発電 및 限界費用의 解析的 推定法에 관한 研究

論 文
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An Analytic Algorithm to Estimate Expected Generation and Marginal Costs

朴 永 文* · 徐 輔 爀**
(Young-Moon Park, Bo-Hyeok Seo)

Abstract

This paper derives the algorithm to estimate the operating cost, its marginal cost, and the reliability indices for the long term planning of power system.

Treating the load duration curve and the system in the stochastic sense takes the place of the inverted load duration curve, effective load duration curve, and the numerical integration in the conventional methods.

The time and accuracy of computation are substantially improved due to the fact that all expressions are represented by simple analytic form instead of the existing recursive form.

1. Introduction

An exact and comprehensive method to estimate the expected cost of energy procuction is an important but difficult aspect of system planning.

In conventional methods, the computation of the operating cost and reliability indices starts with the load duration curve (LDC), which is inverted to produce the inverted load duration curve (ILDC) by means of some appropriate techniques. The concept of the effective load duration curve (ELDC) is a very useful tool to treat the probabilistic behavior of the system and every unit associated with the forced outage rate (FOR) of it.

Units are added on in their proper merit order to obtain the corresponding ELDC.

In an i unit system when k units are considered, the ELDC_k is obtained by the famous recursive convolution equation (1). Then the expected energy by the (k+1)th unit is calculated with the numerical integration of equation (2) [6][7][9][10][12][16]

$$\mathrm{ELDC}_k(L)=p_k\cdot \mathrm{ELDC}_k(L)+q_k\cdot \mathrm{ELDC}_k(L-C_k) \quad (1)$$

$$E_{k+1}=p_{k+1}\cdot\int_{\alpha}^{\beta}\mathrm{ELDC}_k(L)\,dL\cdot T_L \quad (2)$$

*正 会 員：서울大 工大 電気工學科 教授・工博
**正 会 員：서울大 大學院 電気工學科 博士課程
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where $\alpha=\sum_{j=1}^{k}C_j$, $\beta=\alpha+C_{k+1}$

and p_j is the availability of capacity C_j, T_L is the period hours of the LDC.

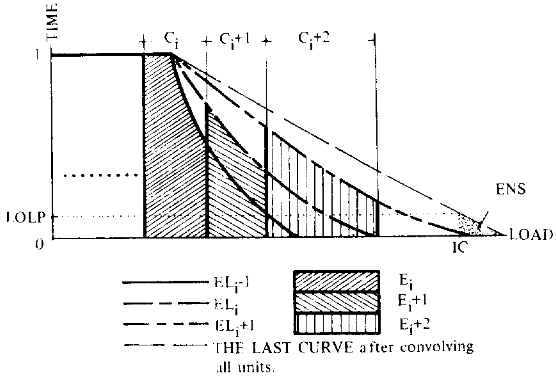

Fig. 1. The conventional ELDC and energy production

For the reliability indices of the system, the loss of load probability (LOLP) and the energy demanded but not served (ENS) are computed by

$$\mathrm{LOLP}=\mathrm{ELDC}_i(IC) \quad (3)$$

$$\mathrm{ENS}=\int_{IC}^{\infty}\mathrm{ELDC}_i(L)\,dL\cdot T_L \quad (4)$$

where IC is the installed capacity of the system.

These expressions of E_k, LOLP, and ENS are exact ones, but in actual applications the recursive computation of ELDC's and the numerical integration to obtain the energy are not only tedious but also errorneous because the procedures accumulate the errors.[7][17] In reality, for the system consisted of a large number of units the computation becomes almost useless.

The proposed new method uses the Gaussian density functions to represent the characteristics of the load and the system instead of generating the effective load duration curves, and resulted in very simple analytic expressions.

The evaluation of the incremental cost is also investigated.

2. The Proposed New Algorithm

This paper does not use the inverted load duration curve nor ELDC because of their disadvantages of inaccuracy and much requirements of computer resources. Instead, the joint Gaussian density function of the load and system characteristics are used on the whole simulation.

2-1. Probabilistic Characteristics of Generation System

Considering an i unit generation system, every unit is considered to be operated in independent fashion with respect to the others.

Moreover it is possible to regard the system to have the Gaussian characteristics which is represented by the Gaussian density function instead of the real density of binomial density when i is an enough large number.[1][2][3][5]

Then the expected generation and the variance of the system are represented by

$$ i_{\bar{g}} = E[i_g] = \sum_{j=1}^{i} \bar{C}_j = \sum_{j=1}^{i} P_j \cdot C_j \qquad (5) $$

where $i_{\bar{g}}$; the expected generation capacity of the system.

i_g ; the installed capacity of the system.

$\bar{C}_j$; the expected generation of unit j

P_j ; the availability of unit j.

and

$$ {}^{i}S_g^2 = \sum_{j=1}^{i} S_g^2{}_j = \sum_{j=1}^{i} \{ E[C_j^2] - E^2[C_j] \} $$

$$ = \sum_{j=1}^{i} P_j q_j C_j^2 \qquad (6) $$

where

${}_iS_g^2$; the variance of the generation of the system.

$S_g^2{}_j$; the variance of capacity C_j.

q_j ; the FOR of unit j.

2-2. Probabilistic Characteristics of Load Duration Curve

When a load curve is represented by a series of discrete load data (which may not ordered in their magnitude), an expected load value $\bar{L}_t$ and its variance S_{Lt}^2 can be determined in every time interval t. And if the Gaussian density is assumed, the load curve is characterized by $N(\bar{L}_t, S_{Lt}^2)$ in any interval t, and by $N(\bar{L}, S_L^2)$ when the full load duration period is chosen as the interval.

For the simplest case, let $L_1, L_2, L_3, \cdots L_m$ be the given data. Then obviously $\bar{L}_t = E[L_t] = L_t$ and $S_{Lt}^2 = 0$ which is a special case of the Gaussian density function that has zero variance.[4] And if the same length of all time intervals are assumed, the total (popular) mean and variance are[Appendix. 1].

$$ \bar{L} = \frac{1}{m} \sum_{t=1}^{m} L_t \ , \ S_L^2 = \frac{1}{m} \sum_{t=1}^{m} [L_t^2] - \bar{L}^2 \qquad (7) $$

In general, when m_t data are in an interval t, and L_{tj}, $j = 1, 2, \ldots, m_t$ are those load data in the corresponding interval whose length is I_t, $t = 1, 2, \cdots$, m, the expected load value at t $\bar{L}_t$, its variance S_{Lt}^2, the popular mean $\bar{L}$, and its variance S_L^2 are represented by [Appendix. 1]

$$ \left.\begin{aligned} &\bar{L}_t = \frac{1}{m_t} \sum_{j=1}^{m_t} L_{tj}, \quad S_{Lt}^2 = \frac{1}{m_t} \sum_{j=1}^{m_t} L_{tj}^2 - \bar{L}_t^2 \\ &\bar{L} = \sum_{t=1}^{m} m_t \cdot I_t \cdot \bar{L}_t \Big/ \sum_{t=1}^{m} m_t \cdot I_t \\ &S_L^2 = \Big\{ \sum_{t=1}^{m} m_t \cdot I_t \cdot S_{Lt}^2 + \sum_{t=1}^{m} m_t \cdot I_t \cdot \bar{L}_t^2 \Big\} \Big/ \sum_{t=1}^{m} m_t \cdot I_t - \bar{L}^2 \\ \text{or} \quad &S_L^2 = \sum_{t=1}^{m} m_t \cdot I_t \cdot S_{Lt}^2 \Big/ \sum_{t=1}^{m} m_t \cdot I_t + S_{\bar{L}}^2 \end{aligned}\right\} \qquad (8) $$

where $S_{\bar{L}}^2$ is the variance of $\bar{L}_t$'s.

These are the characterizations of the discrete load curve. But in the conventional methods as in [6][12], in LDC approaches when the coefficients of the LDC are given or determined, generation of the discrete LDC with the mean values and those variances from the curve is very simple.

Let the LDC be represented by [7]

$$L(X)=\sum_{j=1}^{6} a_j X^{j-1} \quad (9)$$

Then the mean load $\bar{L}_t$, $\bar{L}$ and the variances S_{Lt}^2, S_L^2 are given by

$$\bar{L}_t=\frac{1}{\Delta t}\int_\alpha^\beta L(X)\,dx = \frac{1}{\Delta t}\Big[\sum_{j=1}^{6} a_j X^j/j\Big]\Big|_\alpha^\beta \quad (10)$$

$$S_{Lt}^2=\frac{1}{\Delta t}\int_\alpha^\beta L^2(X)dx-\Big[\frac{1}{\Delta t}\int_\alpha^\beta L(X)\,dx\Big]^2$$

$$=\frac{1}{\Delta t}\Big(\sum_{j=1}^{11} a_j' X^j/j\Big)\Big|_\alpha^\beta-\bar{L}_t^2 \quad (11)$$

and

$$\bar{L}=\int_0^1 L(X)\,dx=\Big(\sum_{j=1}^{6} a_j X^j/j\Big)_0^1 \quad (12)$$

$$S_L^2=\int_0^1 L^2(X)\,dx-\Big[\int_0^1 L(X)\,dx\Big]^2$$

$$=\Big(\sum_{j=1}^{11} a_j' X^j/j\Big)\Big|_0^1-\bar{L}^2 \quad (13)$$

where $\alpha=(t-1)\Delta t$, $\beta=t\cdot\Delta t$
$\Delta t=1/m$.

a_j', $j=1,2,\cdots$, are the coefficients of

$$L^2(X)=\sum_{j=1}^{11} a_j' X^{j-1}$$

The combination of these two cases proposes a possible way to consider multiple load curves, that is, n curves at once by the mean values and the variances.

For example, let $L^s(X)$ for spring, $L^m(X)$ for summer, $L^f(X)$ for fall, and $L^w(X)$ for winter be considered, for the combined load curve $L(X)$, the mean load $\bar{L}_t$, the variance S_{Lt}^2 or $\bar{L}$ and S_L^2 representing the 4-seasons can be determined in the probabilistic sense using the equation(8)[App. 1].

2-3. Expected Cost of Energy Production and Not Served Energy

To calculate the expected value of served and not served energy by a system, the concept of the Gaussian density is very useful.

This paper assumes the generation system and the load curve have the Gaussian densities and it is the only assumption in the paper.

Let $f_{ig}(z)$ and $f_{Lt}(Z)$ be the density function of the i unit system and load at time t respectively, and let a random variable z be defined as the possible generation of the system having $f_Z(Z)$ as its density function.

Then

$$Z \triangleq \min\left[{}^i g,\ L_t\right] \quad (14)$$

with

$$f_{ig}(Z)=\exp\Big\{-\frac{(Z-{}^i\bar{g})^2}{2\,{}^iS_g^2}\Big\}\Big/\big(\sqrt{2\pi}\cdot{}^iS_g\big) \quad (15)$$

$$f_{Lt}(Z)=\exp\Big\{-\frac{(Z-\bar{L}_t)^2}{2\,S_{Lt}^2}\Big\}\Big/\big(\sqrt{2\pi}\cdot S_{Lt}\big) \quad (16)$$

For the density of z, Papoulis [1] gives

$$f_z(Z)=\big[1-F_{Lt}(Z)\big]f_{ig}(Z)+\big[1-F_{ig}(Z)\big]f_{Lt}(Z) \quad (17)$$

where

$$F_{ig}(Z)=\int_{-\infty}^{z} f_{ig}(X)\,dx=\frac{1}{2}+\mathrm{erf}\Big(\frac{Z-{}^i\bar{g}}{{}^iS_g}\Big)$$

$$F_{Lt}(Z)=\int_{-\infty}^{z} f_{Lt}(X)\,dx=\frac{1}{2}+\mathrm{erf}\Big(\frac{Z-\bar{L}_t}{S_{Lt}}\Big)$$

and

$$\mathrm{erf}(X)\triangleq\frac{1}{\sqrt{2\pi}}\int_0^x \exp\{-u^2/2\}\,du$$

The expected energy production by the i unit system at time t, ${}^i\bar{E}_t$ is given by

$${}^i\bar{E}_t=\Delta t\cdot E\big[Z\big]=\Delta t\cdot\int_{-\infty}^{\infty} Z\cdot f_z(Z)\,dz \quad (18)$$

which results in [Appendix 2]

$${}^i\bar{E}_t=\Delta t\cdot\Big[\frac{1}{2}({}^i\bar{g}+\bar{L}_t)-(\bar{L}_t-{}^i\bar{g})\cdot \mathrm{erf}\Big(\frac{\bar{L}_t-{}^i\bar{g}}{\sqrt{{}^iS_g^2+S_{Lt}^2}}\Big)-\frac{\sqrt{{}^iS_g^2+S_{Lt}^2}}{\sqrt{2\pi}}\exp\Big\{-\frac{(\bar{L}_t^2-{}^i\bar{g})^2}{2({}^iS_g^2+S_{Lt}^2)}\Big\}\Big] \quad (19)$$

Therefore, the expected energy generated by the system is

$$ {}^{i}\bar{E} = \sum_{t=1}^{m} {}^{i}\bar{E}_t \cdot T_L \qquad (20) $$

Similarly, by defining the random variable z as the shortage of the generation, the expected energy demanded but not served (ENS) is formulated. Starting with

$$ Z_t \triangleq L_t - {}^{i}g \qquad (21) $$

or

$$ \bar{Z}_t = \bar{L}_t - {}^{i}\bar{g} \ , \ S_{Zt}^2 = S_{Lt}^2 + {}^{i}S_g^2 \qquad (22) $$

the density function of z becomes

$$ f_Z(Z) = \frac{1}{\sqrt{2\pi}\, S_Z} \cdot \exp\left\{ -\frac{(Z-\bar{Z}_t)^2}{2 S_{Zt}^2} \right\} \qquad (23) $$

then the expected ENS ($\bar{N}$) is given by [App. 2]

$$ \bar{N} = E\,[\mathrm{ENS}] = \sum_{t=1}^{m} [\bar{N}_t] $$

$$ = \sum_{t=1}^{m} \Big[\Big\{ \bar{Z}_t \cdot \Big[\frac{1}{2} + \mathrm{erf}\Big(\frac{\bar{Z}_t}{S_{Zt}}\Big)\Big] + \frac{S_{Zt}}{\sqrt{2\pi}} \cdot \exp\Big[-\frac{\bar{Z}_t^2}{2 S_{Zt}^2}\Big] \Big\} \cdot \Delta t \Big] \cdot T_L \qquad (24) $$

Some short calculations show that the sum of ${}^{i}\bar{E}$ and $\bar{N}$ meets the expected energy demand $\bar{L}$ That is,

$$ {}^{i}\bar{E} + \bar{N} = \bar{L}, \text{ or } {}^{i}\bar{E} = \bar{L} - \bar{N} \qquad (25) $$

where $L = T_L \cdot \int_0^1 L(X)\, dx$

which is a very natural and basic requirement.

Although there are many other approaches leading to the results of equation (19), (20), and (24), some of them are exposed in Appendix 2.

Anyway, since the expected energy production of i unit system is known, the expectation of energy for i-1 unit system in which unit i is unload, is represented only by ${}^{i-1}E$. Then the difference of them is the expected energy produced by unit i.

$$ \bar{E}^i = {}^{i}\bar{E} - {}^{i-1}\bar{E} \qquad (26) $$

Repeating these procedures for $i-1, i-2, \cdots \cdots 1$ determines the expected energy by every unit except unit 1.

$$ \bar{E}^k = {}^{k}\bar{E} - {}^{k-1}\bar{E}, \ k = 2, 3, \cdots, i-1, i. \qquad (27) $$

And for unit 1, since the expected energy generated by the 1 unit system is due to the unit itself,

$$ \bar{E}^1 = {}^{1}\bar{E} \qquad (28) $$

Then the operating cost of the system which is the production cost, is the simple sum of each energy multiplied by the corresponding production cost per unit energy.

In the conventional methods as in WASP [7] or others, as far as the effective load curve or convolutions of load curve are concerned, the more of units not only introduces the less accuracy but also requires the more time to compute because the ELDC's must always be reconstructed whenever a unit is added.

The main advantages of the new method are due to the fact that the more units results in the better accuracy with almost non-additional computation requirements, since as more units are included the density fashion of the system becomes more closed to the real Gaussian one and since the calculations are performed with "ANALYTIC" functions.

However using $\bar{L}$ and S_L^2 which are $\bar{L}_1$, S_{L1}^2 for $m=1$, will positively reduce the computation time but of course, will introduce some increased errors caused by the differences between the assumed Gaussian density and the real density fashion of the load curve in the duration period.

2-4. Calculation of LOLP

In the conventional approaches, the LOLP's are determined from the last ELDC after all units are convolved. That's why for a large number of units the LOLP, which should be calculated at the near-end of the curve becomes very inaccurate [7]. But the Gaussian density makes the computation a very simple thing and moreover analytic.

Defining a random variable Z as

$$ Z_t = L_t - {}^{i}g, \text{ for i unit system} \qquad (29) $$

the positive value of z is the shortage of generation. And the distribution of z for $z \geq 0$ is the very LOLP of the system. That is,

$$ \mathrm{LOLP}_t = \int_0^{\infty} f_{Zt}(Z_t)\, dz_t \cdot \Delta t $$

$$= \left[\frac{1}{2} + \mathrm{erf}\left(\frac{\bar{Z}_t}{S_{Zt}}\right)\right] \cdot \Delta t \qquad (30)$$

and

$$\mathrm{LOLP} = \sum_{t=1}^{m} \mathrm{LOLP}_t \cdot T_L \qquad (31)$$

where

$$\bar{Z}_t = \bar{L}_t - {}^{i}\bar{g},\ S_{Zt}^2 = S_{Lt}^2 + {}^{i}S_g^2$$

2-5. Formulations of the Incremental Cost

The incremental energy of a k unit system caused by the unit increment of the capacity C_j is the defferentiation of the energy ${}^{k}E$ with respect to the capacity or $\partial^k E / \partial C_j$. Differentiating the equation(19) leads to

$$\frac{\partial^k \bar{E}_t}{\partial C_j} = \frac{\partial}{\partial C_j}\left[\frac{1}{2}(\bar{L}_t + {}^{k}\bar{g}) - (\bar{L}_t - {}^{k}\bar{g}) \cdot \mathrm{erf}\left(\frac{\bar{L}_t - {}^{k}\bar{g}}{\sqrt{S_{Lt}^2 + {}^{k}S_g^2}}\right) - \frac{\sqrt{S_{Lt}^2 + {}^{k}S_g^2}}{\sqrt{2\pi}} \cdot \exp\left\{-\frac{(\bar{L}_t - {}^{k}\bar{g})^2}{2(S_{Lt}^2 + {}^{k}S_g^2)}\right\}\right] \cdot \Delta t \qquad (32)$$

and since

$$\partial^k \bar{g} / \partial C_j = \{ 0, \text{for } k < j;\ P_j, \text{ for } k \geq j \} \qquad (33)$$

$$\partial^k S_g^2 / \partial C_j = \{ 0, \text{ for } k < j;\ \frac{\partial}{\partial C_j} \sum_{i=1}^{k} P_i\, q_i C_i^2 = 2 P_j q_j C_j, \text{ for } k \geq j \} \qquad (34)$$

$$\frac{\partial}{\partial C_j}\left[\mathrm{erf}\left(\frac{\bar{L}_t - {}^{k}\bar{g}}{\sqrt{S_{Lt}^2 + {}^{k}S_g^2}}\right)\right] = \{0, \text{ for } k < j;\ -\frac{1}{\sqrt{2\pi}} \cdot \exp\left\{-\frac{(\bar{L}_t - {}^{k}\bar{g})^2}{2(S_{Lt}^2 + {}^{k}S_g^2)}\right\} \cdot \frac{\{P_j \cdot ({}^{k}S_g^2 + S_{Lt}^2) + (\bar{L}_t - {}^{k}\bar{g}) \cdot P_j q_j C_j\}}{({}^{k}S_g^2 + S_{Lt}^2)^{3/2}}, \text{ for } k \geq j\} \qquad (35)$$

$$\frac{\partial}{\partial C_j}\left[\frac{\sqrt{S_{Lt}^2 + {}^{k}S_g^2}}{\sqrt{2\pi}}\right] = \{ 0, \text{ for } k < j;\ \frac{P_j q_j C_j}{\sqrt{2\pi(S_{Lt}^2 + {}^{k}S_g^2)}}, \text{ for } k \geq j\} \qquad (36)$$

$$\frac{\partial}{\partial C_j}\left[\exp\left\{-\frac{(L_t - {}^{k}g)^2}{2(S_{Lt}^2 + {}^{k}S_g^2)}\right\}\right] = \{ 0, \text{ for } k < j;\ \exp\left\{-\frac{(L_t - {}^{k}g)^2}{2(S_{Lt}^2 + {}^{k}S_g^2)}\right\} \cdot \left\{\frac{(\bar{L}_t - {}^{k}\bar{g}) \cdot P_j \cdot ({}^{k}S_g^2 + S_{Lt}^2) + (\bar{L}_t - {}^{k}\bar{g})^2 P_j q_j C_j}{({}^{k}S_g^2 + S_{Lt}^2)}\right\}, \text{ for } k \geq j\} \qquad (37)$$

the equation (32) results in

$$\partial^k \bar{E}_t / \partial C_j = 0, \text{ for } k < j \qquad (38)$$

or

$$\partial^k \bar{E}_t / \partial C_j = \left[P_j/2 + P_j \cdot \mathrm{erf}\left(\frac{\bar{L}_t - {}^{k}\bar{g}}{\sqrt{{}^{k}S_g^2 + S_{Lt}^2}}\right) - \frac{P_j q_j C_j}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{S_{Lt}^2 + {}^{k}S_g^2}} \cdot \exp\left\{-\frac{(\bar{L}_t - {}^{k}\bar{g})^2}{2(S_{Lt}^2 + {}^{k}S_g^2)}\right\}\right] \cdot \Delta t, \text{ for } k \geq j \qquad (39)$$

and

$$\partial^k \bar{E} / \partial C_j = \sum_{t=1}^{m} (\partial^k \bar{E}_t / \partial C_j) \cdot T_L \qquad (40)$$

The knowledge of the incremental energy of a k unit system informs of the energy increment of unit k as in the case of the expected energy.

Let the energy increment of unit k due to the capacity C_j be represented by $\partial E^k / \partial C_j$, then

$$\partial \bar{E}^k / \partial C_j = \partial({}^{k}\bar{E} - {}^{k-1}\bar{E}) / \partial C_j \qquad (41)$$

and the incremental cost is defined by

$$J_{inc} = \sum_{k=1}^{i} \mathrm{HR}_k \cdot \frac{\partial}{\partial C_j}({}^{k}\bar{E} - {}^{k-1}\bar{E}), \text{ with } \frac{\partial}{\partial C_j}({}^{o}\bar{E}) = 0 \qquad (42)$$

where HR_k represents the heat rate of unit k.

3. Results of Computation

Case studies were carried out using the real load curve from KEPCO.

The generation system consists of 50 units with an installed capacity of 1.3 Gw. Each unit is assumed to have the same capacity of 26 Mw with 20% of FOR.

For the purpose of comparison, the number of units of 1, 5, 10, 20 and 50 were also tested with the fixed IC of 1.3 Gw. Table 1 shows the results of computation of the expected energy production and ENS of each system. The results were compared with the output of the WASP package with the same load duration curve and systems.

Table 1. Comparison with the WASP results

SAMPLE SYSTEM			SIMULATION RESULTS					
SYSTEM	GROUP #	No. of Units (Unit CAP.) (GW)	WASP		Proposed New Method			
			Expected Energy Generated (GWH)	ENS (GWH)	Expected Energy Generated (when m=1) (GWH)	ENS (when m=1) (GWH)	Expected Energy Generated (when m=100) (GWH)	ENS (when m=100) (GWH)
A	1	1(1.3)	5510.65	966.9	5927.89	960.5	5928.84	959.5
B	1	5(.26)	6640.82	244.8	6686.41	201.9	6692.39	196.0
C	1	10(.13)	6790.69	97.5	6797.84	90.5	6807.49	80.4
D	1	10(.065)	4530.15		4538.64		4528.45	
	2	10(.065)	2323.48	34.7	2308.92	40.8	2331.33	28.6
E	1	10(.026)	1822.08		1822.08		1822.08	
	2	10(.026)	1821.17		1821.45		1821.03	
	3	10(.026)	1741.03		1761.06		1740.17	
	4	10(.026)	1209.31		1180.53		1209.92	
	5	10.(.026)	288.29	6.5	286.01	17.2	289.81	5.4

*NOTE: The FOR of all units are assumed to be 20 % for simplicity.

The incremental energy was calculated for the system 5, in which all units are assumed to have the same heat rates.

And in WASP, the incremental energy $\partial\bar{E}/\partial C_{j}$, $j=1, 2, 3, \cdots\cdots 5$ were calculated by increasing the capacity of unit 10, 20, 30, 40 and 50 respectively by 1 Mw.

The energy increment of every group for every j is also shown in Table 2.

Another case study was carried out using the sample system and discrete load data from the EPRI final report [17]. The results are compared with those of the EPRI report in Table 3.

The EPRI REPORT calculates the energy under the load duration curve as 14,293.44 GWH. However, the load data listed in the REPORT produce 15,041.35 GWH, and there exists serious energy unbalance of 747.91 GWH.

Since conventional methods use the load duration curve approximated as a 5-th order polynomial [6][7][9][10][12][16] the energy unbalance as shown above is unavoidable.

In the new algorithm, the addition of the produced energy and the not served energy is 15,041.33 GWH, and is unbalanced only by 0.02 GWH.

The fact proves the accuracy of the new algorithm.

The simulation procedures of the new one is shown in Fig. 2.

4. Conclusion

A new method to evaluate the expected energy production and the reliability indices of a generation system has been proposed. The new algorithm is based on the concept of the Gaussian density.

This method is several times faster than the conventional techniques and can be easily extended to the multi-block system.

The most prominent thing is that all expressions included in the paper are analytic, which has seemed impossible.

Table 2. Computation results of incremental cost

$\frac{\partial \bar{E}}{\partial C_j}$	$\frac{\partial \bar{E}^k}{\partial E_j}$	Energy Increment in WASP (GWH)	Incremental Energy in New Method (GWH)	Computation Conditions in WASP	
$\partial \bar{E}^k / \partial C_1$	$\partial \bar{E}^1 / \partial C_1$	7.01	7.01	$j = 1$, $C_{10} = 27\ MW$	$\partial \bar{E}^k / \partial C_j$ is the sum of the increment energies of each unit in the group and ($\partial \bar{E} / \partial C_j$) is the energy incremented of the system.
	$\partial \bar{E}^2 / \partial C_1$	-0.05	-0.04		
	$\partial \bar{E}^3 / \partial C_1$	-0.80	- .81		
	$\partial \bar{E}^4 / \partial C_1$	-3.40	-3.40		
	$\partial \bar{E}^5 / \partial C_1$	-2.63	-2.65		
$\partial \bar{E} / \partial C_1$		0.12	0.11		
$\partial \bar{E}^k / \partial C_2$	$\partial \bar{E}^1 / \partial C_2$	0.	0.	$j = 2$, $C_{20} = 27\ MW$	
	$\partial \bar{E}^2 / \partial C_2$	6.97	6.97		
	$\partial \bar{E}^3 / \partial C_2$	-0.40	-0.81		
	$\partial \bar{E}^4 / \partial C_2$	-3.40	-3.40		
	$\partial \bar{E}^5 / \partial C_2$	-2.63	-2.65		
$\partial \bar{E} / \partial C_2$		0.12	0.11		
$\partial \bar{E}^k / \partial C_3$	$\partial \bar{E}^1 / \partial C_3$	0.	0.	$j = 3$, $C_{30} = 27\ MW$	
	$\partial \bar{E}^2 / \partial C_3$	0.	0.		
	$\partial \bar{E}^3 / \partial C_3$	6.11	6.16		
	$\partial \bar{E}^4 / \partial C_3$	-3.40	-3.40		
	$\partial \bar{E}^5 / \partial C_3$	-2.63	-2.65		
$\partial \bar{E} / \partial C_3$		0.12	0.11		
$\partial \bar{E}^k / \partial C_4$	$\partial \bar{E}^1 / \partial C_4$	0.	0.	$j = 4$, $C_{40} = 27\ MW$	
	$\partial \bar{E}^2 / \partial C_4$	0.	0.		
	$\partial \bar{E}^3 / \partial C_4$	0.	0.		
	$\partial \bar{E}^4 / \partial C_4$	2.74	2.76		
	$\partial \bar{E}^5 / \partial C_4$	-2.63	-2.65		
$\partial \bar{E} / \partial C_4$		0.13	0.11		
$\partial \bar{E}^k / \partial C_5$	$\partial \bar{E}^1 / \partial C_5$	0.	0.	$j = 5$, $C_{50} = 27\ MW$	
	$\partial \bar{E}^2 / \partial C_5$	0.	0.		
	$\partial \bar{E}^3 / \partial C_5$	0.	0.		
	$\partial \bar{E}^4 / \partial C_5$	0.	0.		
	$\partial \bar{E}^5 / \partial C_5$	0.12	0.11		
$\partial \bar{E} / \partial C_5$		0.12	0.11		

In the conventional algorithm, the numerical integration should be repeated till all of the generation units are included. Moreover, the LDC has to be computed recursively at every step and the coefficients of the curve [7] or the values of the curve at every point [16] must be reculated.

But the new algorithm calculates the expected energy production by a unit directly from the resulted simple equation.

For the incremental cost or energy, the whole procedures must be repeated for the same system increased by unit capacity as many times as the number of units, because there has been no way to find out the derivatives of the numerical, recursive energy or cost function. But the new algorithm, owing to the analytic form, makes it a simple calculation of only one equation.

For more fast computation with less accuracy, the use of approximations of the error function [13] [14] [15] are recommended.

Table 3. Simulation results for the sample system (System I-1) of the EPRI report

THERMAL GENERATION DATA FOR THE SYSTEM								ENERGY PRODUCED	
GROUP #	STATION #	NAME	# of Units in Station	CAP. of Ea. Unit (MW)	FUEL COST ($/ MBtu)	Heat Rate (Btu/ KWH)	FOR (%)	EPRI Report (GWH)	The New Algorithm (GWH)
1	1	NU12	6	1200.	.76	10400	15	4950.4	4950.4
	2	NUC8	1	800.	.76	10400	15		
2	3	COL8	1	320.	2.0	9625	24	6185.8	6859.9
	4	COL6	3	150.	2.0	10814	21		
	5	COL4	5	100.	2.0	10674	13		
	6	COL2	33	50.	2.0	11581	8		
3	7	OIL8	1	320.	3.75	10010	24	2845.23	2916.08
	8	OIL6	3	150.	3.75	11300	21		
	9	OIL4	2	100.	3.75	11148	13		
	10	OIL2	23	50.	3.75	12068	7.4		
4	11	CT50	96	50.	4.39	14000	24	298.19	301.98
Simulation Execution Times in seconds.								0.32	0.11
TOTALS(GWH)								14279.63	15028.45
LOLP (P.U.)								0.0241	0.02073
ENS (GWH)								14.11	12.8805

*Note: Thermal Station Loading Order Corresponds to Order of Table Entries.
*NU: Nuclear Station　　COL: Coal Station
OIL: Oil Station　　CT: Combustion Turbines
*The Execution Times of the new algorithm includes the calculation of the marginal energy.

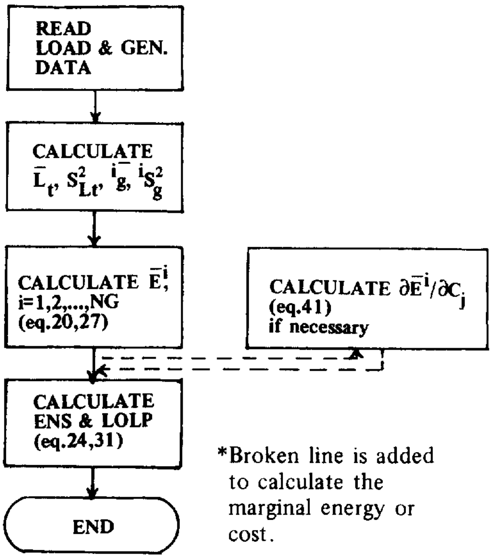

Fig. 2 Simulation flowchart of the proposed algorithm

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Appendixes

Appendix 1: Popular mean, variance from sample mean and variance.

Let the length of i-th interval be l_i and the number of data in the interval be m_i, the number of intervals be N, then the mean and the variance in the interval are given by

$$\bar{L}_i = \frac{1}{m_i} \cdot \sum_{j=1}^{m_i} L_{ij} \qquad \text{(A.1-1)}$$

$$S_i^2 = \frac{1}{m_i} \cdot \sum_{j=1}^{m_i} L_{ij}^2 - \bar{L}_i^2 \qquad \text{(A. 1-2)}$$

where $L_{ij}, j=1,2,\dots,m_1, i=1,2,\dots, N$ are the given load data.

Then the popular mean (expectation) becomes

$$\bar{L} = \sum_{i=1}^{N} \sum_{j=1}^{m_i} L_{ij} \cdot l_i / \sum_{i=1}^{N} l_i \cdot m_i$$
$$= \sum_{i=1}^{N} l_i m_i \bar{L}_i / \sum_{i=1}^{N} l_i m_i \qquad \text{(A. 1-3)}$$

and if $l_i=l$, $m_i=m$, for all i

$$\bar{L} = \frac{1}{N} \cdot \sum_{i=1}^{N} \bar{L}_i \qquad \text{(A. 1-4)}$$

And the popular variance is given by

$$S^2 = \frac{1}{\sum_{i=1}^{N} l_i m_i} \sum_{i=1}^{N} \sum_{j=1}^{m_i} l_i L_{ij}^2 - \bar{L}^2$$
$$= \frac{1}{\sum_{i=1}^{N} l_i m_i} \left[\sum_{i=1}^{N} l_i m_i S_i^2 + \sum_{i=1}^{N} l_i m_i \bar{L}_i^2 \right] - \bar{L}^2 \qquad \text{(A. 1-5)}$$

which, when $l_i=l$, $m_i=m$, for all i, becomes

$$S^2 = \frac{1}{N} \left[\sum_{i=1}^{N} (S_i^2 + \bar{L}_i^2) \right] - \bar{L}^2 \qquad \text{(A. 1-6)}$$

Appendix 2: Proofs of Equation (19) and (24)

Proof 1;

Defining a random variable z by the possible supply as

$$Z \triangleq \min [g, L] \qquad \text{(A. 2-1)}$$

with

$$f_g(Z) = \frac{1}{\sqrt{2\pi}\, S_g} \cdot \exp\left\{ -\frac{(z-g)^2}{2 S_g^2} \right\}$$
$$f_L(Z) = \frac{1}{\sqrt{2\pi}\, S_L} \cdot \exp\left\{ -\frac{(z-L)^2}{2 S_L^2} \right\} \qquad \text{(A. 2-2)}$$

for the density function of z, Papoulis [1] gives

$$f_Z(Z) = [1-F_L(Z)]\, f_g(Z) + [1-F_g(Z)]\, f_L(Z) \qquad (A.2-3)$$

where

$$F_g(Z) = 1/2 + \mathrm{erf}\left[(z-\bar{g})/S_g\right]$$
$$F_L(Z) = 1/2 + \mathrm{erf}\left[(z-\bar{L})/S_L\right]$$

then

$$\bar{E} = \int_{-\infty}^{\infty} z \cdot f_Z(Z)\cdot dz \qquad (A.2-4)$$

$$= \int_{-\infty}^{\infty} z\cdot[1-F_L(Z)]\cdot f_g(Z)\cdot dz + \int_{-\infty}^{\infty} z\cdot[1-F_g(Z)]\cdot f_L(Z)\cdot dz$$

$$= \frac{\bar{g}+\bar{L}}{2} - \int_{-\infty}^{\infty} z\cdot \mathrm{erf}\left(\frac{z-\bar{L}}{S_L}\right)\cdot \frac{1}{\sqrt{2\pi}\,S_g}\exp\left\{-\frac{(z-\bar{g})^2}{2S_g^2}\right\} dz - \int_{-\infty}^{\infty} z\cdot\mathrm{erf}\left(\frac{z-\bar{g}}{S_g}\right)\cdot\exp\left\{-\frac{(z-\bar{L})^2}{2S_L^2}\right\} dz \qquad (A.2-5)$$

$$= \frac{\bar{g}+\bar{L}}{2} - (\bar{L}-\bar{g})\cdot\mathrm{erf}\left(\frac{\bar{L}-\bar{g}}{\sqrt{S_g^2+S_L^2}}\right) - \frac{\sqrt{S_g^2+S_L^2}}{\sqrt{2\pi}}\cdot\exp\left\{-\frac{(\bar{L}-\bar{g})^2}{2(S_g^2+S_L^2)}\right\} \qquad (A.2-6)$$

Proof 2;

Defining a random variable z by the shortage of supply as

$$Z \triangleq L - g \qquad (A.2-7)$$

that is,

$$\bar{Z} = \bar{L}-\bar{g},\quad S_Z^2 = S_L^2 + S_g^2 \qquad (A.2-8)$$

and

$$f_Z(Z) = \frac{1}{\sqrt{2\pi}\,S_Z}\cdot\exp\left\{-\frac{(z-\bar{z})^2}{2S_Z^2}\right\} \qquad (A.2-9)$$

then, z for $z>0$ is the shortage of generation g. And the expectation of Not Served Energy $\bar{N}$ is given by

$$\bar{N} = E[Z] \qquad (A.2-10)$$

Since the power requirement is a constant $\bar{L}$ the supplied energy becomes

$$\bar{E} = \bar{L} - \bar{N} \qquad (A.2-11)$$

and

$$\bar{N} = E[Z] = \int_0^{\infty} z\cdot f_Z(Z)\, dz \qquad (A.2-12)$$

$$= \int_0^{\infty}(z-\bar{z})\cdot\frac{1}{\sqrt{2\pi}\,S_Z}\cdot\exp\left\{-\frac{(z-\bar{z})^2}{2S_Z^2}\right\} dz + \int_0^{\infty}\bar{z}\cdot\frac{1}{\sqrt{2\pi}\,S_Z}\cdot\exp\left\{-\frac{(z-\bar{z})^2}{2S_Z^2}\right\} dz \qquad (A.2-13)$$

$$= \frac{S_Z}{\sqrt{2\pi}}\cdot\exp\left\{-\frac{\bar{z}^2}{2S_Z^2}\right\} + \bar{z}\left[1/2+\mathrm{erf}\left(\frac{z}{S_Z}\right)\right] \qquad (A.2-14)$$

Substraction $\bar{N}$ from $\bar{L}$ gives the equation(A.2-6).

Proof 3;

The Served Energy $\bar{E}$ is evaluated directly from the physical meaning.

Assuming the chracteristics of load and generation to have the Gaussian density function $f_L(L)$ and $f_g(g)$ respectively. When they are jointed, the expected Served Energy becomes

$$\bar{E} = \int_{-\infty}^{\infty} f_L(L)\left[\int_{-\infty}^{L} g\cdot f(g)\, dg + \int_L^{\infty} L\cdot f_g(g)\, dg\right] dL \qquad (A.2-15)$$

Substitutions of $f_g(g)$ and $f_L(L)$

$$f_L(L) = \frac{1}{\sqrt{2\pi}\,S_L}\cdot\exp\left\{-\frac{(L-\bar{L})^2}{2S_L^2}\right\}$$
$$f_g(g) = \frac{1}{\sqrt{2\pi}\,S_g}\exp\left\{-\frac{(g-\bar{g})^2}{2S_g^2}\right\} \qquad (A.2-16)$$

leads to the same result of equation(A.2-6).

Since the detail derivations of these equations are very long and complex they are omitted, but the main key-points are all included above.