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A Recursive Restoration Scheme of B-Scan Ultrasonographic Images in Noisy Case

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=**Abstract**=

The objective of this phantom study is to develop a digital method for improving the lateral resolution of B-scan ultrasonographic images in medical application of ultrasound. By utilizing a discrete state-space modeling approach and Kalman-Buch method for analysis of the transducer's beam profile and the measurement and sampling noise, a stable recursive restoration of the object image was obtained for improved lateral resolution. The point spread function(PSF) was measured for the reflective signals after scanning the small pins located along the depth of interest. One major advantage of the present recursive scheme over the transform method is in its applicability for the space-variant imaging, such as in the case of the rotational movement of transducer.

1. Introduction

In B-scan ultrasonographic imaging system for medical application, the finite beam width of the transducers and spatial variation of the beam profile across the scanning area can cause lateral blurring in the recorded image. A posteriori image processing techniques, such as inverse filtering methods, have been used to improve the lateral resolution[1,2]. In these methods, however, the modifications of inverse filter functions were necessary to avoid zeros at some spatial frequencies or to eliminate peaks in the high frequency ranges. Thus, the restored image was an approximation of the object image. Furthermore, in the presence of noise, the linear least square error filter methods are known to be more effective in restoration of the object image than the inverse filter method[3].

In our previous paper[4], we proposed a recursive scheme to improve lateral resolution in B-scan ultrasonography, where the method was based upon a discrete-time, state-space model of the image degradation. In that scheme, the point spread function(PSF) was assumed to be measured exactly, and the restoration was obtained for linear space-invariant and noise-free case.

However, in actual medical B-scan ultrasonographic images, the effects of the electronic noise and the quantization error in analog-to-digital conversion of the echo signals cannot

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be neglected. Also, it is difficult to obtain a correct estimation of PSF in noisy case. These problems in noisy case can produce unstability in the recursive restoration of the object images due to inexact PSF and accumulation of errors[5,6].

In the present study, we have used Kalman-Buch filter approach to restore the original phantom image in a sense of optimal linear least square estimation for noisy case.

2. Analysis

As we presented in the previous paper[4], a linear model for describing image degradation in noise-free and space-invariant blurring case can be expressed in the following convolution summation;

$$R(n,k)=\sum_{i\in M}H(i)\;Q(n,\;k-i) \qquad (1)$$
$$n,k=1,2,\;\cdots,N$$

Where R(n,k) is the intensity of the recorded image at point k on line n, and Q(n,i) is the object image located at a horizontal position i and on the vertical spatial location n. M indicates the extent of the blur at line n and N is the number of pixels in stored image frame. The values of H(i) for $i\in M$ define PSF, and can be obtained from the measured signal amplitude by scanning a point-like reflector.

In Eq. (1), the PSF H(i) is noncausal operators, as the blurring caused by the beam profile of ultrasonic transducer is noncausal function. In this case, for simplicity, we can construct causal moving average operator by advance shifting. Then, the degraded process can be represented by a linear difference equation with a single input-single output of order M;

$$\begin{aligned}&x(n,k+1)=A_1x(n,k)+B_1Q(n,k)\\&R(n,k)=C_1x(n,k)+dQ(n,k) \qquad (2)\\&n,k=1,2,\;\cdots,N\\&x(n,1)=Xn\end{aligned}$$

where system matrices A_1,B_1,C_1, a constant d, and initial condition can be obtained from the knowledge of PSF and the background intensity[7].

For the object image, it is possible to develop another difference equation model under the assumption that the object image may be modeled as a zero mean, stationary random field with known statistics[8].

$$\begin{aligned}&y(n,k+1)=A_2y(n,k)+B_2u(n,k)\\&Q(n,k)=C_2y(n,k) \qquad (3)\\&n,k=1,2,\cdots,N\end{aligned}$$

where $E\{u(n,k)\}=0$

$$\begin{aligned}&E\{u(n_1,\;k_1)u'\;(n_2,\;k_2)\}\\&=K\triangle(n_1-n_2)\triangle(k_1-k_2) \qquad (4)\end{aligned}$$

and $\triangle(.)$ is a Kronecker delta function.

The above two models of Eq. (2) and (3) can be combined to form a total image blur model where the augmented vector z(k) is defined as;

$$z(k)=\begin{bmatrix}x(k)\\y(k)\end{bmatrix} \qquad (5)$$

and these augmented z(k) can be related as follows[5,6,9];

$$\begin{aligned}&z(n,k+1)=Az(n,k)+Bu(n,k)\\&R(n,k)=Cz(n,k) \qquad (6)\\&n,k=1,2,\cdots,N\end{aligned}$$

where $A=\begin{bmatrix}A_1&B_1C_2\\O&A_2\end{bmatrix}\quad B=\begin{bmatrix}O\\B_2\end{bmatrix}\quad C=[C_1\;dC_2]$

When the white noise with zero mean value is added to the output of Eq. (6) to represent measurement and sampling noise, one can get the overall image degradation model as follows;

$$\begin{aligned}&z(n,k+1)=Az(n,k)+Bu(n,k)\\&R(n,k)=Cz(n,k)+v(n,k) \qquad (7)\\&n,k=1,2,\cdots,N\end{aligned}$$

where $E\{v(n,k)\}=0$

$$\begin{aligned}&E\{v(n_1,\;k_1)v'(n_2,k_2)\}\\&=\sigma^2\triangle(n_1-n_2)\triangle(k_1-k_2) \qquad (8)\end{aligned}$$

For the above discrete model of the image degradation, a recursive estimation process can be applied using the previously reported Kalman-Buch filtering method[10] as follows;

$$\hat{z}(n,k)=[I-F(k)C]Az(n,k-1)$$
$$+F(K)R(n,k)$$
$$F(k)=P(k)C[CP(k)C'+\sigma^2]^{-1} \qquad (9)$$
$$P(k+1)=A[I-F(k)C]P(k)A,+BKB'$$
$$n,k=1,2,\cdots,N$$
$$\hat{z}\ (n,\ 1)=Zn$$
$$P(1)=Pn$$

Where P(k) is error covariance matrix and F(k) is Kalman gain matrix. The initial conditions P(1) and z(n,1) are assumed to be given as a part of available data.

Then, the optimal linear mean square estimation of the object image can be obtained as follows:

$$\hat{Q}(n,k)=[O\quad C_2]\ \hat{z}(n,k) \qquad (10)$$

In the above analysis, only the horizontal scan is considered for simplification of the presentation. However, any perpendicular movement of the transducers with respect to the scanning direction would not affect the present analysis, since the change of PSF along the vertical direction was shown to be small[1].

3. Experiment and Result

An ultrasonography Digital Imager (Picker, Model 80L-DI) interfaced with DEC MINC-11 computer was used for processing of the 480 ×480 pixel data with 16 grey levels[11]. Scanning was performed using a 5MHz nonforcused disc type transducer (19mm diameter), and a typical profile of beam pattern is shown in Fig. 1. The average value of several scans of point-like vinyl pins (0.6mm diameter) located at 6cm depth in a water phantom, was used for determing the PSF and A_1, B_1, C_1, d matrices with zero initial state (zero background).

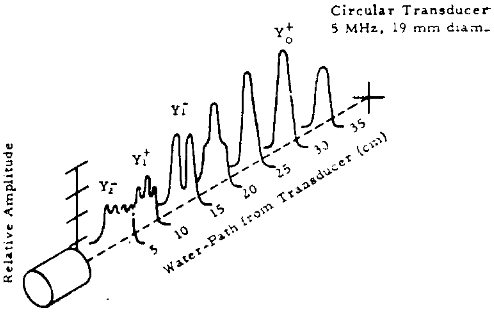

Fig. 1. The transducer beam profile

Fig. 2 Shows the average value of lateral profiles of the center line of the above single pin. This profile was used as the measured point spread function for computation. Fig. 3 shows the image of two pins (0.6mm diameter) located 3mm apartat 6cm depth in water and it's corresponding lateral profile.

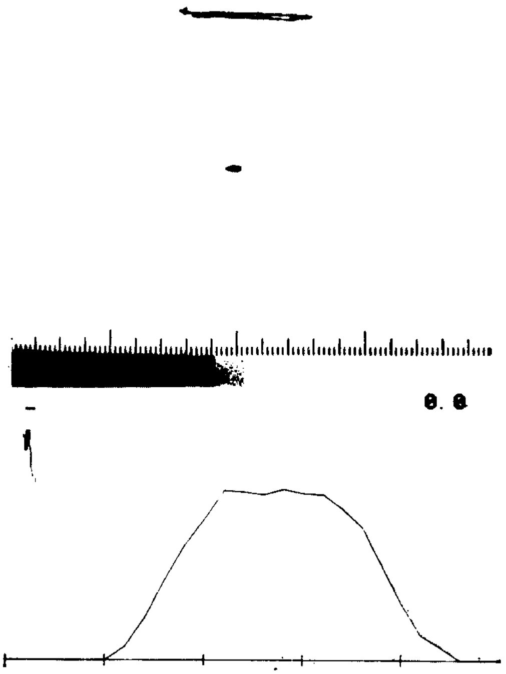

Fig. 2. (Top) An image obtained by scanning a point-like reflector located at 6cm depth in water.
(Btottom) The corresponding digital value of point spread function averaged from ten scans.

Previously reported methods[10, 12] were used to compute the matrix parameters A_2, B_2, C_2

Fig. 3. (Top) Image obtained by scanning two pins 3mm apart located 6cm depth. Two dots shown just above the object image are the digital caliper output indicating the distance between two points on image surface shown in the lower right corner. (Bottom) The corresponding digital values of the gray level at the center line of the two pins.

and K. In the present study, we used a prototype image data from the above two pins' phantom image to obtain the parameters of A_2, B_2, C_2 and K. The other estimation parameters, σ^2 and $z(n,1)$, were determined empirically from the measured data. The steady state values of P(k) in Eq. (9) were found to be independent of the initial conditions, and object image was estimated using these final steady state values of P(k).

Fig. 4 Shows the lateral profile of the same two pins after the present restoration process. While the centers of the two pins could not be separated in the original blurred profile of Fig. 3, the two pins could be clearly separated in Fig. 4. after the digital recursive processing. Also, it can be shown that the Kalman-Buch filter approach could eliminate the ringing effect near the object, which was observed in the previous noisy-free analysis and caused by error accumulation.

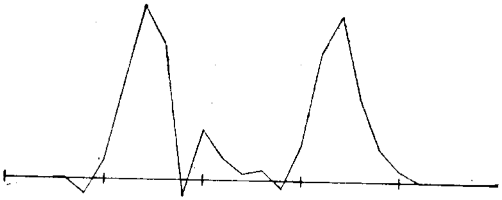

Fig. 4. Recursively estimated profile(relative amplitude) of the above pin image shown in Fig. 3.

4. Discussion

In this paper of the phanto mstudy, the statistical information of the object image was assumed to be known, and the pin image was evaluated in a single horizontal line. In actual case of restoring the two-dimensional object images, it will be simpler to use steady state value of P(k) and the fixed gain F(k) to obtain suboptimal estimation[5,6,10]. Also, it should be noted that reinitilization at each line could adversely affect the restoration process of Eq. (10) at each line edge.

Presently, we are evaluating the following problems to be solved before application of the present method in clinical case;

(1) For actual ultrasonographic image restoration, we need a two-dimensional image restoration method, compared with the present one-dimensional restoration of the lateral blurring.

(2) In scanning of the abdominal areas, a rotational scan is most widely used, as compared with the present horizontal scanning method. In this case, the blurring process becomes space-variant, and the spatial coordinate transform method, used in the photographic image restoration[13], can be applied by approximation of the scanning direction as a

circular surface. However, in this case also, any perpendicular movement of the transducer with respect to the scanning direction would not affect the blurring process.

(3) Another complex problem is how to identify the in-vivo point spread function from the measured in-vitro point spread function and the degraded image itself.

Among the above three problems, the first two problems were previously evaluated in the photographic image processing. Thus, our present main research interest is on the third identification problem of the point spread function. Once the above problems can be solved for actual clinical application, the improved lateral resolution will increase the accuracy of the B-scan ultrasonographic scanning in diagnosis of small size lesions.

5. Conclusion

In this paper, a recursive scheme is introduced to improve lateral resolution of B-scan ultrasonic images blurred by finite beam width of the transducer and noise. By utilizing a discrete state-space modeling approach for the transducer's beam profile and the measurement errors, a stable recursive restoration of the object image was obtained in a sense of optimal linear least square estimation in noisy case. In a phantom study, two pins located 3mm apart could be separated after recursive processing without any ringing effect near the object, while it was difficult to separate two pins in normal B-scan images of preprocessing.

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▨ 국문 초록 ▨

雜音을 考慮한 回歸方法에 依한 超音波 診斷器의 畫像改善

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본 논문은 의학용 진단기로 널리 쓰여지고 있는B-모드 초음파 진단기의 측면해상도 개선에 목적이 있다. 초음파 변환기의 초음파 방사형태와 계측 및 신호 추출 오차에 의한 화질 저하를 이산적 상태 방정식으로 모델화하고 여기에 칼만 여과기를 적용시켜 안정된 회귀적 화면 재구성을 통해 최적의 개선된 측면해상도를 얻을 수있다. 이때 점확산 함수는 특정 깊이에 놓여 있는 작은 핀을 주사함으로써 얻어진다. 이 방법은 또한 시변 화면에도 적용시킬 수 있는 장점을 가지고 있다.

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