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Dynamic Buckling of Orthotropic Cylindrical Shells

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直交異方性 圓筒殼의 動的挫屈

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抄 錄

갑자기 작용하는 外壓을 받는 直交異方性圓筒殼의 動的挫屈을 解析하였다. Donnell-Kármán 形의 非線型方程式을 유도하였으며 殼의 初期不完全性도 고려하였다. Galerkin의 方法을 사용하여 運動方程式을 구하고 Runge-Kutta 數值解法으로 非線型方程式을 풀었다. 殼의 直交異方性特性이 처짐-荷重 關係式의 非線型성에 미치는 영향을 검토하였으며 動的挫屈荷重의 判別法을 정의하였다. 本 研究의 結果, 直交異方性圓筒殼은 殼의 初期不完全성에 그리민감하지 않음을 보여주었다.

Nomenclature

b^2	: Elastic parameter($b^2 = E_y/G_{xy} - 2\nu_{yx}$)	w_0	: Initial inward radial displacement
c^2	: Bending rigidity parameter ($c^2 = D_{xy}/D_x$)	x, y, z	: Axial, circumferential, and inward radial coordinates on the median surface
D_x, D_y, D_{xy}	: Bending rigidities of orthotropic shell	Z	: Batdorf parameter($Z = L^2/Rh(1-\nu^2)^{1/2}$)
E_x, E_y, G_{xy}	: Elastic constants of orthotropic shell	β	: Geometrical parameter
$F(x, y, t)$: Stress function	$\epsilon_x, \epsilon_y, \gamma_{xy}$: Axial, circumferential, and shear strains, respectively
h	: Shell thickness	ν_{xy}, ν_{yx}	: Poisson's ratios in orthotropic shell
k^2	: Orthotropic parameter($= E_y/E_x$)	ρ	: Mass density of shell material
k_y	: Nondimensional pressure parameter	ζ, ζ_0	: Nondimensional amplitude of radial and initial displacements, respectively
L	: Shell length	σ_0	: Nondimensional circumferential compressive stress
m, n	: Number of axial and circumferential waves, respectively	$\sigma_x, \sigma_y, \tau_{xy}$: Axial, circumferential, and shear stresses, respectively
P	: Nondimensional load parameter	τ	: Nondimensional time($\tau = \frac{t}{R} \sqrt{\frac{E_x}{\rho}}$)
P_{cr}	: Critical static pressure		
R	: Radius of the shell		
t	: Time		
u, v, w	: Axial, circumferential, and radial displacements of the median surface,		

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1. Introduction

The study of the dynamic stability of struc-

tural elements has been an interested subject nearly forty years. In two survey articles^{1,2} Hoff traced the development of the various problems and made classifications according to the method of loading. One particular type of dynamic loading of practical interest is that applied rapidly. This type of loading has been used in the analysis of columns³, rectangular orthotropic plates⁴, and isotropic shells⁵. A rapidly applied load differs from an impact load in that the time required to reach the critical load is greater than the time required for a pressure wave to travel from one end of the structure to the other. Therefore, for a rapidly applied load, inertial effects in the middle surface of the shell are negligible and only the motion normal to the middle surface needs to be considered.

In recent years the use of anisotropic materials, both homogeneous and composite, has increased. Many applications involve thin plates and shells under loadings which may be failed by buckling. The static stability of orthotropic shells has been analyzed by numerous investigators and representative results are contained in References⁶⁻⁹. However, the dynamic stability of orthotropic cylindrical shells has received very little attention.

It is the purpose of this paper to study the elastic stability of a simply-supported orthotropic cylindrical shell with various initial imperfections under a step-pressure. Since the postbuckling behavior is to be analyzed, the large-deflection shell equations are used. The effect of orthotropy on nonlinearity of deflection-load relation is discussed and the criterion of dynamic buckling load is defined.

2. Equations of Motion

The nonlinear equations of motion are based

on the assumptions commonly used in a Donnell-type formulation which is valid for moderate-length cylinder. In the formulation of these equations the following assumptions are made; a) Donnell's nonlinear shell theory is applicable; b) longitudinal and tangential inertia terms are lower order of importance compared to normal inertia.

Fig. 1 shows the cylindrical shell geometry and coordinate system. Let (u,v) represent the displacement components in the direction of the coordinate axes (x,y) , respectively. The notation w is the inward normal displacement. Also, the stress and strain components are denoted by $(\sigma_x, \sigma_y, \tau_{xy})$ and $(\epsilon_x, \epsilon_y, \gamma_{xy})$, respectively.

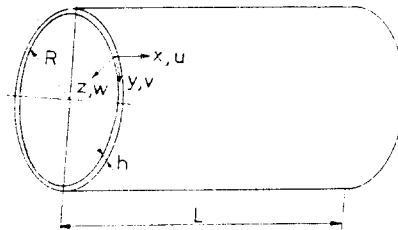


Fig. 1 Shell geometry and coordinates.

Assume the natural axes of the material coincide with the coordinate axes, so that the middle-surface stress-strain relations may be written as

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{1}{1-\nu_{xy}\nu_{yx}} \begin{pmatrix} E_x & \nu_{xy}E_y & 0 \\ \nu_{yx}E_x & E_y & 0 \\ 0 & 0 & (1-\nu_{xy}\nu_{yx}) \end{pmatrix} \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix} \quad (1)$$

and

$$\begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 1/E_x & -\nu_{yx}/E_y & 0 \\ -\nu_{xy}/E_x & 1/E_y & 0 \\ 0 & 0 & 1/G_{xy} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} \quad (2)$$

where E_x, E_y , and G_{xy} denote Young's moduli in the x and y directions and the shear modulus, respectively; ν_{xy} represents the relative contraction in the y direction influenced by the tension in the x direction. Apparently, the

relation $E_x\nu_{yx} = E_y\nu_{xy}$ holds.

The critical load for cylindrical shell under axial compression has been known to be sensitive to initial imperfections in the shell. In order to investigate the effect of the initial imperfections one has to observe the postbuckling behavior and should include the second-order strain-displacement relations

$$\begin{aligned}\epsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \\ \epsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 - \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \\ &\quad - \frac{w}{R} + \frac{w_0}{R}\end{aligned}\quad (3)$$

In Eq.(3) $w(x,y,t)$ is the total displacement normal to the middle surface and $w_0(x,y)$ is the initial displacement normal to the middle surface.

The equation of motion in the direction normal to the middle surface yields

$$\begin{aligned}\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \sigma_x h \frac{\partial^2 w}{\partial x^2} + \sigma_y h \left(\frac{1}{R} + \frac{\partial^2 w}{\partial y^2} \right) \\ + 2\tau_{xy} h \frac{\partial^2 w}{\partial x \partial y} + q - \rho h \frac{\partial^2 w}{\partial t^2} = 0\end{aligned}\quad (4)$$

where ρ denotes the mass density of the material of the shell, t is the time and

$$\begin{aligned}Q_x &= -\frac{\partial}{\partial x} \left(D_x \frac{\partial^2 \bar{w}}{\partial x^2} + D_{xy} \frac{\partial^2 \bar{w}}{\partial y^2} \right) \\ Q_y &= -\frac{\partial}{\partial y} \left(D_{xy} \frac{\partial^2 \bar{w}}{\partial x^2} + D_y \frac{\partial^2 \bar{w}}{\partial y^2} \right)\end{aligned}\quad (5)$$

with $\bar{w} = w - w_0$,

$$\begin{aligned}D_x &= E_x h^3 / 12 (1 - \nu_{xy} \nu_{yx}) \\ D_y &= E_y h^3 / 12 (1 - \nu_{xy} \nu_{yx}) \\ D_{xy} &= \frac{1}{2} (D_x \nu_{yx} + D_y \nu_{xy}) + 2(G_{xy} h^3 / 12)\end{aligned}\quad (6)$$

Now one may represent the stress components in terms of a stress function $F(x,y,t)$ in the form as

$$\begin{aligned}\sigma_x &= \partial^2 F / \partial^2 y & \sigma_y &= \partial^2 F / \partial^2 x \\ \tau_{xy} &= -\partial^2 F / \partial x \partial y\end{aligned}\quad (7)$$

These automatically satisfy the equations of

equilibrium in the plane tangent to the middle surface of the shell when body force is not included.

Thus the compatibility equation yields

$$\begin{aligned}\frac{\partial^4 F}{\partial x^4} + \left(\frac{E_y}{G_{xy}} - 2\nu_{xy} \frac{E_y}{E_x} \right) \frac{\partial^4 F}{\partial x^2 \partial y^2} \\ + \frac{E_y}{E_x} \frac{\partial^4 F}{\partial y^4} = E_y \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right. \\ \left. - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y} \right)^2 \right. \\ \left. + \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} + \frac{1}{R} \frac{\partial^2 w_0}{\partial x^2} \right]\end{aligned}\quad (8)$$

and the equation of motion is

$$\begin{aligned}D_x \frac{\partial^4 \bar{w}}{\partial x^4} + 2D_{xy} \frac{\partial^4 \bar{w}}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 \bar{w}}{\partial y^4} \\ = h \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 F}{\partial y^2} + \left(\frac{1}{R} + \frac{\partial^2 w}{\partial y^2} \right) \frac{\partial^2 F}{\partial x^2} \right. \\ \left. - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{q}{h} \right] \\ - \rho h \frac{\partial^2 w}{\partial t^2}\end{aligned}$$

The deflection function $w(x,y,t)$ is chosen in the separable form

$$w(x,y,t) = f(t) \sin(m\pi x/L) \sin(ny/R) \quad (10)$$

where $f(t)$ is the time-varying amplitude of w , and m and n represent the numbers of longitudinal half-wave and circumferential wave, respectively. This form satisfies the boundary conditions of simply-supported edges.

In analyzing the effects of initial imperfections, the initial and final shapes are usually assumed to be the same basic form¹⁰. Thus, the initial shape of the cylindrical shell will be taken as

$$w_0(x,y) = f_0 \sin(m\pi x/L) \sin(ny/R) \quad (11)$$

where f_0 is the amplitude. This also satisfies the boundary conditions.

Substituting Eqs. (10) and (11) into the compatibility equation, Eq.(8), a particular solution is obtained for a cylinder under external pressure q only:

$$F(x,y,t) = E_y \left[\frac{\beta^2}{32m^2} (f^2 - f_0^2) \cos \frac{2m\pi x}{L} \right.$$

$$\begin{aligned}
 & + \frac{m^2}{32k^2\beta^2} (f^2 - f_0^2) \cos \frac{2ny}{R} \\
 & + \frac{m^2 L^2 (f - f_0)}{R\pi^2 (m^4 + b^2 m^2 \beta^2 + k^2 \beta^4)} \\
 & \times \left[\sin \frac{m\pi x}{L} \sin \frac{ny}{R} \right] - \sigma_0 \frac{x^2}{2} \quad (12)
 \end{aligned}$$

where $\sigma_0 = qR/h =$ nondimensional circumferential compressive stress,

$$\begin{aligned}
 b^2 &= E_y/G_{xy} - 2\nu_{yx}, \\
 k^2 &= E_y/E_x = D_y/D_x, \text{ and} \quad (13) \\
 \beta &= nL/\pi R.
 \end{aligned}$$

Substitution of Eqs. (10), (11) and (12) into Eq. (9) and application of Galerkin's method yield the following nonlinear differential equation of the second order for the time-dependent function f :

$$\begin{aligned}
 \frac{d^2 f}{dt^2} + A(f - f_0) + Bf + H(f^2 - f_0^2)f \\
 = 0 \quad (14)
 \end{aligned}$$

where the following notations are used:

$$\begin{aligned}
 A &= \frac{D_x}{\rho h} \frac{\pi^4}{L^4} (m^4 + 2m^2 c^2 \beta^2 + k^2 \beta^4) \\
 B &= \frac{E_y}{\rho R^2} \frac{m^4}{(m^4 + b^2 m^2 \beta^2 + k^2 \beta^4)} - \frac{\sigma_0}{\rho} \left(\frac{n}{R} \right)^2 \\
 H &= \frac{\pi^4 E_y (m^4 + k^2 \beta^4)}{16\rho L^4}, \quad c^2 = D_{xy}/D_x \quad (15)
 \end{aligned}$$

Omitting the inertia and nonlinear terms and setting $f_0 = 0$, then the resulting equation is the corresponding static buckling equation using linear theory which yields the critical circumferential stress

$$\sigma_{ycr} = (qR/h)_{cr} = \frac{\pi^2 E_x}{12(1 - \nu_{xy}\nu_{yx})} (h/L)^2 k_y \quad (16)$$

where

$$\begin{aligned}
 k_y &= \frac{m^4 + 2c^2 m^2 \beta^2 + k^2 \beta^4}{\beta^2} \\
 & + \frac{12(1 - \nu_{xy}\nu_{yx}) k^2 L^4}{\pi^4 R^2 h^2} \beta^2 \\
 & \times \frac{(m^4 + b^2 m^2 \beta^2 + k^2 \beta^4)}{m^4} \quad (17)
 \end{aligned}$$

If the shell is isotropic, then $\nu_{xy} = \nu_{yx} = \nu$, $E_x = E_y = E$, $c^2 = 1$, $k^2 = 1$, $b^2 = 2$, and the Eq. (16) becomes

$$\begin{aligned}
 \sigma_{ycr} &= \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{h}{L} \right)^2 \left[\frac{(m^2 + \beta^2)^2}{\beta^2} \right. \\
 & \left. + \frac{12Z^2}{\pi^4} \frac{m^4}{\beta^2 (m^2 + \beta^2)^2} \right] \quad (18)
 \end{aligned}$$

where

$$Z = \frac{L^2}{Rh} (1 - \nu^2)^{1/2} \quad (19)$$

Equation (18) is exactly the same as the one from the classical linear theory¹¹. The least critical load, with $m = 1$, is

$$\begin{aligned}
 \sigma_{ycr} &= \frac{\pi^2 E_x}{12(1 - \nu_{xy}\nu_{yx})} \left(\frac{h}{L} \right)^2 \\
 & \times \left[\frac{1 + 2c^2 \beta^2 + k^2 \beta^4}{\beta^2} + \frac{12(1 - \nu_{xy}\nu_{yx})}{\pi^4} \right. \\
 & \left. \times \frac{k^2 L^4}{R^2 h^2} \frac{1}{\beta^2 (1 + b^2 \beta^2 + k^2 \beta^4)} \right] \quad (20)
 \end{aligned}$$

The large deflection equation describing the static postbuckling behavior is obtained by omitting $d^2 f/dt^2$ term from Eq. (14) with $f_0 = 0$:

$$\begin{aligned}
 \left[\frac{D_x}{h} \frac{\pi^4}{L^4} (m^4 + 2c^2 m^2 \beta^2 + k^2 \beta^4) + \frac{E_y}{R^2} \right. \\
 \left. \frac{(m^4 + b^2 m^2 \beta^2 + k^2 \beta^4)}{m^4} - \sigma_y \left(\frac{n}{R} \right)^2 \right] \\
 + \frac{\pi^4 E_y}{16L^4} (m^4 + k^2 \beta^4) f^2 = 0 \quad (21)
 \end{aligned}$$

For real value of f , the quantity in the bracket should be positive.

The equation of motion, Eq. (14) will be in a more convenient form if the following nondimensional parameters are introduced:

$$\begin{aligned}
 P &= q/P_{cr}, \quad \zeta = f/h, \quad \zeta_0 = f_0/h, \\
 \tau &= \frac{1}{R} \sqrt{\frac{E_x}{\rho}} t \quad (22)
 \end{aligned}$$

where P_{cr} is the critical static pressure on an orthotropic cylindrical shell. With these substitution, Eq. (14) becomes

$$\begin{aligned}
 \frac{d^2 \zeta}{d\tau^2} + \frac{\pi^4}{12(1 - \nu_{xy}\nu_{yx})} \frac{R^2 h^2}{L^4} \\
 \times (m^4 + 2m^2 c^2 \beta^2 + k^2 \beta^4) (\zeta - \zeta_0) \\
 + \left[\frac{(m^4 + b^2 m^2 \beta^2 + k^2 \beta^4)}{k^2 m^4} \right. \\
 \left. - n^2 P (P_{cr}/E_x) (R/h) \right] \zeta \\
 + (\pi^4/16) (R^2 h^2/L^4) (m^4 + k^2 \beta^4) \\
 \times (\zeta^2 - \zeta_0^2) \zeta = 0 \quad (23)
 \end{aligned}$$

3. Numerical Examples

To illustrate the application of Eq.(23),

numerical solutions are made for boron/epoxy cylindrical shells of two kinds of orthotropy and at a specified dynamic loading condition. The material properties used⁽¹²⁾ are shown in Table 1.

Table 1 Material properties of the orthotropic cylindrical shells.

Orthotropy I		Orthotropy II	
$E_x=40 \times 10^6$ psi	$\nu_{yx}=0.025$	$E_x=4 \times 10^6$ psi	$\nu_{yx}=0.25$
$E_y=4 \times 10^6$ psi	$k^2=0.1$	$E_y=40 \times 10^6$ psi	$k^2=10.0$
$G_{xy}=1.5 \times 10^6$ psi	$c^2=0.0995$	$G_{xy}=1.5 \times 10^6$ psi	$c^2=0.9953$
$\nu_{xy}=0.25$ psi	$b^2=2.6167$	$\nu_{xy}=0.025$	$b^2=26.1667$

Numerical results are presented for the cylindrical shell with $R/h=100$, $L/R=2$, and $P_{cr}=32.6662$ psi and 126.8827 psi for Orthotropy I and II, respectively.

The loading condition assumed in the present study is a step-pressure. Its time variation is considered as

$$P=0 \text{ at } \tau \leq 0$$

$$P=P_0 \text{ at } \tau > 0$$

and is shown in Fig. 2.

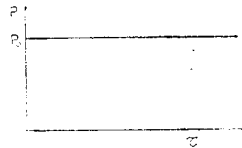


Fig. 2 Shape of loading.

Equation(23) is solved numerically using the fourth-order Runge-Kutta method. The values of initial imperfection $\zeta_0=0.001, 0.01, 0.05,$ and 0.1 are used. The initial conditions for the problem are $\zeta=\zeta_0$ and $d\zeta/d\tau=0$ at $\tau=0$.

Assuming that the mode shapes of the linear problem are also relevant in the nonlinear problem of the imperfect cylinders⁽¹³⁾, the critical number of circumferential waves is chosen to be 6⁽¹⁴⁾ and an asymmetric mode with $m=1$ is selected in the numerical calculations.

For the determination of the critical dynamic step-pressure, the deflection response curves are plotted. In Fig.3, the typical response curves of the orthotropic cylindrical shell under step-pressure are shown. An examination of Fig.3 shows that there is no definite change of stability as also observed in the static analysis of the cylindrical shells. The nonlinear term in Eq.(23) gives greater influence upon the response when the deflection is large. In Fig. 4 and 5, the amplitude-load curves are shown. In these figures one can find the strong(hardening)

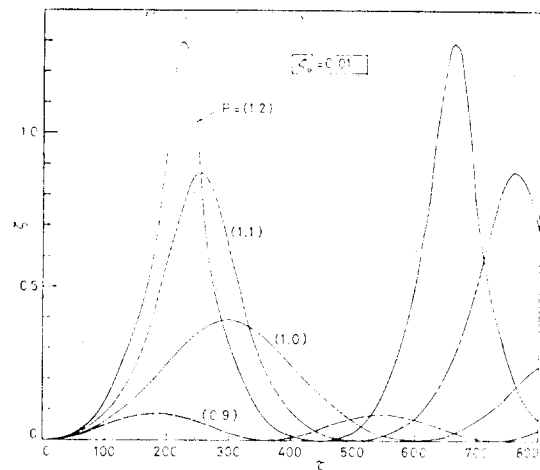


Fig. 3 Nonlinear response curves of the orthotropic cylindrical shell under step-pressure.

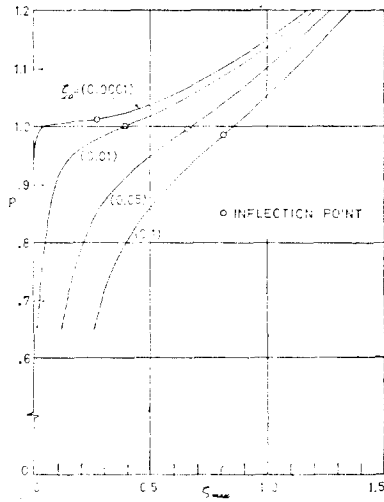


Fig. 4 Amplitude-load curves for a cylindrical shell under step-pressure(orthotropy I).

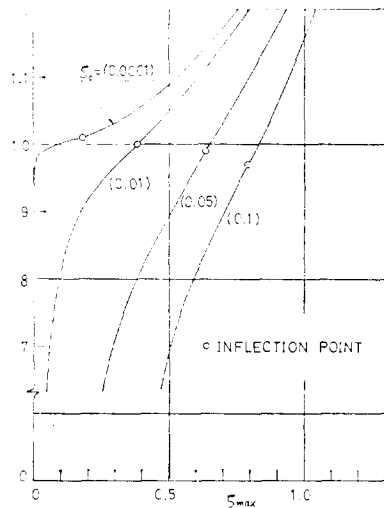


Fig. 5 Amplitude-load curves for a cylindrical shell under step-pressure(orthotropy II).

nonlinearity and the inflection points which show the change of slope. These inflection points are the critical step-pressures of the orthotropic cylindrical shells with initial imperfections. The inflection points are checked with respect to the linear response curve analysis.

In Fig. 6, typical linear midsection-deflection response curves of an orthotropic cylindrical shell under step-pressure are shown. Using the linear theory one can observe that the amplitude of the shell increases with time without bound when the dynamic load is equal to or beyond the inflection-point load.

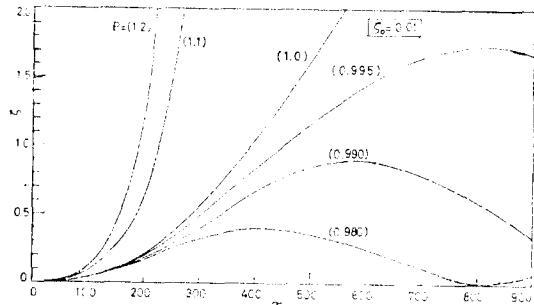


Fig. 6 Linear response curves of the midsection of the orthotropic cylindrical shell under step-pressure.

Table 2 shows the approximate critical values of ratios between the dynamic pressure and the static pressure on the orthotropic cylindrical shells. It is based on the Figs. 4 and 5.

4. Discussion and Conclusion

In the present paper the criterion of dynamic

Table 2 Critical values of pressure ratio.

Initial imperfection	p_{cr}	
	Orthotropy I	Orthotropy II
0.0001(quasi-perfect)	1.010	1.010
0.01	1.000	1.000
0.05	0.995	0.990
0.10	0.985	0.970

buckling load is defined by the inflection point on the amplitude-load curve. This criterion can be applied only to the shell with hardening nonlinearity. To the shell with softening nonlinearity one can observe the limit load⁽¹⁰⁾. The characteristic of nonlinearity depends mainly on the deflection function. Since the deflection function is chosen to approximate the shape of the deformation and to satisfy the boundary conditions, the nonlinearity therefore differs with the loading conditions and shell geometries.

The degree of nonlinearity depends upon the material properties of the orthotropic shell, especially the value of $E_y/E_x=k^2$. For these numerical examples the values of k^2 for the orthotropy II is 100 times greater than that of orthotropy I.

From Table 2, one can observe that the effect of initial imperfection on the shell under step-pressure is not very significant. The phenomenon of the hardening nonlinearity of deflection are shown in Fig.4 and 5. Between the two types of orthotropy, one can find that the nonlinearity is sensitive to the initial imperfection. The orthotropy has same effect on the dynamic as well as the static critical pressure which is about 4 times greater in Case II than Case I. The magnitude of critical pressure ratio between the dynamic and the static shell is less than that of the isotropic cylindrical shell which is given $P_{cr}=1.07^{(15)}$.

In conclusion, a nonlinear shell theory has been employed to investigate the dynamic response of orthotropic cylindrical shells under step-pressure. The shell is permitted to have various initial imperfections. A criterion for the dynamic buckling of the orthotropic cylindrical shells under step-pressures is established in connection with the characteristics of nonlinearity. The numerical results indicate that the buckling load of a cylindrical shell under step-

pressure can be approximated by the static buckling load. The dynamic effect of loading for the orthotropic cylindrical shell is smaller than that of the isotropic cylindrical shell with loading and geometry conditions assumed in the present study.

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