan, 1981.

(5) V. K. Prabhu, "The detection efficiency of 16-ary QAM, Bell System Technical Journal, vol. 59, no. 4, April 1980.

(6) D. Middleton, "Statistical-physical models of electromagnetic interference," IEEE Trans. Electromag. Compat., vol. EMC-19, no. 3, pp. 106-127, Aug. 1977.

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