〈研 究 論 文〉

A STUDY ON A PROBABILISTIC MULTI-LOCATION PROBLEM IN A TWO-ECHELON LOGISTIC SYSTEM FOR DETERIORATING ITEMS

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ABSTRACT

A special case of the probabilistic multi-location problem is studied in a two-echelon logistic system for deteriorating items. The objective is to determine the location of the minimum number of supply centers among a discrete set of location sites of supply centers, such that the probability each retailer being covered by some supply center is not less than a specified value.

A logistic cost is introduced as performance measure of the system, and leads us to analyze the impact of deterioration rate on the location problem.

The results obtained from numerical examples are discussed, which provides effective guidelines that can be used for the logistic managerial decisions.

1. INTRODUCTION

Since 17th century when Fermat [6] problem was proposed, a great many papers have been published on the subject of facility location and layout. These have been applied in variety of contexts; product delivery [2], network [3], truck dispatching [4], assembly line balancing [8], warehouse location [9] and distribution [7].

Most of these studies are concerned with deterministic formulation of problems so called "central gravity method", "set covering problem" and "p-center problem", and a distance norm or traveling time was considered as a performance measure of the system.

In this study we consider a special case of supply center location problem based upon a discrete solution space under the following fundamental structure:

1) a two-echelon logistic system is considered,
2) the item to be shipped deteriorates in accordance with exponential distribution,
3) a logistic cost is considered as the performance measure of this system,
4) some probabilistic factors are introduced,
5) the rectilinear distance measure is assumed.

The impact of deterioration rate on the logistic cost cannot be neglected in locating the upper-echelon facilities (the supply centers) for the long range of operational performance of the system. Unfortunately, little attention has been given to this special case of location problems.

This problem arises naturally for some public systems as follows:

1) military supply systems for deteriorating items,
2) fuel storage site location problems,
3) some distribution systems of National Agricultural Cooperation Union for the agricultural and aquatic products,
4) the other public supply systems handling deteriorating items.

2. MATHEMATICAL MODELS

In this study we consider a multi-location problem in a two-echelon logistic system for deteriorating items of which the upper echelon implies supply centers and the lower echelon represent all the retailers.

A set of finite number of potential sites for supply centers is assumed to be given as the discrete solution space.

Our problem is to determine the minimum number of supply centers in which the probability of each retailer being covered by some supply center is not less than a specified value defined as critical logistic cost.

First, we consider a problem in which the covering coefficient is considered deterministically and then extend it to the probabilistic problem.

Notations used for the model development are as following:

R_i (a_i, b_i) =the location of retailer R_i,
S_j (x_j, y_j) =the location of supply center S_j,
F_{ij} (R_i, S_j) =the logistic cost per unit period incurred between retailer R_i and supply center S_j,
A_i =the allowable service level of the logistic cost for retailer R_i,
D_i =the amoung of demand per unit time for retailer R_i,
c_i =the transportation cost per unit amount per unit distance for retailer R_i,
d_i =the cost of unit deteriorated item for retailer R_i,
α_i =the deterioration rate of items for retailer R_i,
$d(S_j, R_i)$ =the distance norm between supply center S_j and retailer R_i,
$t(S_j, R_i)$ =the in-transit time between supply center S_j and retailer R_i, $t(S_j, R_i) = K + V.d(S_j, R_i)$, where K and V are costants whose values depend on the state of the road conditions and average speed of vehicle.

2.1 LOGISTIC COST

We define the logistic cost, F_{ij} as the sum of the transportation cost of one period's demand and the deterioration cost incurred during the intransit time for this period for delivery from S_j to R_i.

To compute the amount to be shipped from S_j to R_i, we consider a deteriorating inventory system depicted as in Figure 1 in which only the depletion due to deterioration is allowed.

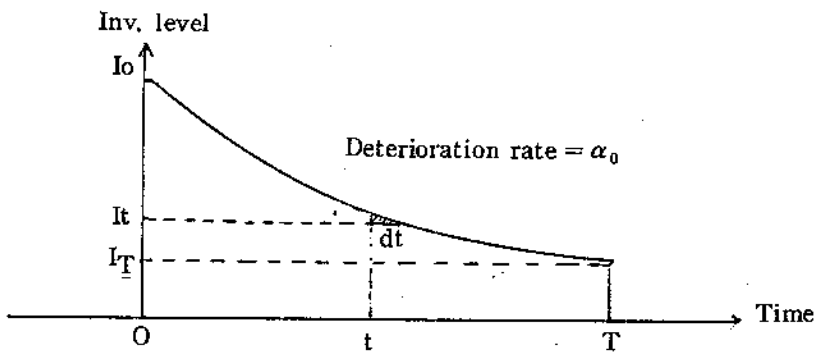

Figure 1. A simple deteriorating inventory system.

When the initial inventory level is I_o, the inventory level at time t, I_t is obtained by solving the following differential equation.

$$I_t \cdot d_t + I_t \cdot \alpha_0 = 0.$$

By solving it,

$$I_t = I_0 \cdot e^{\alpha_0 t},$$

thus,

$$I_0 = I_t \cdot e^{\alpha_0 t}.$$

From the above result, F_{ij} becomes

$$F_{ij} = c_i \cdot d(S_j, R_j) \cdot D_i \cdot e^{\alpha_i t_{S_j,R_i}} + d_i D_i (e^{\alpha_i t_{S_j R_i}} - 1) \quad (1)$$

Let, $d(S_j, R_i) = |x_j - a_i| + |y_i - b_i|$,

$t(S_j, R_i) = K + V(|x_j - a_i| + |y_j - b_i|)$, then equation (1) becomes

$$F_{ij} = (c_i D_i d(S_j, R_i) + d_i D_i)\, e^{\alpha k_i d(S_j, R_i)} - d_i D_i \qquad (2)$$

2.2 DETERMINISTIC FORMULATION

We consider a well known deterministic set covering problem for the purpose of comparison with the probabilistic one. This is given as

D1. Minimize $\sum_{j=1}^{n} X_j$

Subject to $\sum_{j=1}^{n} a_{ij} X_j \geqslant 1$, for all $i = 1, \ldots, m$

$X_j = (0, 1)$, for all $j = 1, \ldots, n$

where m = the number of retailers,
n = the number of potential sites of supply centers,
a_{ij} is the covering coefficient defined as

$$a_{ij} = \begin{cases} 1, & \text{if } F_{ij} \leq A_i, \\ 0, & \text{otherwise.} \end{cases}$$

This is a location problem of finding minimum number of supply centers which is to be determined such that the logistic cost incurred by a supply center is not greater than specified critical value.

In problem D1, the covering coefficient, a_{ij} is treated deterministically.

2.3 PROBABILISTIC FORMULATION

For the probabilistic version of the covering coefficient, the logistic cost F_{ij} is assumed to be a random variable with known distribution. This assumption is valid when D_i is considered as a random variable.

Aly [1] formulated a probabilistic set covering problem under the following assumptions: 1) the supply centers are always in available state and 2) the traveling time is considered as random variable with known distribution.

When we consider the logistic cost as a random variable with the supply centers being always in available state, a probabilistic formulation noted P1 is given as:

P1. Minimize $\sum_{j=1}^{n} X_j$,

Subject to Probe $(F_{ij} \leqslant A_i) \geqslant r_i$,
for some $j \in \theta(x)$; $i = 1, \ldots, m$,
$X_j = (0, 1)$, for all $j = 1, \ldots, n$,

where r_i is the minimum allowable probability that R_i is covered by a supply center under the assumption that all supply centers are in available state,
and $\theta(x)$ is the set of sites where a supply center can be located, i.e., $\theta(x) = (j \mid X_j = 1, j = 1, \ldots, n)$

The supply centers satisfying the first constraint for the retailer i are said to cover R_i. In this case it is assumed that whenever there are requests from the retailers, the supply centers can satisfy these demands without any delay due to shortage or waiting for action.

The first constraint of P1 is transformed to an equivalent but computationally more efficient form as:

P2. Minimize $\sum_{j=1}^{n} X_j$,

subject to $\sum_{j=1}^{n} a_{ij} X_j \geqslant 1$, for all $i = 1, \ldots, m$

$X_j = (0,1)$, for all $j = 1, \ldots, n$

where

$$a_{ij} = \begin{cases} 1, & \text{if Prob } (F_{ij} \leqslant A_i) \geqslant r_i, \\ 0, & \text{otherwise.} \end{cases}$$

Now P2 can be solved by a 0-1 programming algorithm. And the covering coefficient a_{ij} in P2 is determined under the assumption that a supply center is always in available state but in most real situations this is not always true. So the asumption can be relaxed by introducing the probability of a supply center being in available state.

Let b_j = Prob (S_j is in available state), and

P_{ij} = Prob (R_i is covered by S_j),

then, $P_{ij} = a_{ij} b_j$,

and $q_{ij} = 1 - P_{ij}$,

thus, $$\text{Prob}\left\{\begin{array}{l}\text{a } R_i \text{ is covered by some of} \\ \text{the available supply centers}\end{array}\right\} = 1 - \prod_{j \in \theta(\lambda)} q_{ij}$$

where $\theta(\lambda) = (j \mid X_j = 1,\ j = 1, \ldots, n)$.

The equation (3) leads us to develop another formulation P3 as follow:

P3. Minimize $\sum_{j=1}^{n} X_j$,

subject to $1 - \prod_{j \in \theta(\lambda)} q_{ij} \geq l_i$ for i = 1, . . . , m

$X_j = (0, 1)$, for j = 1, . . . , n

where l_i is the probability that R_i is covered by some available supply center, with its maximum equal to $(1 - \prod_{j \in \theta(\lambda)} q_{ij})$.

To make the solution procedure of P3 more manageable the first constraint can be transformed as follows:

$$1 - \prod_{j \in \theta(\lambda)} q_{ij} = 1 - \prod_{j=1}^{n} (q_{ij})^{X_j} \geq l_i, \text{ for } R_i,$$

thus, $$\prod_{j=1}^{n} (q_{ij})^{X_j} \leq 1 - l_i.$$

Thaking a logarithmic transformation, equation (4) becomes

$$\sum_{j=1}^{n} (\log q_{ij}) X_j \leq \text{Log}\,(1 - l_i).$$

Let $S_{ij} = -\log q_{ij}$,

and $W_i = -\log (1 - l_i)$,

then, P3 can be reformulated as

P4. Minimize $\sum_{j=1}^{n} X_j$,

subject to $\sum_{j=1}^{n} S_{ij} X_j \geq W_i$, for i = 1, . . . , m

$X_j = (0, 1)$, for J = 1, . . . , n

This is the form of standard 0-1 programming and can be solved using an appropriate algorithm [5].

3. COMPUTATIONAL RESULTS

An example problem is solved with ten potential locations of supply centers and seventeen retailers. Without loss of generality the coordinates of these locations and the demand rates for each retailers are generated from the uniform distribution in the open interval (0, 50) and (0, 10) respectively.

For all i, the transportation costs are given as c_i = ₩3.00 per unit per travel distance and the deterioration costs as d_i = ₩50.00 per unit deteriorated. the constants K and V are given by 1.5 and 0.03 respectively.

The deterioration rate and the critical value of the logistic costs are varied from 0.00 to 0.30 and

200 to 3,000 respectively by appropriate increments, and the logistic costs, F_{ij}, are assumed to be normally distributed with standard deviation equal to $0.01 \cdot F_{ij}$.

For the sensitivity analysis 189 problems were solved with various combination of the above parameter values. The results obtained from the model D1 and P3 are shown in Table 1 and Table 2 respectively and also depicted in Figure 2 and Figure 3. In these tables two numbers in each box represent the number of the supply center required and the number of alternatives of the optimal solution. For instance, with A_i equal to 600 and equal to 0.05, the number of alternative of the optimal solution is 4 and the number of supply center required is 2.

For clarity, the extra column of the left of Figure 2 and 3 represents number of suppliers when no deterioration occurs. By observing the above tables and figures it can be seen that the number of supply centers required to cover all the retailers increases as the value of deterioration rate increases but it decreases as the critical logistic cost increases. As it seems to be expected, with the deterioration rate and the number of the supply centers fixed, the number of the alternative optimum solutions increases as the critical logistic cost increases.

The probabilistic formulation requires larger number of supply centers compared with that from a deterministic formulation. When the number of supply centers required are the same for both cases, the former has the smaller number of alternatives of the optimal solutions.

From a managerial point of view, the above results can be effectively used for the following problems:

1) find $\sum_j X_j$, given A_i,

and 2) find A_i, given $\sum_j X_j$.

4. SUMMARY AND CONCLUDING COMMENT

A special case of location problem was studied with the emphasis of deterioration rate for both deterministic as well as probabilistic problem under the discrete solution space. In the probabilistic case, the model is extended by introducing the supply center availability.

A numerical problem was solved with various combinations of given parameters, which leads us to helpful guidelines which can be effectively used for logistic managerial decisions.

Table 1. The result obtained from solving 91 deterministic

(Σx_j, number of alt.)

A_i \ α_i	0.00	0.05	0.10	0.15	0.20	0.25	0.30
200	5,2	5,1	-,-	-,-	-,-	-,-	-,-
300	3,2	5,8	5,2	5,1	-,-	-,-	-,-
400	2,1	2,1	4,2	5,4	5,2	5,1	-,-
500	2,4	2,1	2,1	3,1	5,8	5,2	5,1
600	2,10	2,4	2,2	2,1	3,2	5,8	5,2
700	2,15	2,10	1,4	2,1	2,1	3,2	5,8
800	1,1	2,14	2,10	2,4	2,1	2,1	3,2
900	1,3	1,1	2,10	2,5	2,4	2,1	2,1
1000	1,6	1,1	2,15	2,10	2,4	2,3	2,1
1200	1,8	1,3	1,1	2,14	2,10	2,10	2,4
1500	1,9	1,8	1,6	1,3	1,1	2,14	2,10
2000	1,10	1,9	1,9	1,7	1,5	1,3	2,15
3000	1,10	1,10	1,10	1,9	1,9	1,8	1,6

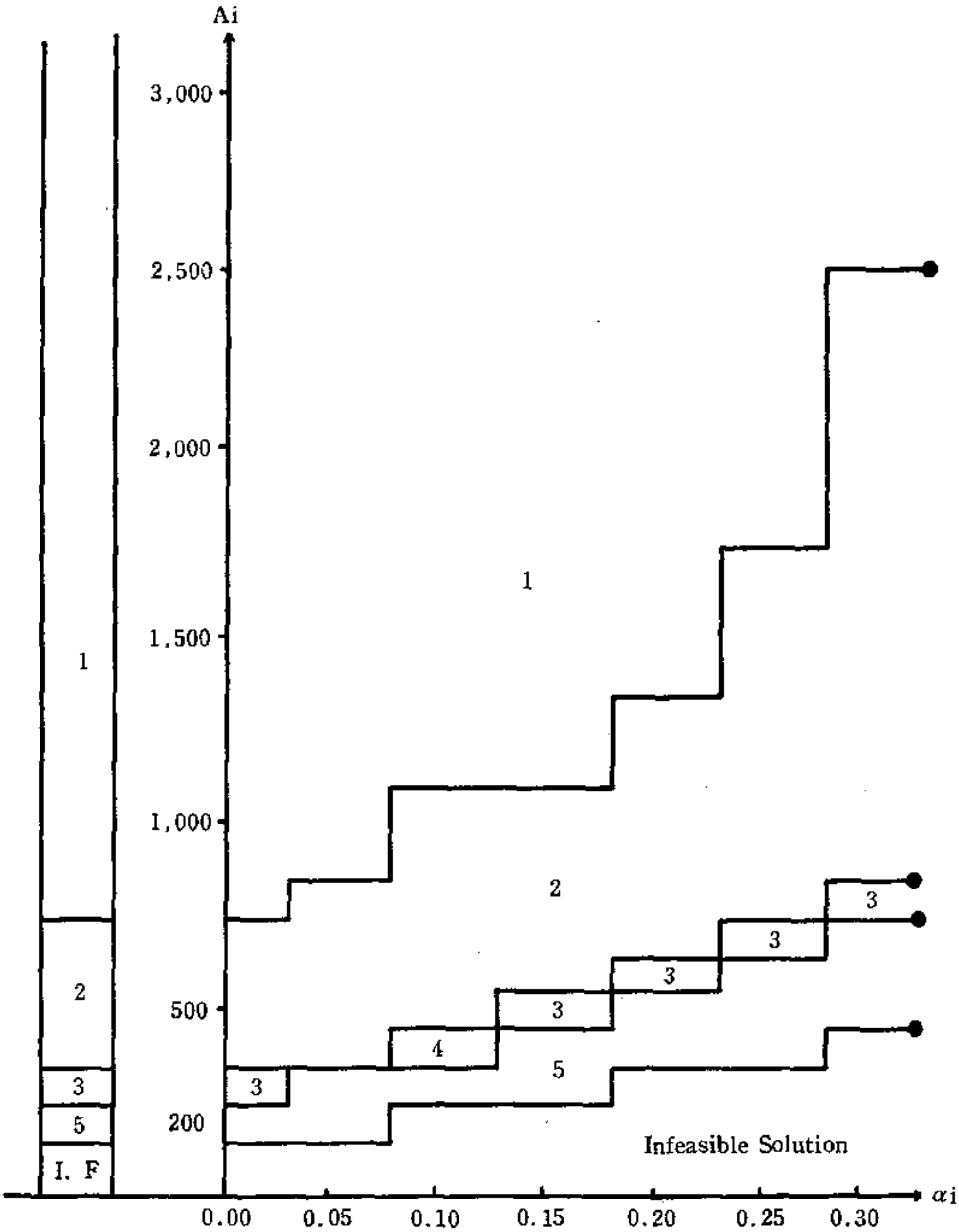

Figure 2. The impact of both the deterioration rate and the critical logistical cost A_i on the supply centers required (the result obtained from the solution of deterministic problems)

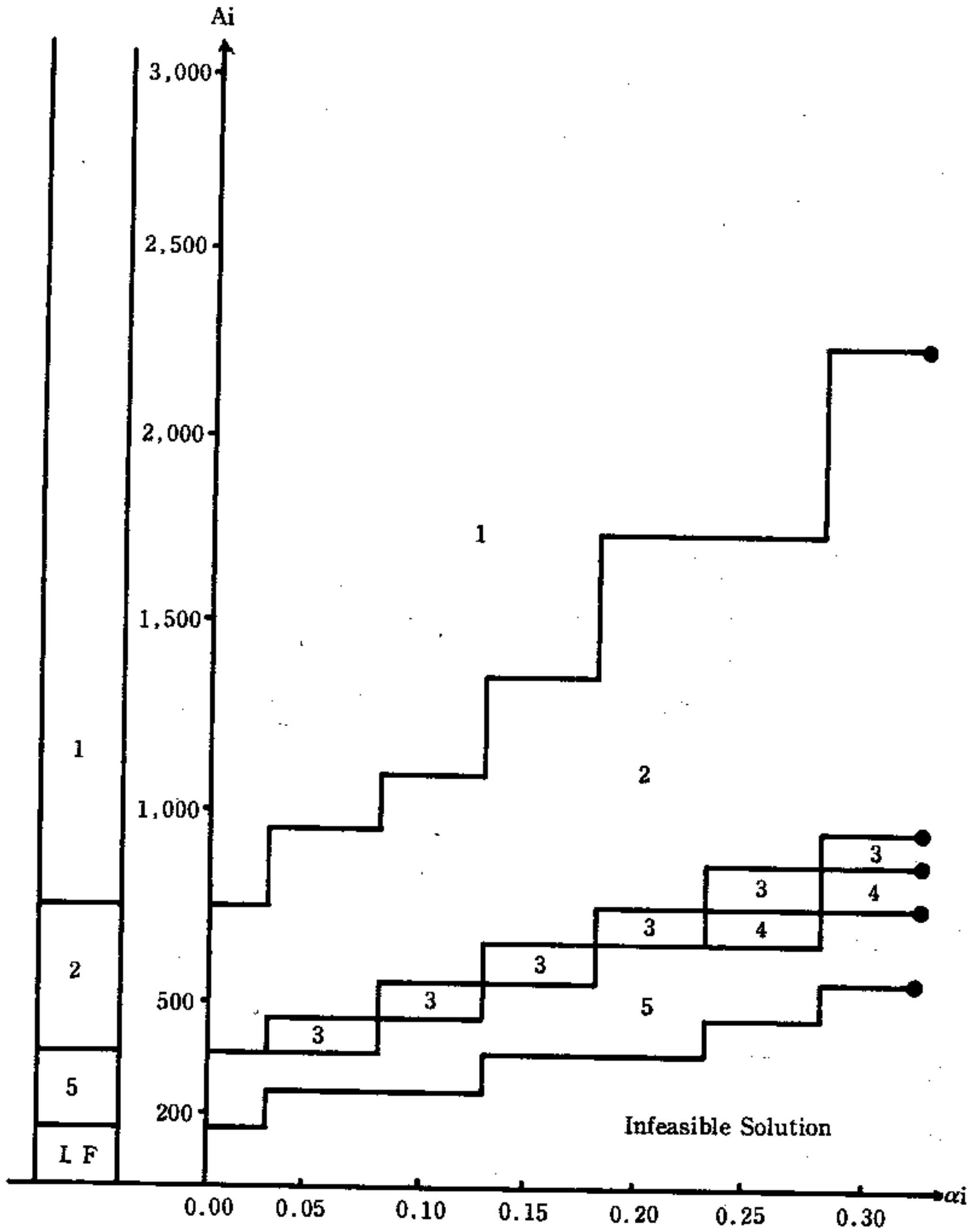

Figure 3. The impact of both the deterioration rate and the critical logistic cost A_i on the supply center required given parameters by ; $a_i = 0.9$, $b_i^j = 0.9$ for all i and j, i=1, ……, m, j = 1, ……, n.

Table 2. The result obtained from solving 98 probabilistic problems.
(ℓ_i=0.90, and b_j=0.90, for for all i and j)

(Σx_i, number of alt.)

A_i \ α_i	0.00	0.05	0.10	0.15	0.20	0.25	0.30
200	5,1	-,-	-,-	-,-	-,-	-,-	-,-
300	5,8	5,2	5,1	-,-	-,-	-,-	-,-
400	2,1	3,1	5,8	5,2	5,1	-,-	-,-
500	2,3	2,1	3,2	5,8	5,2	5,2	-,-
600	2,5	2,3	2,1	3,5	58	5,4	5,1
700	2,10	2,5	2,2	2,1	3,5	4,2	5,4
800	1,1	2,10	2,4	2,2	2,1	3,2	4,2
900	1,1	2,14	2,10	2,4	2,1	2,1	3,2
1000	1,3	1,1	2,10	2,5	2,4	2,1	2,1
1200	1,6	1,3	1,1	2,10	2,10	2,4	2,1
1500	1,9	1,6	1,3	1,1	2,15	2,10	2,4
2000	1,10	1,9	1,8	1,6	1,3	1,1	2,14
2500	1,10	1,10	1,9	1,9	1,6	1,4	1,1
3000	1,10	1,10	1,9	1,9	1,9	1,6	1,4

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