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PRE-GALACTIC CONSTRAINTS ON THE GALACTIC EVOLUTION

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ABSTRACT

The characteristic size and mass of galaxies as pre-galactic constraints on the Galactic evolution are reviewed and the general constraints for their existence in gravitationally bound systems are examined. Implications on the self-similar gravitational clustering are also discussed.

I. INTRODUCTION

In the study of our Galactic evolution, the usual approach is "from center outward" in the sense that we proceed from our vantage point of the solar neighborhood with observational constraints as "boundary conditions" to be checked with the final results.

The present paper is motivated by the recent semi-quantitative arguments (Gott and Thuan 1976, Rees and Ostriker 1977, Silk 1977, Binney 1977) for the existence of the characteristic radius($\lesssim$75kpc) and/or mass($\lesssim 10^{12} M_s$), M_s being the solar mass, of galaxies on the basis of the fragmentation theory of cosmic gas clouds separated from the cosmic substratum after the epoch of recombination $z \sim 1000$ as claimed in the standard Friedmann cosmology.

Since these values are in crude but suggestive accordance with those of actual galaxies, we might as well adopt them as "pre-galactic constraints" on the galactic evolution.

In the present work, the arguments for the characteristic size R_o and mass M_o of galaxies are briefly reviewed in the hierarchical fragmentation picture of Hoyle (1953) in section II, then the general constraints to the existence of R_o and M_o in gravitationally bound systems are examined in section III and their implications on the self-similar gravitational clustering are discussed in section IV.

II. THE CHARACTERISTIC SIZE AND MASS OF GALAXIES

Galaxies are now known to have typical radii $\sim 10^{23}$cm($\sim 10^2$ kpc) and masses $\sim 10^{45}$g($\sim 10^{12}$ M_s) which cannot be determined by purely gravitational process. Einstein's equations of gravitation contain no characteristic scales. Hence it is relevent to recall the Jeans characteristic size or mass of collapsing gas cloud, λ_J and $M_J \simeq \rho \lambda_J^3$ where

$$M_J \simeq (\pi k / G m_p \mu)^{3/2} T^{3/2} \rho^{-1/2} \qquad (1)$$

is the maximum stable mass against the gravitational instability, where m_p is the proton mass and μ the mean molecular weight.

For the gas sphere of mass $M > M_J$, gravitational collapse follows. If the gas, being heated adiabatically during the collapse, can radiate efficiently enough for $T^{3/2}\rho^{-1/2}$ to decrease as ρ rises, then the collapsing gas sphere would undergo "hierarchical fragmention" into smaller and smaller masses until individual fragments became opaque enough to trap their radiation. (Hoyle, 1953; Rees, 1976)

Assuming $T \propto \rho^a$, then $M_J \propto T^{3/2} \rho^{-1/2} = \rho^{3/2(a-1/3)}$ and the fragmentation is possible if $a <$ 1/3, e.g. the isothermal ($a=0$) or cooled collapse ($a<0$), but not the adiabatic collapse ($a=2/3$). The process will stop at the stage when the fragment approaches the black body due to its increased opacity such that radiation rate from the fragment becomes equal to gravitational heating rate or

$$f 4\pi R^2 \sigma T^4 \simeq (G\rho)^{1/2} G M^2 / R \qquad (2)$$

where $f(\leq 1)$ denotes the relative emissivity of the fragment as compared to a black sphere of radius R, and $(G\rho)^{-1/2} \sim$ time scale of gravitational collapse to divide the gravitational binding energy of the fragment $\sim GM^2/R$ (Rees, 1979).

This equation gives the opracity limited Jeans mass M_F as

$$M_F \simeq M_c \mu^{-9/4} f^{-1/2} (kT/m_p c^2)^{1/4} \quad (3)$$

where

$$M_c = (\hbar c/G m_p^2)^{3/2} m_p = \alpha_G^{-3/2} m_p (\sim M_s) \quad (4)$$

and

$$\alpha_G = G m_p^2/\hbar c = 10^{-38.2} \quad (5)$$

This is the gravitational analogue of the fine structure constant $\alpha = e^2/\hbar c$. M_F depends on f and T, and for $T = 10 - 10^4 K$ relevant to the clouds it gives

$$M_F \simeq (1-6) \times 10^{-3} f^{-1/2} \mu^{-9/4} M_c \quad (3\text{-}1)$$

which may be of stellar order for not two small values of f, and also of galactic order for very small f. Each case would be important for the star and galaxy formation respectively.

For the galaxy formation, we start with a cosmic gas cloud with gravitational energy considerably in excess of its thermal energy which separated from the overall cosmic expansion after the epoch of recombination (redshift $z \sim 1000$): hence it is neutral initially, but heated up to ionize during ensuing adiabatic collapse. Now the problem to consider is: What is the necessary condition for a diffuse collapsing gas cloud to be capable of getting rid of the compressional heating of the initially adiabatic collapse to become a gravitationally bound system?

Assuming a uniform cloud of pure hydrogen of temperature T and density ρ, the relevant cooling rate will be expressed for free-free (ff) and free-bound (fb) emission of hydrogen as (Silk, 1977)

$$\Lambda_H = (A_{ff} T^{1/2} + A_{fb} T^{-1/2}) \rho \text{ erg. gr.}^{-1} s^{-1} \quad (6)$$

where

$$A_{ff} = (2^{9/2} \pi^{1/2} e^4 \alpha k^{1/2}) / (3^{3/2} m_e^{3/2} m_p^2 c^2) \quad (7)$$

$$\left.\begin{array}{l} A_{fb} = \alpha^2 m_e c^2 k^{-1} A_{ff} \phi_1(T) \\ \alpha = e^2/\hbar c, \phi_1(T) \sim 1 \end{array}\right\} \quad (8)$$

(slowly varying function of T)

and the competing gravitational heating rate Γ_G (equation (2) or (15)) needs to be compared. Alternatively, the cooling and free-fall (heating) time scale t_c and t_f may be used, where t_c= energy per gram$/\Lambda_H$ and t_f=energy per gram $/\Gamma_G \propto (G\rho)^{-1/2}$, hence

$$t_c/t_f = \Gamma_G/\Lambda_H \quad (9)$$

From equations (2) or (15) and (6)

$$t_c/t_f = \Gamma_G/\Lambda_H \propto \rho^{-1/2} \quad (10)$$

hence at sufficiently low densities, $t_c/t_f \gg 1$, and the cloud will be heated up adiabatically, but as the collapse proceeds $t_c < t_f$ from some instant, the cooling is getting more and more effective, and fragments are capable of surviving and will individually continue to cool and collapse, ultimately fragmenting into stars.

Hence the condition $t_c \sim t_f$ marks a critical epoch in the early collapse. From equations (2), (6) and (9) we have

$$M_J = (k/\mu)^{1/2} (\pi/G)^2 (t_c/t_f) (A_{ff} T + A_{fb}) \quad (11)$$

At $t_c = t_f$ and for $T \leqslant A_{fb}/A_{ff} = T^* = \alpha^2 m_e c^2 k^{-1} = 10^{5.5} K$ (i.e. fb emission is dominant) gives

$$M_o = (k/\mu)^{1/2} (\pi/G)^2 A_{fb} \simeq 5 \times 10^{11} M_s \quad (12)$$

where $\mu = \frac{1}{2} m_p$ and $\phi_1 = 1$ are assumed.

In the other extreme case of $T \gg T^*$ (i.e. ff emission is dominant)

$$M_J = (k/\mu)^{1/2} (\pi/G)^2 A_{ff} T \quad (13)$$

and if we use the virial theorem $T \sim (GM_J/2kR_J)$ at $t_c = t_f$, we have the mass-independent radius (Gott and Thuan 1976)

$$R_o = 73 \text{kpc}. \quad (14)$$

III. CHARACTERISTIC SIZE AND MASS OF GRAVITATIONALLY BOUND SYSTEM

What are the constraints for the gravitationally bound systems to have a characteristic size or mass? Since the gravitation is a long range force following the inverse square law, this force alone can give no characteristic scale or range. If we take the pressure force into account, we get Jeans characteristic size and mass, R_J, $M_J \sim \rho R_J^3$ as the minimum size or mass for the gravitational instability.

But these are not "absolutely" characteristic in the sense that $R_J \propto v_s (G\rho)^{-1/2} \propto (T/\rho)^{1/2}$ and $M_J \propto (T^3/\rho)^{1/2}$ contain 2 variables, hence it changes with T and ρ, where $v_s = (kT/\mu m_p)^{1/2}$ is the sound velocity. To get the real characteristic values, we need 2 more constraints, namely, 1) the cooling rate (Λ)=the gravitational heating rate (Γ_G) (i.e. $t_c = t_f$) 2) the optical thickness across $R_J = 1$ as in the opacity-limited minimum Jeans mass (Low and Lynden-Bell, 1976; Silk, 1977).

For spherically symmetric free fall, the rate of compressional heating per unit mass Γ_G is, neglecting the heating by dissipation of kinetic energy of bulk motion, given by

$$\Gamma_G=(kT/\mu m_p)/t_f \simeq (kG^{1/2}/\mu m_p)\, T\rho^{1/2} \quad (15)$$

In the case of cosmic gas clouds, H-cooling mechanism is due to the binary process; hence $\Lambda_H \propto \rho$ (per gram), but with different T-dependence, i.e. for *ff* emission $\Lambda_H \propto T^{1/2}$ and *fb* emission $\Lambda_H \propto T^{-1/2}$, both having the relevant temperature range (or mass-dependent). It is a curious "coincidence" that $\Lambda_H \propto T^{\pm 1/2}$ for the main cooling mechanism of hydrogen since M-R relation derived from $\Lambda_H=\Gamma$ or $t_c=t_f$ is $R^{1+2\alpha} \propto M^{-(1-2\alpha)}$, for $\Lambda_H \propto \rho T^{\alpha}$, $\Gamma_G \propto (G\rho)^{1/2}$ and $\Lambda_G=\Gamma_G$ give $\rho^{1/2} \propto T^{-\alpha}$ but $\rho \sim M/R^3$, $T \propto M/R$ resulting with the above relation (Rees & Ostriker, 1977).

In a sense, H-cooling of the cosmic cloud seems to conspire in favor of establishing a characteristic size R_o for $\alpha=1/2$ (*ff* emission) and a characteristic mass M_o for $\alpha=-1/2$ (*fb* emission).

On the other hand, in the star formation from the sub-galactic clouds, the cooling mechanism or the opacity-dependence is rather complicated to predict exactly the characteristic size and mass of the last fragment. However, it generally turned out to be roughly of stellar order (Rees, 1976).

In quite a different region ($M \sim M_s$) of M-R diagram, we have M-R relation ($M \sim R^{-3}$) for completely degenerate stars where the degenerate pressure of the electron or the neutron ($P \propto \rho^{1+(1/n)}$, $n=3/2$ for non-relativistic and $n=3$ for relativistic) is dominant over the thermal pressure, and we also have a similar coincidence for $n=3$, R being eliminated in the relation $M^{(n-1)/n}R^{(3-n)/n}=$constant, thus obtaining Chandrasekhar's mass ($\sim 1.4M_s$) for the white dwarf and Oppenheimer-Volkoff mass $0.7-2M_s$, depending on the equation of the state, for the neutron star.

When the effect of general relativity comes inot play, we have another $M-R$ relation for Schrvarzschild radius $R=2GM/C^2$ which may be thought as the limit of virial therem ($2T+\Omega=0$) when $kT \to mc^2$, the gravitation overriding any kind of pressures.

IV. DISCUSSION

The distribution of galaxies in our universe has often been interpreted in terms of a hierarchy of clustering (Layzer, 1954; de Vaucouleurs, 1971).

The statistical data of de Vaucouleurs (1970) shows the internal density-radius relation $\bar{\rho}(r) \propto r^{-\theta}$ with $\theta=1.7$ in the range $r=10^{20}-10^{27}$cm, while the correlation analysis of galaxy correlation function has power-law form $\xi(r) \propto r^{-\gamma}$, with index $\gamma=1.77$ and no preferred scale in the range $r=0.1\,h_o^{-1}-10h_o^{-1}$ Mpc (h_o is Hubble's constant in unit of $100\text{km}s^{-1}$ Mpc^{-1}). The absence of a preferred clustering scale suggests that the hierarchy is "self-similar".

Although a self-similar distribution of matter may be a result of the predominance of scale-free gravitational force over the pressure force at larger scales, there are still several possibilities (Efstathiou, Fall and Hogan 1979):

a) The present distribution reflects initial conditions, but the initial conditions were scale-free: as, for example, with a power law distribution of isothermal perturbations at recombination (Peebles 1974; Press & Schechter 1974; Davis & Peebles 1977).

b) The present distribution reflects the tendency for the matter distribution to evolve, by relaxation and disruption processes, to a self-similar form which is independent of initial conditions (Press & Lightman 1978; Silk & White 1978).

The chronological order in the large scale hierarchy, namely, the evolution from larger clusterings to smaller ones or the other way, has not been settled (Zeldovich, 1970; Press & Schechter, 1974).

The characteristic size and mass of galaxies seem to indicate the self-similarity of the large scale hierarchy to break down at the galactic scales, while the main reason for the self-similarity in the cosmological clustering has been attributed to the near constancy of two dimensionless parameters

$$q=4\pi m\; nG/3H^2,\quad N_J=n(v/H)^3 \quad (16)$$

where m and n are the typical mass and density of gas particles, v the peculiar velocity; thus q represents the deceleration and N_J related to the number of particles in M_J (Press & Schechter, 1974).

It is conjectured that the most readily observable parameters characterizing the large scale clusterings—the masses and radii of galaxies and clusters—are determined by gas-dynamical processes occuring after the separation of cosmic clouds from general expansion of substratum rather than being sensitive to the precise spectrum of irregularities that survives through recombination (Rees & Ostriker, 1977).

Peebles (1974) showed, for the assumed power-law spectrum of initial irregularities with diameter$\sim k^{-1}$ in the Einstein-de Sitter universe

$$(\delta\rho/\rho)_k^2 \propto k^{n+3} \quad (17)$$

that the covariance function and the internal density of size r are also of power-law form $\xi(r) \propto \bar{\rho}(r) \propto r^{-\gamma}$ with

$$\gamma = (9+3n)/(5+n). \quad (18)$$

For $n=0$ (i.e. the flat spectrum), $\gamma=1.8$, but some authors have suggested slightly different value of $n=1$ (Gott & Rees, 1975).

In this context, it is to be noted that the free-fall of a gas sphere with initially uniform density tends to a self-similar collapse asymptotically at $t \rightarrow t_f$ (just before singularity) with a density profile $\rho(r) \propto r^{-12/7}$ (Penston, 1969), where the latter is rather close to the observed internal density-radius relation $\bar{\rho}(r) \propto r^{-1.77}$. But this resemblance might be only superficial, because of assumption of spherical symmetry of the free fall and, above all, the neglect of fragmentation which is essential for the hierarchical evolution.

Although we now have an understanding on the existence of characteristic size and mass of galaxies, the less readily observed features of the large scale structure, such as the luminosity function and mass spectrum of galaxies, and their possible relations with the conditions in pre-galactic evolution etc. are largely left for current and future investigations.

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