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《**Original**》

Børrensen Model Computation for Neutronic Benchmark Problems

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Neutronic Benchmark 문제에 대한 Børrensen 모델응용

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Abstract

Bφrrensen proposed a coarse mesh, three-dimensional one-and-half group diffusion scheme for computing the gross power distribution in light water reactors as an alternative to the conventional fine mesh finite difference approach in dealing with three dimensional problems, which require a prohibitively long computing time. The method reported takes extremely small execution time. However, its computational accuracy has not been investigated yet. The Bφrrensen method is revised in this work and both efficiency and accuracy are examined by applying it to IAEA benchmark problem and RISØ benchmark problem. It is found that two modifications on core-reflector boundary conditions and Bφrrensen's model constants may improve computational accuracy of power distribution calculation.

요 약

Bφrrensen은 3차원적 노심분석에 긴 시간이 요구되는 유한차분법의 대안으로서 경수형 원자로의 전체출력분포를 계산하는데 한가지의 coarse mesh 방법을 제안하였다. 이 방법은 계산시간이 매우 짧은 것으로 알려져 있지만, 그 계산의 정확도에 대한 것은 아직까지 알려져 있지 않다. 본 논문에서는 Bφrrensen의 방법을 IAEA benchmark 문제와 RISØ benchmark 문제에 적용시켜서 계산시간과 정확도에 대해서 고찰하였다. 두 문제에서 노심—반사체와의 경계조건과 Bφrrensen의 모델 상수들의 개선으로 출력분포 계산의 정확도를 상당히 높힐 수 있었다.

Nomenclature

a ; the weighting factor for the fast group flux
a' ; the weighting factor for the thermal group flux
b ; the constant derived from 'a'
b' ; the constant derived from 'a''
c ; another constant derived from 'a'
c' ; another constant derived from 'a''
D ; diffusion coefficient
f ; coarse mesh factor
h ; mesh size to the horizontal direction
i ; index of nodes
j ; index of nodes
J ; current of neutrons

k ; mesh size to the vertical direction, effective multiplication factor
L ; diffusion length
R ; the constant, h^2/k^2
S ; contacting area with reflector of node
V ; volume of node
α ; albedo values
Δ ; reflector thickness
ϕ ; neutron flux
Σ_a ; macroscopic absorption cross section
Σ_r ; macroscopic removal cross section
Σ_f ; macroscopic fission cross section
ν ; neutron numbers emitted per fission
1 ; lst group or the fast group (subsript)
2 ; 2nd group or the thermal group (subsript)

1. Introduction

Nuclear analysis of a large light water reactor (LWR) generally includes numerical calculation of few group neutron diffusion equations. The straightforward and reliable method for the solution of the diffusion equation is the conventional finite difference method. However, for many applications this solution technique is unfavourable, since too many flux points and thereby too long computation time are necessary.

Various approximate methods may also be utilized-; the nodal method[1]; the finite element method[2]; the response matrix method[3]; the flux expansion method[4]; and other coarse mesh finite difference methods. In this paper the applicability of the coarse mesh finite difference method[5], proposed by Bϕrrensen, is discussed.

This method has already been used to the three dimensional program PRESTO, but numerical investigation on its computational accuracy has not been clear yet. In a recent paper[6], it is reported that the maximum error of the power distribution for 3-D IAEA benchmark problem is approximately 12%, compared to PDQ fine mesh calculation.

In the present paper both efficiency and accuracy of this method is investigated and an attempt is made to improve the method by replacing the empirical core-reflector boundary condition with the analytical boundary condition proposed by Kalambokas and Henry[7,8] To improve the computational accuracy of the power distributions, we modify the constants of the Bϕrrensen's model.

Calculations are perfomed for two test cases, i.e. three dimensional IAEA benchmark problem[6] with its related two dimensional problem and three dimensional RISΦ problem[9].

Investigations are carried out concerning the differences from other methods in the accuracy and computational time.

2. Theory

2.1. Bϕrrens n model

Bϕrrensen proposed a one and half group coarse mesh diffusion scheme for calculating the gross power distribution in a boiling water reactor core. It is based on two group diffusion equation such as

$$-\nabla\cdot(D_1\nabla\phi_1)+(\Sigma_{a1}+\Sigma_r)\phi_1 = \frac{1}{k}(\nu\Sigma_{f1}\phi_1+\nu\Sigma_{f2}\phi_2) \quad (1)$$

$$-\nabla\cdot(D_2\nabla\phi_2)+\Sigma_{a2}\phi_2=\Sigma_r\phi_1 \quad (2)$$

Then a finite difference formulation is used for the fast flux equation, whereas a smoothed asymptotic distribution is assumed for the thermal flux.

In solving the finite difference equation, the following approximation is introduced to reduce the computer memory.

$$\frac{D_iD_j}{D_i+D_j}\simeq\frac{1}{2}\sqrt{D_i}\sqrt{D_j} \quad (3)$$

And an improved formula is obtained by expressing the average fast flux in a node as the sum of a weighted average of the mid point flux in that node and the flux values on the six interfaces.

$$\bar{\phi}_i = b\phi_i + 2c\left(\sum_{4j}\phi_i^j + R\sum_{2j}\phi_i^j\right) \tag{4}$$

where

$$b = 3a \cdot \frac{1}{3a+(1-a)(R+2)},$$

$$c = \frac{1-a}{4} \cdot \frac{1}{3a+(1-a)(R+2)},$$

$$R = h^2/k^2,$$

and

$$\phi_i^j \simeq \frac{\sqrt{D_i}}{2\sqrt{D_j}}\phi_i + \frac{\sqrt{D_j}}{2\sqrt{D_i}}\phi_j$$

Here ϕ_i^j is determined using the continuity conditions between node i and node j, and 'a' is called the weighting factor.

Bφrrensen proposed that this weighting factor for fast group is close to 0.3 from his model cell analysis, and another weighting factor for the thermal group, defined similar to the factor of the fast group, is 0.7. And he used these values to all nodes whose sizes are equal to the width of fuel assemblies.

But in the assemblies with control rods, there are strong absorption of the thermal neutrons, and thereby the self-shielding effects occur. So the effect of the interface flux on the averaged thermal neutron flux is diminised, and accordingly the weighting factor for thermal group should be changed. Unfortunately the factor is not derived in the explicit form, so we choose it as one fitting parameter for calculating the gross power distribution of the core.

2.2. Boundary conditions

Kalambokas and Henry derived the general analytical expression for core-reflector boundary conditions involving the properties of the reflector material alone. These analytical expressions are employed in Bφrrensen's model.

For the one-dimensional homogeneous reflector, the boundary conditions are given by

$$\phi_1 = \alpha_{11}J_1 \tag{5}$$

$$\phi_2 = \alpha_{21}J_1 + \alpha_{22}J_2 \tag{6}$$

where

$$\alpha_{11} = \frac{L_1}{D_1}\tanh\left(\frac{\Delta}{L_1}\right)$$

$$\alpha_{22} = \frac{L_2}{D_2}\tanh\left(\frac{\Delta}{L_2}\right)$$

$$\alpha_{21} = \frac{\Sigma_r}{D_1\Sigma_2 - D_2\Sigma_1}(D_1\alpha_{11} - D_2\alpha_{22})$$

For the 90°-wedge homogeneous reflector with infinite width, these become

$$\phi_1\left(\frac{y}{L_1}\right) \simeq \frac{L_1}{D_1}\left\{1+\exp\left(-\frac{y}{L_1}\right)\right\} \cdot J_1\left(\frac{y}{L_1}\right) \tag{7}$$

$$\phi_2\left(\frac{y}{L_2}\right) \simeq \frac{L_2}{D_2}\left\{1+\exp\left(-\frac{y}{L_2}\right)\right\} \cdot J_2\left(\frac{y}{L_2}\right) + \frac{\Sigma_r/D_1 \cdot D_2}{\frac{1}{L_2^2}-\frac{1}{L_1^2}} \cdot \left\{L_1\left\{1+\exp\left(-\frac{y}{L_1}\right)\right\} - L_2\left\{1+\exp\left(-\frac{y}{L_2}\right)\right\}\right\} J_1\left(\frac{y}{L_1}\right) \tag{8}$$

where y is the distance from the corner.

2.3. Application of boundary conditions

In the case of one dimensional homogeneous reflectors, the fast group (eq. (5)) is fomulated as

$$\phi_{i+\frac{1}{2}} \simeq \alpha_{11}D_1\frac{\phi_i - \phi_{i+\frac{1}{2}}}{\frac{h}{2}} \tag{9}$$

where subscript $i+\frac{1}{2}$ denotes the core-reflector interface and i the node center point.

But the approximation is very poor due to a coarse mesh model and the fact that for most cases the fast group flux gradient

is decreasing in the vicinity of the reflector. Thus the fast flux at the interface, $\phi_{i+\frac{1}{2}}$, may be underestimated using the above approximation and the average flux value of boundary assemblies may also be expected to be underestimated. To compensate this effect, we proposed the coarse mesh factor,[10] f, defined as

$$\phi_{i+\frac{1}{2}}=\alpha_{11}D_1 f\frac{\phi_i-\phi_{i+\frac{1}{2}}}{\frac{h}{2}} \tag{10}$$

where f is usually greater than 1.0. Considering that the neutron flux behaves similarly in the node adjacent to the reflector, we assume this factor is nearly constant over the nodes and choose it as another fitting parameter.

But for the thermal group (eq. (6)), a difficulty is arised in direct formulation of the finite difference, because the Bφrrensen's model is one and half group whereas the boundary conditions are deduced from two groups. So we applied the conditions by following procedures. If there is no thermal neutron leakage to the reflector region, the average thermal neutron flux is expressed as

$$\phi_2=b'\phi_i^*+2c'(\sum_j\phi_i^{j*}+R\sum_j\phi_i^{j*}) +2c'(n+R\cdot m)\phi_i^* \tag{11}$$

where i denotes node center, n the numbers of adjacent reflector nodes in horizontal direction, m the numbers in vertical direction and ϕ_i^* asymptotic thermal flux defined as $\frac{\Sigma_r}{\Sigma_{a2}}\phi_{1i}$. Next considering thermal leakage to the reflector, the relation is formed by

$$\Sigma_{a2}\tilde{\phi}_2V=\Sigma_{a2}\bar{\phi}_2V-J_2\cdot S \tag{12}$$

where $\tilde{\phi}_2$ denotes the averaged thermal flux in the node considering the thermal leakage. And the thermal current J_2 can be expressed as a function of α_{22}, α_{21} and the fast flux ϕ_{1i} by using the boundary condition eq. (6), and asymptotic thermal flux assumption. Substituting this value and rearranging give

$$\tilde{\phi}_2=\bar{\phi}_2+(A_x+A_y+A_z)\phi_1 \tag{13}$$

$$A_i=\begin{cases}0; & \text{no reflector in the } i \text{ direction}\\ \left(\frac{1}{h\Sigma_{ar}}\cdot\frac{1}{\alpha_{22}+\frac{h}{2D_2}}\left(\frac{\alpha_{21}}{\alpha_{11}}\cdot\frac{\alpha_{11}D_1}{\alpha_{11}D_1+\frac{h}{2}}-\frac{\Sigma_r}{\Sigma_a}\right)\right)_i; & \text{in presence of the reflector}\end{cases}$$

$i=x,y,z$

Its expression is developed for a coarse mesh only, so if the coarse mesh factor is introduced, we substitute α_{11} by $\alpha_{11}f$ and α_{21} by $\alpha_{21}f$.

In case of 90° wedge reflector with the finite width, we modify eq. (7) and (8) as

$$\phi_1\left(\frac{y}{L_1}\right)\simeq\alpha_{11}\left\{1+\exp\left(-\frac{y}{L_1}\right)\right\}J_1\left(\frac{y}{L_1}\right) \tag{14}$$

$$\phi_2\left(\frac{y}{L_2}\right)\simeq\alpha_{22}\left\{1+\exp\left(-\frac{y}{L_2}\right)\right\}J_2\left(\frac{y}{L_2}\right) +\frac{\Sigma_r}{\frac{1}{L_2^2}-\frac{1}{L_1^2}}\left\{\frac{\alpha_{11}}{D_2}\left\{1+\exp\left(-\frac{y}{L_1}\right)\right\} -\frac{\alpha_{22}}{D_1}\left\{1+\exp\left(-\frac{y}{L_2}\right)\right\}\right\}J_1\left(\frac{y}{L_1}\right) \tag{15}$$

With integrating from the origin to the node length, h, and assuming the flux and current are varying slowly, these become

$$\phi_1\simeq\bar{\alpha}_{11}J_1 \tag{16}$$

$$\phi_2\simeq\alpha_{22}J_2+\bar{\alpha}_{21}J_1 \tag{17}$$

where

$$\bar{\alpha}_{11}=\int_0^h\alpha_{11}\left\{1+\exp\left(-\frac{y}{L_1}\right)\right\}\cdot dy,$$

$$\bar{\alpha}_{22}=\int_0^h\alpha_{22}\left\{1+\exp\left(-\frac{y}{L_2}\right)\right\}\cdot dy,$$

and

$$\bar{\alpha}_{21}=\int_0^h\frac{\Sigma_r}{\frac{1}{L_2^2}-\frac{1}{L_1^2}}\left\{\frac{\alpha_{11}}{D_2}\left\{1+\exp\left(-\frac{y}{L_1}\right)\right\} -\frac{\alpha_{22}}{D_1}\left\{1+\exp\left(-\frac{y}{L_2}\right)\right\}\right\}\cdot dy$$

Thus the boundary conditions for 90° wedge reflector are reduced to one dimensional, and the finite difference formulation

and application are followed by the same procedures.

Eventually the theory has two empirical fitting parameters, *i.e.* the weighting factor and the coarse mesh factor.

3. Method of the Calculations

the finite difference form of the diffusion equation can be solved with inner iterations and outer iterations. For inner, Gauss-Seidel method is used. For each outer, the effective multiplication factor can be obtained such as

$$k^{(n)}=k^{(n-1)}\frac{(\underline{\phi}_1^{(n)},\ \underline{\phi}_1^{(n)})+(\underline{\phi}_2^{(n)},\ \underline{\phi}_2^{(n)})}{(\underline{\phi}_1^{(n)},\ \underline{\phi}_1^{(n-1)})+(\underline{\phi}_2^{(n)},\ \underline{\phi}_2^{(n-1)})} \tag{18}$$

where $\underline{\phi}_g^{(n)}$; column vector of gth group flux defined as

$$\{\phi_{gl}^{(n)};\ l=1, 2, \ldots, N\}$$

and the source term is recalculated and extrapolated to improve the convergence rate, that is

$$\bar{S}_i^{(n)}=(\nu\Sigma_{f1}\phi_1^{(n)}+\nu\Sigma_{f2}\phi_2^{(n)})_i \tag{19}$$

$$\bar{S}_i^{(n)}=\frac{1}{k^{(n)}}S_i^{(n)}+\alpha\left\{\frac{S_i^{(n)}}{k^{(n)}}-S_i^{(n-1)}\right\}+\beta\{S_i^{(n-1)}-S_i^{(n-2)}\} \tag{20}$$

where $S_i^{(n)}$ denotes the extrapolated source, n the outer index, i the node index, and N the total node numbers. The extrapolation parameter α and β range between 0 and 1, and can be chosen by using methods based on Chebyshev polynomial interpolation. But in this paper, we choose the optimal values by numerical experiments to avoid the complexity.

Computer code BØR3D for three dimensional calculation is constructed and BØR2D for two dimensional calculation is constructed by modifying BØR3D. In BØR2D, the axial bucklings are assumed to make two dimensional diffusion equations of eq. (1) and eq. (2), and the constant R of eq. (4) is zero because of the infinite axial mesh size.

The flow diagram of BØR3D and BØR2D is shown in Fig. 1.

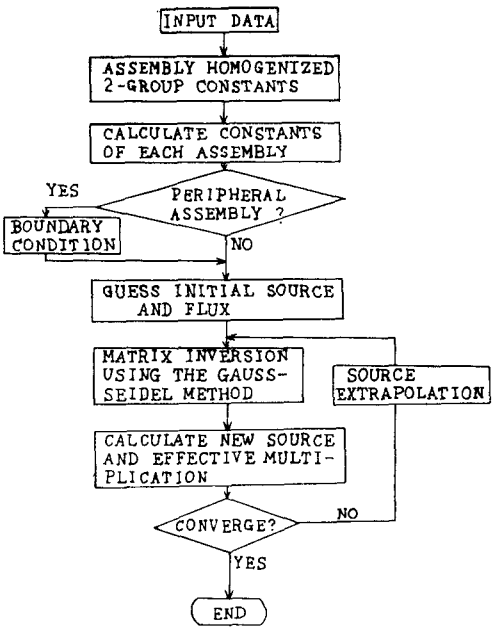


Fig. 1. Flow Diagram of BØR2D and BØR3D

4. Numerical Calculations of the Model

4.1. IAEA benchmark problem

The three dimensional IAEA benchmark problem[6] simulates a quarter clean core of the light water reactor (LWR) with varying enrichment zones and insertion of the control rods. Its configuration and the cross sections for each regions are given in Fig.2 and Table 1. The two dimensional problem is establised with axial bucking of $0.8\times10^{-4}cm^{-2}$ at the midplane of it.

The weighting factor at the assemblies with control rods is chosen by empirical tests, and the coarse mesh factor is also

adjusted by tests. The results are shown in Table 2 for three cases of two dimensional problem, and the comparison with other methods[11] is shown in Table 3. Where A-type represents the original Bφrrensen model, of which the coarse mesh effect is not considered on the boundary conditions. In B-type ($f=1.1$), we consider the coarse mesh effect only. In C-type ($f=1.1$, $a'=0.78$), we consider the weighting factor at the assemblies with control rods and the coarse mesh factor.

Table 1. Assembly Homogenized 2-group Cross Sections of IAEA Benchmark Problem

Region	Group	Diffusion coeff. (cm)	Removal c.x. (cm^{-1})	Absorption c.x. (cm^{-1})	$\nu*$fission c.x. (cm^{-1})
1	1	1.5	0.02	0.01	0.
	2	0.4	—	0.08	0.135
2	1	1.5	0.02	0.01	0.
	2	0.4	—	0.085	0.135
3	1	1.5	0.02	0.01	0.
	2	0.4	—	0.13	0.135
4	1	2.0	0.04	0.	0.
	2	0.3	—	0.01	0.
5	1	2.0	0.04	0.	0.
	2	0.3	—	0.055	0.

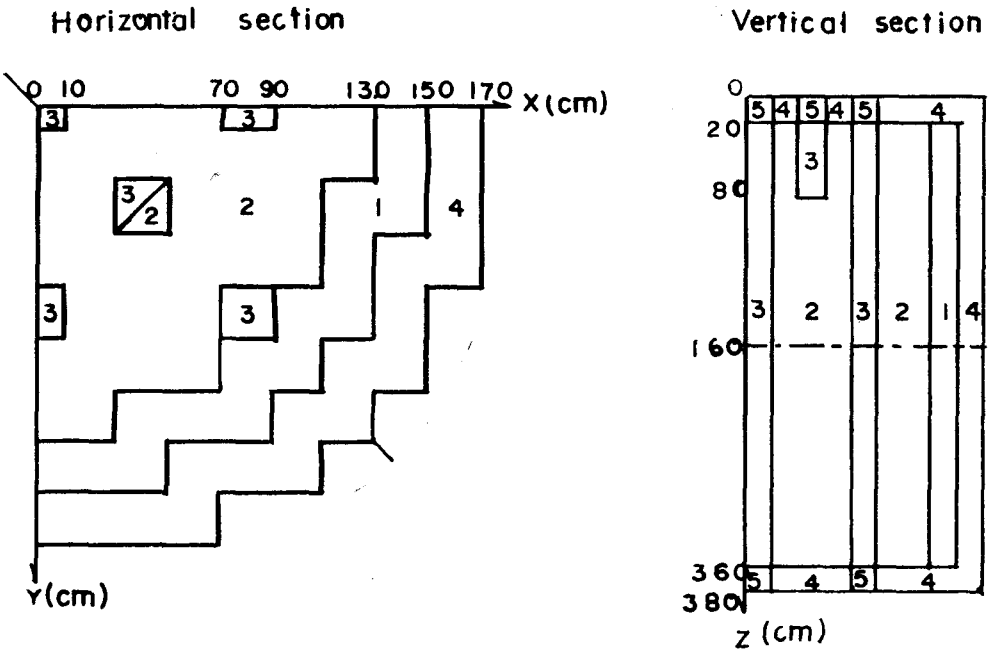


Fig. 2. 3-D Geometry of IAEA Benchmark Problem

1; High k_∞ fuel assembly **2; Low k_∞ fuel assembly**
3; Fuel assembly with control rods **4; Reflector**
5: Reflector with control rods

Table 2. Assembly Averaged Power Distributions for 2-D IAEA Benchmark Problem

Position of assembly center		PDQ reference	BØR2D A-type		BØR2D B-type		BØR2D C-type	
x(cm)	y(cm)	value	value	%	value	%	value	%
*0	0	0.7447	0.8008	7.57	0.7637	2.5	0.7237	−2.82
0	20	1.3042	1.3659	4.73	1.3038	0.0	1.3016	−0.20
0	40	1.4491	1.4606	0.79	1.3991	−3.5	1.4040	−3.11
0	60	1.2065	1.2546	3.99	1.2099	0.3	1.2084	0.16
*0	80	0.6102	0.6461	5.88	0.6332	3.8	0.5994	−1.77
0	100	0.9329	0.9511	1.95	0.9540	2.3	0.9498	1.81
0	120	0.9327	0.8998	−3.53	0.9253	−0.8	0.9273	−0.58
0	140	0.7520	0.6906	−8.16	0.7396	−1.7	0.7414	−1.41
20	20	1.4301	1.4890	4.12	1.4235	−0.5	1.4282	−0.13
20	40	1.4755	1.5018	1.78	1.4408	−2.4	1.4487	−1.82
20	60	1.3114	1.3544	3.28	1.3093	−0.2	1.3135	0.16
20	80	1.0670	1.1009	3.17	1.0800	1.2	1.0770	0.94
20	100	1.0349	1.0494	1.40	1.0519	1.6	1.0530	1.75
20	120	0.9496	0.9239	−2.71	0.9513	0.2	0.9548	0.55
20	140	0.7340	0.6765	−7.83	0.7273	−2.2	0.7300	−0.54
40	40	1.4657	1.4839	1.24	1.4296	−2.5	1.4391	−1.81
40	60	1.3425	1.3576	1.12	1.3182	−1.8	1.3250	−1.30
40	80	1.1780	1.1726	−0.46	1.1548	−2.0	1.1572	−1.17
40	100	1.0709	1.0633	−0.71	1.0705	0.0	1.0737	0.26
40	120	0.9767	0.9597	−1.74	0.9950	1.9	0.9993	2.31
40	140	0.6957	0.6194	−10.9	0.6803	−2.2	0.6875	−1.18
60	60	1.1912	1.2209	2.49	1.1943	0.3	1.1976	0.54
60	80	0.9664	0.9930	2.75	0.9848	1.9	0.9814	1.55
60	100	0.9087	0.9179	1.01	0.9327	2.6	0.9316	2.52
60	120	0.8533	0.7976	−6.53	0.8389	−1.7	0.8416	−1.37
*80	80	0.4729	0.5002	5.77	0.5045	6.7	0.4751	0.47
80	100	0.6891	0.7173	4.09	0.7370	7.0	0.7282	5.67
80	120	0.6073	0.5692	−6.27	0.6034	−0.6	0.6031	0.69
100	100	0.5942	0.5481	−7.76	0.594_2	−0.7	0.5856	−1.45
effective multiplication value		1.0296	1.0294		1.0304		1.0296	

*; fuel assemblies with control rods

%; (BØR2D-PDQ)/PDQ×100

For three dimensional calculations, we recalculated the weighting factors to make the same weighting of node-center fluxes; i.e. the same 'b' values for two dimension in eq. (4). For example, the weighting factor 0.3 of 2-D (R=0) becomes 0.222 for the special 3-D (R=1) and the other factors can also be calculated from 2-D weighting factors. The results and the comparisons with other methods are shown in Table 4.

4.2 RISØ benchmark problem

Another problem, proposed from Danish Atomic Energy Commision,[9] is selected. Its configuration and the cross section data are shown in Fig. 3 and Table 5. The calculations are performed for two cases that the control rods are inserted to 110cm (case 1) and to 269cm (case 2).

Table 3. Comparison of 2-D Results with Other Methods

PROGRAM	PDQ[13]	MEDIUM		IQSBOX[4]		FEM2D[14]		BØR2D	
METHOD	FDM	NCPM	NSM	NEM	NEM	FEM	FEM	B-type	C-type
mesh width (cm)	1	10	coarse; 10	10	20	606 node8	182 nodes	20	20
polynomial	—	—	—	4	4	2	2	—	—
k-eff.	1.0296	1.0292	1.0299	1.0298	1.0300	1.0297	1.0302	1.0304	1.0296
Maximum power difference	reference	17%	1.3%	1.8%	3.4%	3.7%	13.2%	7.0%	5.6%
local error	8×10^{-6}	9×10^{-6}	6×10^{-7}	7×10^{-6}	1×10^{-5}	1×10^{-5}	1×10^{-5}	1×10^{-5}	1×10^{-5}
Cp-time (sec)	400	4.89	15.51	4.17	1.77	35.8	1.98	1.98	1.96
symmetry	1/4	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

*NCPM ; nodal collision probability method
NSM ; nodal synthesis method
NEM ; nodal expansion method
FEM ; finite element method
FDM ; finite difference method
*Calculations are perpormed on CDC6600 computer, except BØR2D on CDC6400.

Table 4. Comparison of 3-D Results with Other Methods

Computer codes	Method, order of approx.	Mesh	Eigenvalue	Peak/ave. power	
				value	location
VENTURE[12]	FDM,	17×17×17	1.02913	2.567	
	mesh	102×102×114	1.02896	2.378	
	centered	extrapolation	1.02903	2.354	
PDQ-7[13]	FDM,	34×34×38	1.03054	2.039	35, 35, 170
	coner mesh	68×68×76	1.02933	2.266	32, 32, 175
FEM-3D[14]	FEM, 2nd order	16×16×13	1.02917	2.298	32, 32, 174
IQSBOX[4]	NEM,	9×9×19	1.02875	2.412	30, 30, 170
	4th order	17×17×21	1.02903	2.356	30, 30, 170
CUBOX[4]	FEM*,	9×9×19	1.02888	2.387	50, 30, 170
	3rd order	9×9×19	1.02895	2.340	50, 30, 170
BØR3D	Børrensen model	8×8×17	1.03019	2.143	40, 20, 190

FEM* ; flux expansion method

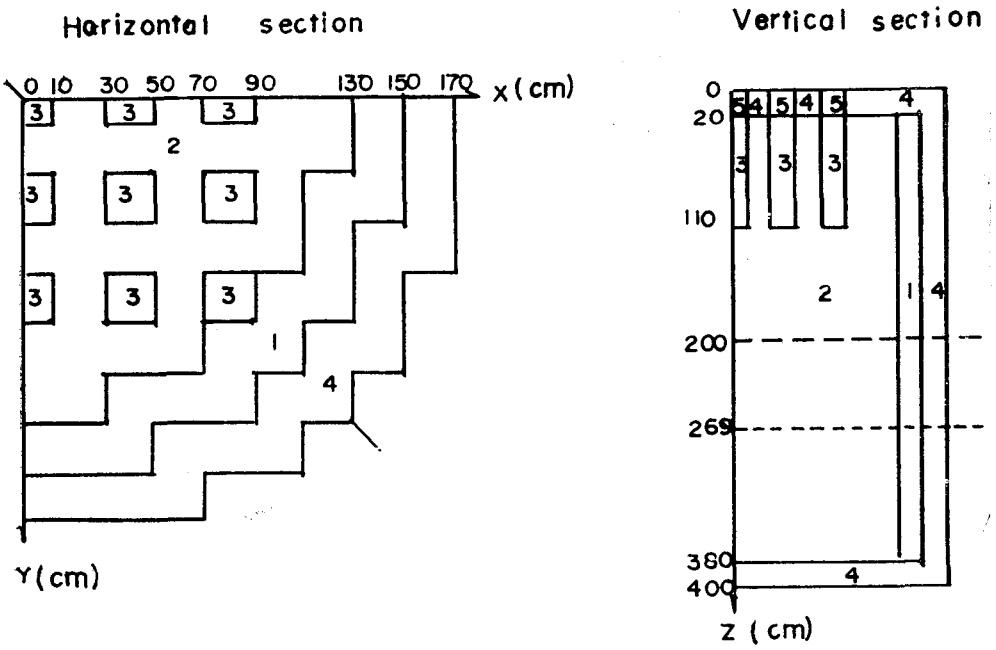
The results and the comparison with other methods are shown in Table 6, and the typical flux distributions are shown in Fig. 4 and Fig. 5; where the numbers of the flux points are 28×28×31 for DC4[15], 27×27×25 for SYNTRON[16], and 8×8×16 for BØR3D, For the coarse mesh factor and the weighting factors, the same values are taken as for IAEA benchmark problems.

5. Results and Discussions

In 2-D IAEA benchmark test, it is found that the direct application of the analytical boundary condition underestimates the power of the boundary region assemblies by 10%. For that reason, we assume the coarse mesh factor and use as a fitting

Table 5. Assembly Homogenized 2-group Cross Sections of RISØ Benchmark Problem

Region	Group	Diffusion coeff. (cm)	Removal c.x. (cm^{-1})	Absorption c.x. (cm^{-1})	ν^* fission c.x. (cm^{-1})
1	1	1.5	0.02	0.01	0.
	2	0.4	—	0.08	0.13145
2	1	1.5	0.02	0.01	0.
	2	0.4	—	0.085	0.13145
3	1	1.5	0.02	0.01	0.
	2	0.4	—	0.13	0.13145
4	1	2.0	0.04	0.	0.
	2	0.3	—	0.01	0.
5	1	2.0	0.04	0.	0.
	2	0.3	—	0.055	0.



1 : High k∞ fuel assembly
2 : Low k∞ fuel assembly
3 : Fuel assembly with Control rods
4 : Reflector
5 : Reflector with Controlrods

Fig. 3. 3-D Geometry of RISØ Benchmark Problem

parameter. The modification of the constants of Bφrrensen's model is needed for the assemblies in which control rods are inserted. We find the optimal value of the coarse mesh factor for an assemblywise mesh is 1.1 and the thermal weighting factor for assemblies with control rods is 0.78.

Using these values, the maximum difference with PDQ fine mesh calculation is 5.6% in calculations of the assembly averaged power, but the results agree within

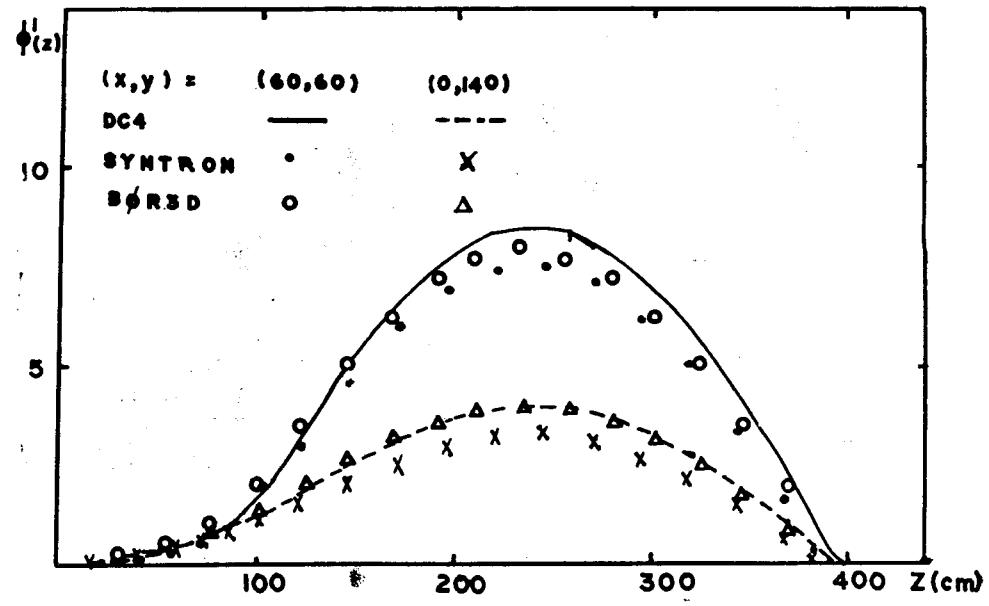


Fig. 4. Axial Fast Flux Distributions (Case 1); Maximum fast flux point is set equal to 10.

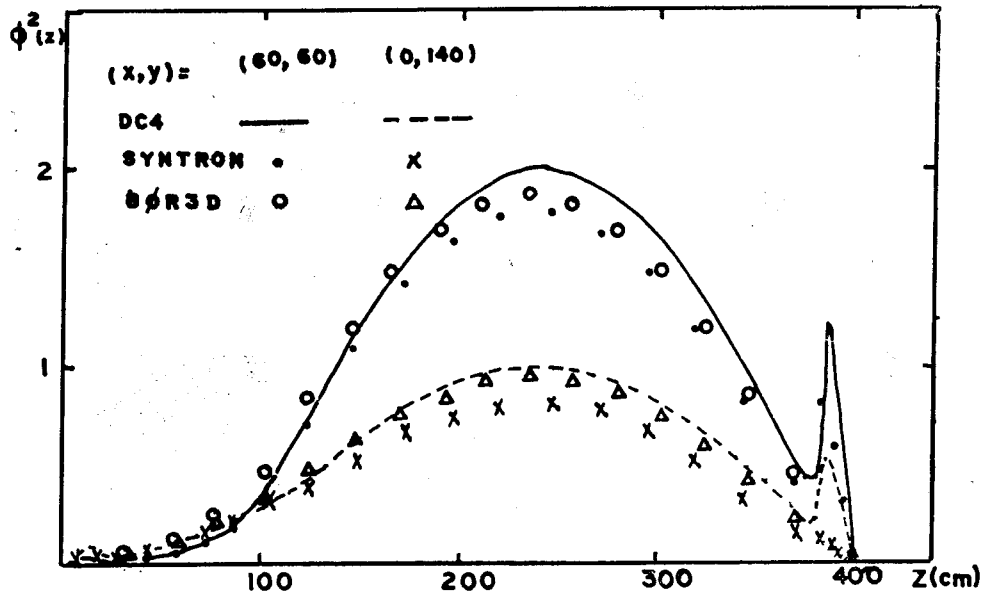


Fig. 5. Axial Thermal Flux Distributions (Case 1); Maximum fast flux point is set equal to 10.

3.1% except this one assembly. The effective multiplication factor agrees well with PDQ. And the BØR2D requires about 2 seconds for the processing on CDC 6400 at KAERI in Korea. It is very small compared to the time of PDQ, because the ratio of numbers of mesh points for BØR2D to those for PDQ is about 1/800 and the processing times are approximately proportional to the numbers of mesh points.

In 3-D IAEA benchmark test, the effective multiplication factor agrees within 0.1%, but the position of the peak power is differentiated with 20cm, and the value

Table 6. Comparison of 3-D Results with Other Methods

	Computer codes	Effective multiplication values	Method	Processor time (min)
case 1	DC4[15]	1.0198	finite difference	35
	SYNTRON[16]	1.0170	synthesis	6.1
	BΦR3D	1.0170	Bφrrensen	0.58
case 2	DC4	1.0032	finite difference	45
	SYNTRON	1.0000	synthesis	6.5
	BΦR3D	1.0010	Bφrrensen	1.13

*DC4 and SYNTRON ; On Burroughs B6700 computer
*BΦR3D ; On CDC6400 computer

is 8% underestimated compared with PDQ calculation. BΦR3D is based on the coarse mesh FDM, thus it cannot predict well the sharf change of the flux along the channel with partially-inserted-control rods. Moreover BΦR3D can only predict the averaged values in the mesh box(20cm×20cm×20cm), so the peak position of the power can be located within the mesh box and the value may be higher than that of BΦR3D. The BΦR3D requires about 30 seconds for the processing, but it is still very small compared with the calculations of 3-D clean core due to the coarse mesh.

In 3-D RISΦ problem test, the effective multiplication factor is between the results of the two other computer codes DC4 and SYNTRON, and the flux distributions are also between the predictions of the two. This shows BΦR3D predictions are well compared with the conventional FDM(DC4) or the synthesis method(SYNTRON), whose mesh sizes are about a half of that for BΦR3D. But we cannot compare the processing time precisely, because DC4 and SYNTRON calculations are perpormed on the Burroughs B6700 computer at RISΦ in Denmark whereas BΦR3D calculations on the CDC6400 at KAERI in Korea.

These tests have shown that, although the modified Bφrrensen model needs the empirical coarse mesh factor and the weighting factor, it requires extremly small execution times and has reasonable accuracy.

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