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ON o-SEMIDEVELOPABLE SPACES

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In this paper, we introduce a class of o-semidevelopable spaces and obtain several properties of these spaces. By using these results, we will show that some results in [3] cotain an unnecessary somewhat cumbersome condition.

In [3], Gittings introduced a class of o-semimetrizable spaces and obtained several properties of these spaces;

(1) Let $f: X \to Y$ be a finite to one open map of an o-semimetrizable space X onto a regular space Y. Then Y is o-semimetrizable,

(2) Suppose $U = \{U_\alpha : \alpha \in A\}$ is a point finite open cover of a regular space X. If each U_α is o-semimetrizable, then so is X,

(3) A regular, metacompact, locally o-semimetrizable space X is o-semimetrizable.

A space X is *semimetrizable* if there exists a real valued function d on $X \times X$ such that (1) $d(x, y) = d(y, x) \geqslant 0$, (2) $d(x, y) = 0$ iff $x = y$, (3) for $M \subset X$, $x \in \overline{M}$ iff $d(x, M) = \inf\{d(x, y) : y \in M\} = 0$. If in addition, d satisfies (4) for every $\varepsilon > 0$ and $x \in X$, $S_d(x, \varepsilon) = \{y \in X : d(x, y) < \varepsilon\}$ is an open subset of X, then X is said to be *o-semimetrizable.*

According to Alexander [1], a space X is *semidevelopable* if there exists a sequence $\langle G_n \rangle$ of (not necessarily open) covers of X such that $\{St(x, G_n) : n \in N\}$ is a local system of neighborhood at x. The sequence $\langle G_n \rangle$ is called a *semidevelopment* for X. If in addition, $\{St(x, G_n) : n \in N\}$ is an open base at x, then X is said to be *o-semidevelopable.* The sequence $\langle G_n \rangle$ is called a *o-semidevelopment* for X. In this paper, all mappings are continuous.

In [1], Alexander showed that a space X is semimetrizable if and only if it is a semidevelopable T_0-space. We have the similar result in o-semidevelopable spaces.

THEOREM 1. *A space X is o-semimetrizable if and only if it is o-semidevelopable T_0-space.*

Proof. See Theorem 2.1 in [3].

It is easy to verify that every developable space is o-semidevelopable and, of course, every o-semidevelopable space is semidevelopable. What is sup-

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rising is that neither of the implications are reversible. Because it is clear by Example 1.1 and 1.2 in [3] and Theorem 1 above.

THEOREM 2. *Let $f : X \to Y$ be a finite to one open map of an o-semidevelopable space X onto Y, then Y is o-semidevelopable.*

Proof. Let $\langle G'_n \rangle$ be an o-semidevelopment for X. Then we may assume that $\langle G'_n \rangle$ is a refining o-semidevelopment for X. Because, if it were not, then we let $G_n = G_1' G_2' \cdots G_n' = \{A_1 \cap A_2 \cap \cdots \cap A_n : A_i \in G_i, i=1, 2, \cdots n\}$. It is clear that $\langle G_n \rangle$ is a refining semidevelopment for X. By construction, we have $St(x, G_n) = \bigcap_{i=1}^{n} St(x, G_i')$. Therefore $\{St(x, G_n) : n \in N\}$ is an open base at x. If we let $G_n^* = \{f(B) : B \in G_n\}$, then $\langle G_n^* \rangle$ is an o-semidevelopment for Y. Because, if U is a neighborhood of $y \in Y$, $f^{-1}(U)$ is a neighborhood of $f^{-1}(y) = \{x_1, \cdots, x_n\}$. Therefore there exist $k_i \in N$ such that $St(x_i, G_{k_i}) \subset f^{-1}(U)$. Let $k = \max\{k_i : i=1, 2, \cdots, n\}$. Then $St(f^{-1}(y), G_k) \subset f^{-1}(U)$. Therefore we have $St(y, G_k^*) \subset U$. Since $St(y, G_l^*) = f(St(f^{-1}(y), G_l)) = f(\bigcap_{i=1}^{n} St(x_i, G_l))$ and f is an open map, therefore we have $St(y, G_l^*)$ open for each $l \in N$.

The following Corollary show that Gittings' result (1) contains an unnecessary condition.

COROLLARY 3. *Let $f : X \to Y$ be a finite to one open map of an o-semimetric space X onto Y, then Y is o-semimetrizable.*

Proof. Since X is an o-semimetrizable space, X is an o-semidevelopable T_1-space. Therefore Y is o-semidevelopable by Theorem 2 and it is obivous that Y is T_1. By Theorem 1, Y is o-semimetrizable.

THEOREM 4. *Suppose $\{U_\alpha : \alpha \in A\}$ is a point finite open cover of a space X. If each U_α is o-semidevelopable, then so is X.*

Proof. Let $X^* = \Sigma U_\alpha^*$ be the free union of U_α^*, where $U_\alpha^* = \alpha \times U_\alpha$. Since U_α^* has an o-semidevelopment $\langle G_n^\alpha \rangle$, then $\langle G_n^* \rangle$ is an o-semidevelopment of X^*, where $G_n^* = \cup_\alpha G_n^\alpha$. If we define $f : X^* \to X$ by $f(\alpha, x) = x$ then f is finite to one open map. Therefore X is o-semidevelopable.

The following Corollary show that Gittings' result (2) contain an unnecessary condition.

COROLLARY 5. *Suppose $\{U_\alpha : \alpha \in A\}$ is a point finite open cover of X. If each U_α is o-semimetrizable, then so is X.*

THEOREM 6. *A metacompact locally o-semidevelopable space X is o-semidevelopable.*

Proof. Let $\{U_\alpha : \alpha \in A\}$ be an open cover of X by o-semidevelopable spaces. Since X is metacompact, there is a point finite open cover $V=\{V_\beta : \beta \in B\}$ such that V is a refinement of U. Since the property of being o-semidevelopable is clearly hereditary to open subsets, each V_β is o-semidevelopable. It follows from Theorem 4 that X is o-semidevelopable.

The following Corollary is a generalization of Gittings' result [3];

COROLLARY 7. *A metacompact locally o-semimetrizable space X is o-semimetrizable.*

However, in the above Theorem 6, o-semidevelopablity (o-semimetrizablity) does not imply metacompactness. Woo [4] proved that a locally semidevelopable space is semidevelopable if and only if it is subparacompact. A space is *subparacompact* [2] if every open cover has a σ-discrete closed refinement.

THEOREM 8. *A locally o-semidevelopable space X is o-semidevelopable if and only if it is subparacompact.*

Proof. Only if: Since o-semidevelopable spaces are semidevelopable, it is clear by Theorem 3 in [4].

If: Suppose each point $x \in X$ has an open neighborhood N_x with an o-semidevelopment $\langle G_n(x) \rangle$. Let $P=\bigcup_{n=1}^{\infty} P_n$ be a σ-discrete closed refinement of $\{N_x : x \in X\}$ where each P_n is a discrete collection.

Now fix an arbitary positive integer n. Given $P \in P_n$ let $x(P)$ be a fixed element of X such that $P \subset N_{x(P)}$ and let

$$U(P)=X-\bigcup \{P' \in P_n : P' \neq P\}$$

For $m \in N$ let

$$U_{n,m}(P)=\{U(P)\cap G : G \in G_m(x(P))\}$$

Finally we define

$$U_{n,m}=\{U : U \in U_{n,m}(P), P \in P_n\} \cup \{Q_n\}$$

where $Q_n=X-\bigcup\{P : P \in P_n\}$. Then $U_{n,m}$ is a cover of X for each n, m and we show that $\{U_{n,m} : n, m \in N\}$ is an o-semidevelopment for X.

If we take a point $z \in X$ there is an integer $n \in N$ with some $P \in P_n$ such that $z \in P$. Consequently if O is any open set containing z there is some $m \in N$ such that $St(z, G_m(x(P))) \subset O \cap N_{x(P)}$. By construction z is not contained in any element of $U_{n,m}(P')$ for $P' \in P_n$ such that $P' \neq P$. Thus $St(z, U_{n,m})=St(z, U_{n,m}(P)) \subset St(z, G_m(x(P))) \subset O \cap N_{x(P)} \subset O$. Since $\langle G_n(x) \rangle$ is an o-semidevelopment, $St(z, G_m(x(P))$ is open in $N_{x(P)}$, hence open in X. Therefore $St(z, U_{n,m})=St(z, U_{n,m}(P))=U(P)\cap St(z, G_m(x(P)))$ is open in X for each $n, m \in N$. Thus X is o-semidevelopable.

By Theorem 1, we have the following Corollary;

COROLLARY 9. *A locally o-semimetrizable space is o-semimetrizable if and only if it is subparacompact.*

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