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# The Crushing Behaviour of Thin Plates Subjected to Compression

## —Ultimate Load and Energy Absorption—*

by

Jae Wook Lee**

**Abstract**

An experimental investigation on the ultimate load and energy absorption of thin plates is presented, which enables the damage to a ship involved in a collision to be estimated in terms of the lost kinetic energy.

The derived formulae are based upon experimental analysis and compared with theoretical presentations published by some authors.

A relationship is found between the absorbed energy and the volume of damaged steel plates.

## 1. Introduction

During the last two decades a large number of papers have been published on the collision protection of ships.

These investigations were motivated largely by the necessity to develope reliable protection systems for nuclear-powered ships and by the international concern about environmental pollution.

The ship collision problem is very complicated and the majority of investigations published in this area have been conducted on experimental models.

Most of the theoretical approaches to the energy absorption characteristic of ship structures are based on the assumption that the absorbed energy is essentially proportional to the volume of damaged steel in the striking ship and the struck ship.

Minorsky [1] has made the first approach to estimate the energy absorbing characteristics of ship structures against collision damage. Since then Woisin [2], Akita and Kitamura [3] have done experiments to verify Minorsky's equation. Reckling [4] has performed theoretical investigations to evaluate problems involving safety calculations, namely "accordion" folding of plates with in-plane loading, tearing of in-plane loaded plates and tearing of transversely loaded plates.

These formulae of Reckling have been used to compute energy distributions from known damages, e.g. from model tests.

Vaughan [5] has recently made a dimensional representation that the dissipated energy can be separated into a crushing part and a tearing part. One interesting feature of Vaughan's work is that it can be applied to grounding.

Due to the varied and complex nature of the collision process, all experimental and theoretical approaches rely on a number of simplifications, sometimes contradictory and highly dependent on the nature of the considered problem.

In this connection a fundamental approach to the analysis of the energy absorption characteristic of thin plates is carried out through experimental verification.

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## 2. Experiments and Results

The universal testing machine installed together with all the necessary service and working apparatus is used for the experiment.

The maximum force of the machine (TREBEL UPM 2000) amounts to 2000KN at a maximum hydraulic cylinder stroke of 300mm. The distance of the compression plates is controllable between 50 to 300 mm. The breadth of the compression plates is 400mm by which the maximum size of test model is limited.

The test models are box type cylinders with an edge length of 80, 100, 200 and 300mm and a plate thickness of 1.0, 1.5 and 3.2mm.

Table 1, 2 and 3 contain the dimensions of all the test models and the results obtained.

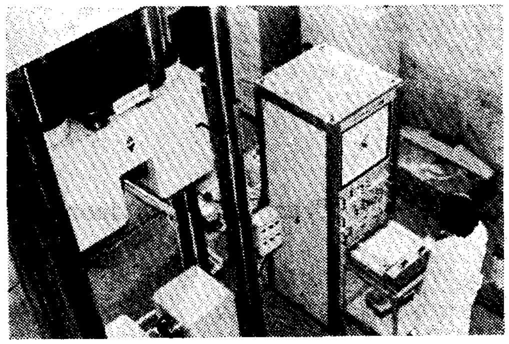

**Fig. 1.** Testing apparatus

For each test run three models were used. Two of the load-deformation curves for a model height of 150mm and a plate thickness of 1.5mm are presented in Fig. 2.

The various load-deformation curves are in very

**Table 1.** Results of tests with the model of edge length 80mm

| plate thickness | mm | 1.0 | | | 1.5 | | |
|---|---|---|---|---|---|---|---|
| edge length | mm | 80.0 | | | 80.0 | | |
| height | mm | 150 | 200 | 240 | 150 | 200 | 240 |
| end length: | | | | | | | |
| measured | mm | 14.05 | 18.70 | 28.0 | 21.2 | 31.50 | 37.25 |
| from recorder | mm | 14.00 | 18.01 | 27.7 | 21.2 | 31.50 | 37.00 |
| ultimate load | N | 45,100 | 41,550 | 41,200 | 85,100 | 87,100 | 90,300 |
| number of fold | | 3 | 4 | 4 | 3 | 4 | 5 |
| energy absorbed before end strengthening | kN cm | 173.0 | 201.2 | 253.4 | 247.5 | 366.9 | 467.1 |
| energy absorbed by end strengthening | kN cm | 129.2 | 163.0 | 214.1 | 147.5 | 171.6 | 196.8 |
| total | kN cm | 302.2 | 364.2 | 467.5 | 395.0 | 538.5 | 663.9 |

**Table 2.** Results of tests with the model of edge length 100mm

| plate thickness | mm | 1.0 | | | 1.5 | | |
|---|---|---|---|---|---|---|---|
| edge length | mm | 100 | | | 100 | | |
| height | mm | 150 | 200 | 250 | 150 | 200 | 250 |
| end length: | | | | | | | |
| measured | mm | 13.7 | 23.7 | 24.5 | 20.9 | 28.6 | 43.5 |
| from recorder | mm | 14.3 | 23.6 | 23.9 | 20.2 | 24.4 | 44.2 |
| ultimate load | N | 46,270 | 53,465 | 51,010 | 92,210 | 98,100 | 89,435 |
| number of fold | | 3 | 3 | 3 | 2 | 3 | 4 |
| energy absorbed before end strengthening | kN cm | 196.6 | 313.2 | 342.5 | 258.5 | 367.5 | 442.3 |
| energy absorbed by end strengthening | kN cm | 104.2 | 123.0 | 184.0 | 218.2 | 278.7 | 207.8 |
| total energy | kN cm | 300.8 | 436.2 | 526.5 | 476.7 | 646.2 | 650.1 |

**Table 3.** Results of tests with model of plate thickness 3.2mm

| plate thickness | mm | 3.2 | | | 3.2 | | |
|---|---|---|---|---|---|---|---|
| edge length | mm | 200 | | | 300 | | |
| height | mm | 400 | 500 | 600 | 600 | 750 | 900 |
| end length: | | | | | | | |
| measured | mm | 40.5 | 49.6 | 67.2 | 81.4 | 93 | 124 |
| from recorder | mm | 40.0 | 48.6 | 68.5 | 80 | 94.2 | 125 |
| ultimate load | N | 469,000 | 475,000 | 465,000 | 430,000 | 451,000 | 454,000 |
| number of fold | | 3 | 4 | 5 | 2 | 3 | 4 |
| energy absorbed before end strengthening | kN cm | 3,870 | 4,980 | 6,702 | 9,690 | 9,777 | 11,290 |
| energy absorbed by end strengthening | kN cm | 3,990 | 4,620 | 4,967 | 5,435 | 6,827 | 9,640 |
| total energy | kN cm | 7,860 | 9,600 | 11,669 | 15,125 | 16,604 | 20,930 |

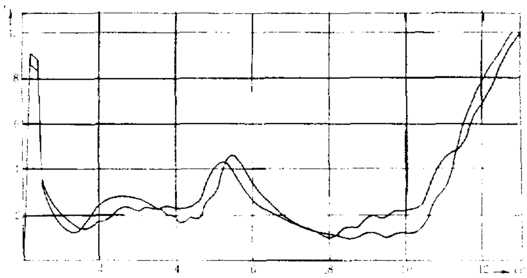

**Fig. 2.** Load-deformation curves for model, 150mm high, 1.5mm thick

good agreement. It is, however, difficult to determine the point at which "end strengthening" of the compressed model begins.

## 3. Ultimate Load and Energy Absorption

There are several ways of analysing the buckling behaviour of a steel structure when some plastic regions exist.

Studies on the ultimate strength of plates have been made by using the concept of the effective width.

The behaviour of a mild steel structure up to its ultimate load can be described by first assuming that it behaves entirely in an elastic manner and then by assuming that it behaves in a rigid-plastic manner.

In the proposed method by Fujita et al. [6] the elastic analysis is based on the Rayleigh-Ritz method. The plastic analysis describes the collapse mechanism. Both analyses treat large deformations.

The ultimate loads obtained from the experiments are evaluated in Fig. 3, in which the theoretical results for the ultimate load of a plate $a/b=1$ by many other authors are presented.

From the experimental results and empirical formula to determine the ultimate load of isotropic square

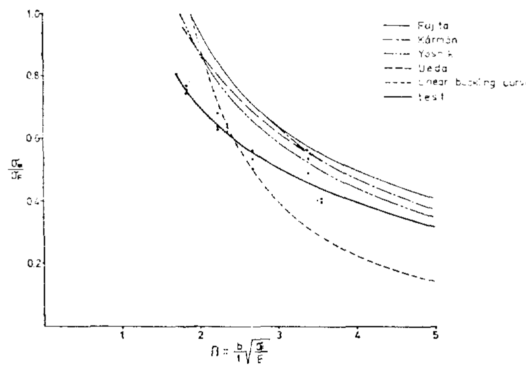

**Fig. 3.** Square plate ultimate strength under compression and theoretical evaluations [6]

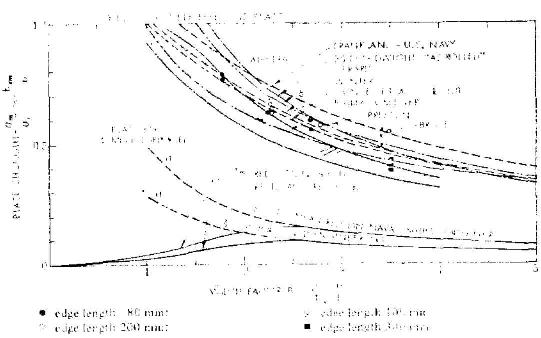

**Fig. 4.** Ultimate load of square plate according to Faulkner [7]

plates under compression can be evaluated:

$$\frac{\sigma_m}{\sigma_y}=\frac{1.8}{\beta}-\frac{0.8}{\beta^2}$$

where $\sigma_m$ : maximum stress

$\sigma_y$ : yield stress

$\beta : \frac{b}{t}\sqrt{\frac{\sigma_y}{E}}$

Another examination of the values for ultimate loads resulting from experiments is presented in the diagram of Faulkner [7] in Fig. 4. The measured ultimate loads lie well above the lower bound of the scattered range. Regarding the energy absorption characteristic of isotropic square plates the tendency is confirmed that the total absorbed energy of the test models increases linearly with increasing model height.

In table 4 is stated the percentage of the compressed length in end status. It offers that the percentage of the compressed length to the initial length is used as abscissa, over which the absorbed energy is plotted.

In this way the compression per percentage of decreased length can be expressed.

On account to the manufacturing imperfections of the models the curves for the absorbed energy in Fig. 5, 6, 7 and 8 show little irregularities.

The curves show a waveform, which correspond to the number of half-waves to the buckling form of the test models and are in parts linear.

A similar connection is found in the investigation of Akita and Vaughan [5], where a linear relation for the absorbed energy of the total bow structure is stated.

As the actual data of ship's collision of Minorsky [1] and the model tests of Akita and Kitamura [3] show, the absorbed energy is dependent on the maximum damaged length.

Further the absorbed energy is dependent on the ultimate load of the plating as the experimental results indicate.

The energy $\Delta E$ absorbed by the model is characterized by:

$$\Delta E=C\cdot T\cdot S$$

where, $T$ : the ultimate load in kN

$S$ : the true damaged length in m

$C$ : a constant, which varies according to the slenderness of the model

Vaughan [5] has evaluated a value of 0.95 for $C$ for the formula, which is derived from the experim-

**Table 4.** Grade of Compression

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| plate thickness | mm | 1.0 | | | 1.5 | | |
| edge length | mm | 80 | | | 80 | | |
| height | mm | 150 | 200 | 240 | 150 | 200 | 240 |
| compression | % | 90.7 | 90.8 | 88.4 | 85.9 | 84.3 | 84.5 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| plate thickness | mm | 1.0 | | | 1.5 | | |
| edge length | mm | 100 | | | 100 | | |
| height | mm | 150 | 200 | 250 | 150 | 200 | 250 |
| compression | % | 90.7 | 88.2 | 90.3 | 86.3 | 86.8 | 82.5 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| plate thickness | mm | 3.2 | | | 3.2 | | |
| edge length | mm | 200 | | | 300 | | |
| height | mm | 400 | 500 | 600 | 600 | 750 | 900 |
| compression | % | 89.9 | 90.2 | 88.5 | 86.6 | 87.5 | 86.2 |

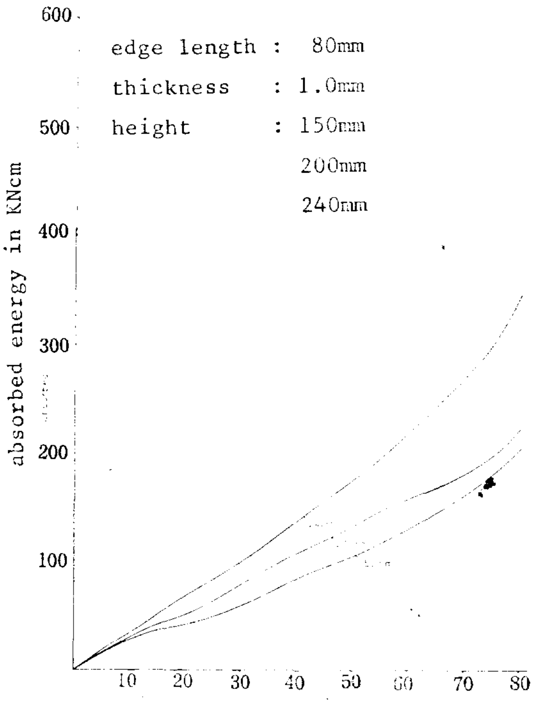


**Fig.** 5. Compression in % model height

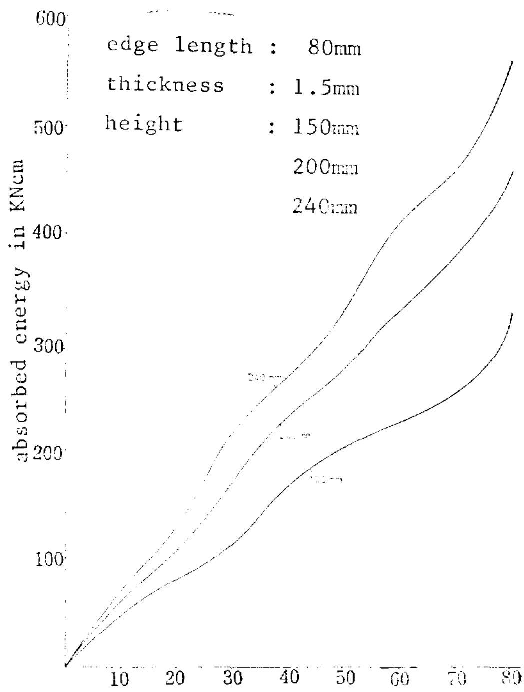


**Fig.** 6. Compression in % model height

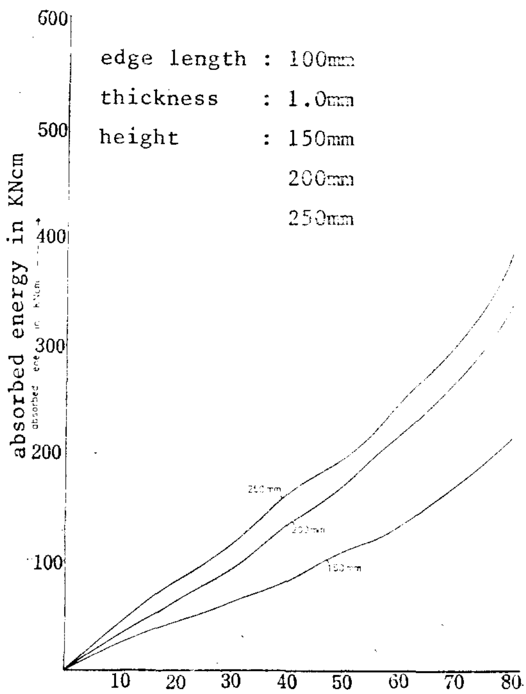


**Fig.** 7. Compression in % model height

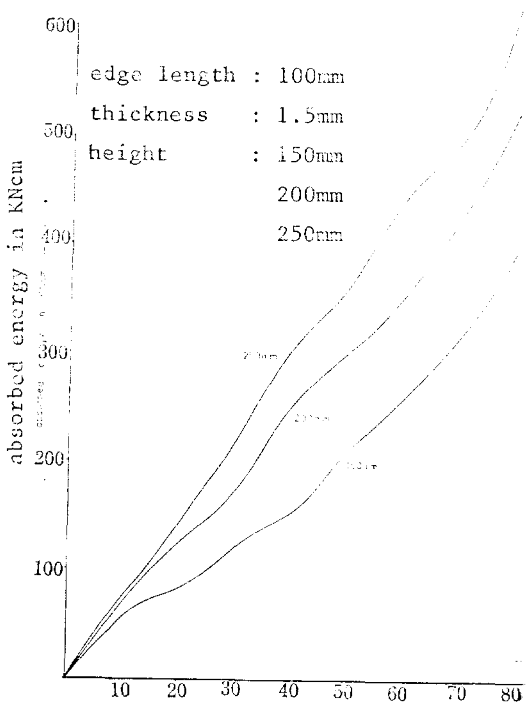


**Fig.** 8. Compression in % model height

**Fig.** 5~8. Curves of absorbed energy over the percentage of the compressed length ental results of Akita and Kitamura. In consequence of the complexity of the structure tested the measured values here scattered around the curve $\Delta E=0.95\cdot T\cdot S$.

On the basis of the test results of isotropic square plates we obtained a clear conclusion that the factor $C$ is to be determined as a function of the buckling load (ultimate load) of the plate.

Faulkner has represented the experimental results to evaluate the ultimate load of square plates over the slenderness-factor. (s. fig 4) An examination of the measured results has shown that the factor $C$ is not constant but obviously dependent on the slenderness-factor $\beta$.

$$\beta=\frac{b}{t}\sqrt{\frac{\sigma_y}{E}}$$

Where, $\beta$ : Slenderness-factor

$b$ : edge length in mm

$t$ : plate thickness in mm

$\sigma_y$ : yield stress in $N/mm^2$

$E$ : Young's modulus in $N/mm^2$

In Fig. 9 and 10 are represented the curves of

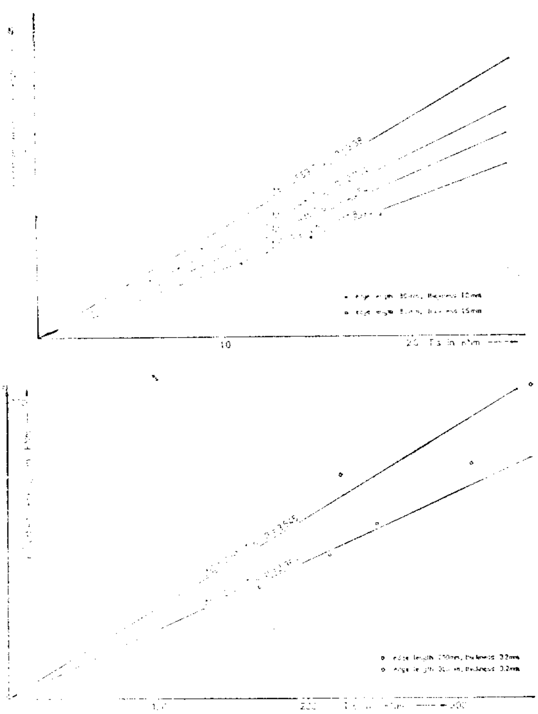

**Fig. 9~10.** Curves of absorbed energy over the factor $T\cdot S$

absorbed energy over the factor $T\cdot S$, where for $S$ the total damaged length is inserted.

The plots show a satisfactory agreement of the measured values with the calculated curves. These measured results have been evaluated from the exp eriments with the calculated curves. These measured results have been evaluated from the experiments with the models of edge length 80mm, 200 and 300 mm.

The total absorbed energy for the model with the edge length 100mm could not be taken into account since with this model only a damaged length was reached of 85%, whereas the empirical $C$-value is based on a damaged length of 90%.

## 4. Conclusion

A relation for the absorbed energy in isotropic square plates under compression is proposed, which is based on experimental results.

The investigation is directed towards the development of an optimized bow structure to absorb the energy occuring in ships collisions.

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