韓國數學敎育學會誌 <數學敎育>
1981. 12. Vol. XX No. 1

Journal of the Korea Society of Mathematical Education
Dec. 1981. Vol. XX No. 1

A Note on the Semigroup with a Right Full set

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Y.K. Chae investigated the properties of semigroups with the full set. In this paper, we investigate some properties of semigroups with the right full set.

Definition. A subset A of a semigroup S is said to be *right full* if and only if $AS=A$. If a subset A of a semigroup S is said to be right and left full, then A is called *full*.

If S is a semigroup with the right identity, then every right ideal of S is right full. But the converse is not true.

Theorem 1. *The minimal right ideal of a semigroup S is right full.*

Proof. Let R be the minimal right ideal of S. Then RS is contained in R. Since RS is a right ideal, $RS=R$. Thus R is right full.

Theorem 2. *The union of any collection of right full sets of a semigroup S is right full.*

Proof. Let R be $\bigcup_\alpha R_\alpha$, where every R_α is right full. Then $RS=(\bigcup_\alpha R_\alpha)S=\bigcup_\alpha (R_\alpha S)=\bigcup_\alpha (R_\alpha)=R$. Thus R is right full.

Definition. An element x of a semigroup S is said to be *periodic* if and only if $x^n=x$ for some positive integer n. S is *pointwise periodic* (PWP) if and only if each element of S is periodic. [4][5]

Theorem 3. *Let S be a PWP semigroup. Then $R^2\subset R$ implies $R^2=R$ for all right full set R.*

Proof. Let R be right full. Since S is PWP, $x^n=x$ for some positive integer n and any x in R. Thus x is in RRS. Since R is right full, x is contained in R^2.

Corollary 1. *Let S be a PWP semigroup. If R is a right ideal, then $R^2=R$.*

Corollary 2. *Let S be a PWP semigroup. Then a right ideal is right full.*

Definition. A semigroup S is called *right stable* if $R^n=R$ for every right ideal R and some positive integer n. We can define left stable and stable for a left ideal and ideal respectiverly. [6]

Theorem 4. *Let every right ideal of a semigroup S be right full. Then S is right stable if and only if S is stable.*

Proof. Suppose that S is stable. Let R be a right ideal. Since SR is a two sided ideal, there exists some integer n such that $(SR)^n=(SR)^{n+1}$. Since $RS=R$, we have that

$$R(SR)^n=(RS)^nR=R^nR=R^{n+1} \text{ and } R(SR)^n=R(SR)^{n+1}=(RS)^{n+1}R=R^{n+1}R=R^{n+2}.$$

Thus $R^{n+1}=R^{n+2}$. Hence S is right stable.

Since every ideal is a right ideal, the converse is true.

Corollary 1. *Let S be a semigroup with the right identity. Then S is right stable if and only if it is stable.*

Corollary 2. *Let S be a semigroup with the identity. Then S is right stable if and only if it is left stable.*

Definition. Let R be a right ideal of a semigroup S. If there exists some positive integer n such that $(RL)^n=(R\cap L)^n$ for every left ideal L. then R is called *power pure*. [6]

Theorem 5. *Let S be a right stable semigroup. Every right ideal in S be right full. Then each right ideal is power pure.*

Proof. Let R be a right ideal in S. Since S is stable and LR is an ideal for all left ideals L of S, by theorem 4, $(LR)^{n-1}=(LR)^n$ for some positive integer n. Thus we have that

$$(RL)^n=R(LR)^{n-1}L=R(LR)^nL=RLR(LR)^{n-1}L=RLR(LR)^nL$$
$$=(RL)^2R(LR)^{n-1}L=\cdots\cdots=(RL)^nR(LR)^{n-1}L=(RL)^{2n}.$$

Thus $(RL)^{2n}\subset(R\cap L)^{2n}=\{(RS\cap L)^2\}^n\subset(RSL)^n\subset(RL)^n=(RL)^{2n}$ for some positive integer n. Hence R is power pure.

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