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# PHYSICAL CONDITIONS IN DARK INTERSTELLAR CLOUDS: MAGNETIC FIELD STRENGTH AND DENSITY*

S. S. HONG
Department of Astronomy, College of Natural Sciences, Seoul National University

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## ABSTRACT

In order to know how the magnetic field increases with density in interstellar clouds, we have analyzed observations of extinction and polarization for stars in the $\rho$ Oph molecular cloud complex. The size of grains in dense parts of the complex is estimated to be larger than the ones in diffuse interstellar clouds by about 15 percent in radii. Employing the Davis-Greenstein mechanism for grain alignment with this estimated grain size, we have put constraints on the exponent in the field-density relation $B\propto n^x$: $1/5\lesssim x\lesssim 1/3$. It is concluded that magnetic field in gravitationally contracting clouds increases less steeply than the classical expectation based on the approximation of isotropic contraction with complete frozen-in flux.

## I. INTRODUCTION

With the problem of angular momentum, the magnetic field has remained as a major stumbling block in understanding stellar formation processes (Mestel, 1977). If a cloud undergoes isotropic contraction with the magnetic flux being completely frozen-in, the strength of magnetic field, B, would increase with density $n$ as $B\propto n^{2/3}$. Since the critical mass for fragmentation in magnetized clouds is proportional to $[B/n^{2/3}]^3$, it is invariant during the gravitational collapse. The existence of magnetic field in interstellar clouds is thus to act as a deterrent for clouds from fragmenting into smaller condensations, until the density becomes as high as $10^9\sim10^{10}$ cm$^{-3}$ where ambipolar diffusion is believed to be effective in decoupling the magnetic field from matter. On the contrary to this theoretical notion, recent infrared observations reveal numerous subcondensations spread all over massive cloud complexes even with densities as low as $10^4\sim10^5$ cm$^{-3}$. Apparently, at such low densities, magnetic field diffuses out and cloud undergoes fragmentation processes. It is, thus, of crucial importance to know how the magnetic field strength varies with the density in gravitationally collapsing clouds.

In principle, observations of Zeeman split in some molecular lines could provide a unique method for direct determination of magnetic field strength. In reality, results are quite conflicting: At one extreme, Verschuur (1971) puts an upper limit of only 4 *micro* Gauss from 21-cm observations of the $\rho$ Ophiuchi cloud. At the other extreme, Clark and Johnson (1974) set 18 *milli* Gauss from SO observations. We may understand the large difference between HI and SO results, because SO observations tend to sample core parts and HI outskirts of the cloud complex. However, with the same OH molecule, Turner and Verschuur (1970) set an upper limit of 130 micro Gauss, and later Crutcher et al. (1975) lowered the limit to 50 micro Gauss.

These conflicting results make it difficult to derive a meaningful relation between $B$ and $n$ directly from observations. One, then, seeks semiempirical means to determine the relation. For example, Scalo (1977) considered the thermal equilibrium of a cloud heated by ambipolar diffusion (magnetic ion slip), and put 0.55 as an upper limit to $x$ in the $B\propto n^x$ relation by demanding the temperature rise due to the field diffusion be consistent with observations of rota-

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tional lines of interstellar molecules.

In the present paper, we shall derive another semiempirical constraints on the field *vs* density relation by analyzing observations of interstellar extinction and polarization. In section II we shall examine optical observations of $\rho$ Ophiuchi cloud complex to determine the dust characteristics and hydrogen number density. With the core-mantle model of interstellar grains by Hong and Greenberg (1980), we shall derive in section III the mangetic field and density relation. In the last section, we shall summarize the results and their implications.

## II. OBSERVATIONS RELATED TO DUST IN $\rho$ OPH CLOUD

The dark cloud near $\rho$ Oph is selected for the present study, for it has been studied most extensively in almost all observational aspects. In particular, Carrasco, Strom and Strom (1973) made a thorough observational investigation on the dust in the region. Here, we summarize their results, and analyze them to derive dust properties.

### a) Extinction Observations

Color excess ratio, $E_{VK}/E_{BV}$, against $E_{VK}$ is plotted in Fig. 1 for 13 stars seen through the $\rho$ Oph cloud. Since the total extinction $A_V$ at the visual wavelength is about 1.1 $E_{VK}$ under normal interstellar conditions, the ratio of total to selective extinction $R=A_V/E_{BV}$ can be approximated by 1.1 $E_{VK}/E_{BV}$. The figure clearly indicates that dust properties vary from place to place in the cloud. However, the total to selective extinction ratio remains more or less constant once the total extinction reaches about 2 magnitudes. Thus, we may adopt $R=3.8$ and $A_V=2.5$ mag as representative values for dust in the region of the cloud where stars can be seen.

### b) Dust Size

Now, we may analyze these observations in terms of a grain model. Using the forward scattering amplitude function based on the ray approximation (van de Hulst, 1957), Hong and Greenberg (1978) gave relations between $R$ and mean size $\langle a\rangle$ of grains for various types of size distribution. They showed that $\exp[-(a/\alpha)^3]$-type distribution best yields the most commonly observed combination of $R=3$ and the maximum polarization wavelength $\lambda_{max}=5500$Å.

Using Fig. 2, which is taken from Hong and Greenberg (1978), one can easily estimate the characteristic size of grains once $R$ is known: $R=3$ corresponds to $\bar{\eta}_V=1.25$ while $R=3.8$ requires $\bar{\eta}_V=1.44$. Therefore, grains in the $\rho$ Oph cloud are generally larger than ones in diffuse interstellar space by about 15 percent in radii.

### c) Hydrogen Number Density

The observationally determined ratio of hydrogen column density, $N_H$, to the total extinction at the visual wavelength has been stated as

$$N_H/A_V=2\times10^{21}\ \mathrm{mag^{-1}\ cm^{-2}}, \tag{1}$$

where $N_H$ includes all forms of hydrogen (Spitz-

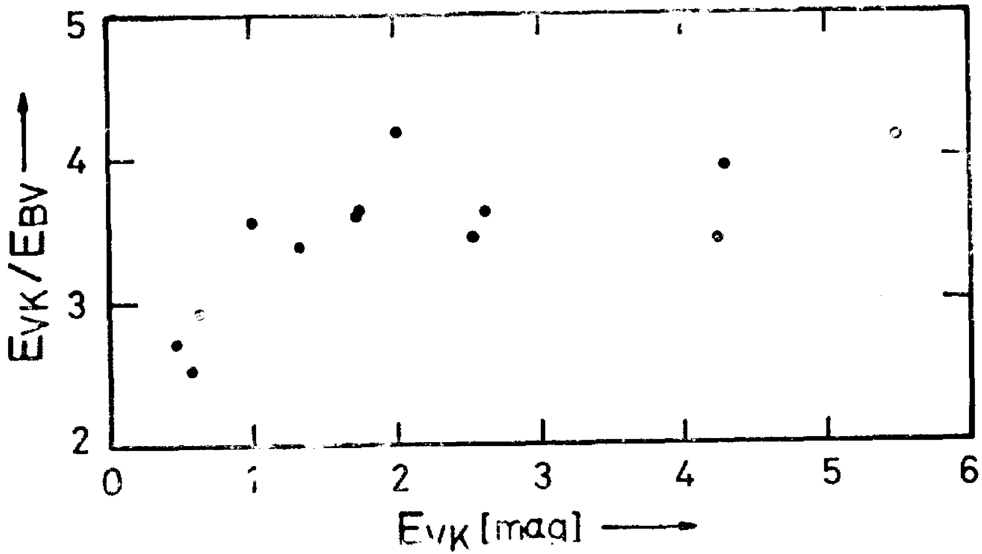

**Fig. 1.** - Color excess ratio $E_{VK}/E_{BV}$ is plotted against $E_{VK}$ for 13 stars given in Carrasco, Strom and Strom (1973). Please note that $A_V\simeq1.1\ E_{VK}$.

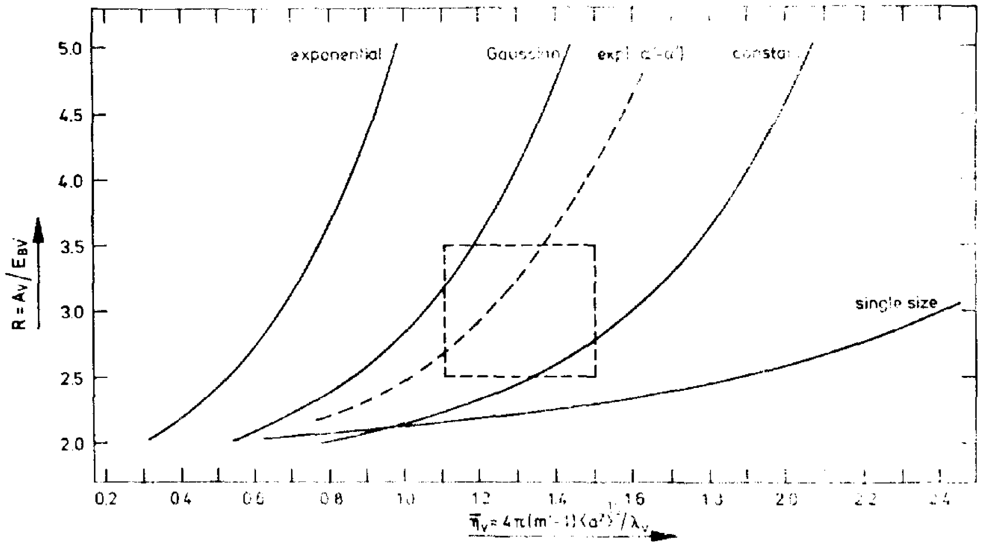

**Fig.** 2. - For various types of size distribution the expected ratio $R$ of total to selective extinction is plotted against the phase lag, $\bar{\eta}_V = 4\pi(m'-1)\langle a^2\rangle^{1/2}/\lambda_V$ of the visual wavelength after passing through the center of grain with mean size $\langle a^2\rangle^{1/2}$. The rectangular box characterizes the diffuse interstellar space.

er, 1978). However, it is not quite right to apply this ratio directly to the dense molecular clouds. One should realize that this ratio is based on observations of mostly diffuse interstellar clouds. Grain characteristics are not the same for both types of cloud.

Fifteen percent increase in grain size should yield an enhancement of the extinction cross-section by about 30 percent. Thus, we revise the ratio as

$$N_H/A_V = 1.5 \times 10^{21} \text{ mag}^{-1} \text{ cm}^{-2}, \qquad (2)$$

and we will use equation (2) in the following analysis. In deriving equation (2) we have assumed that the number of hydrogen per one dust particle is the same for both types of cloud. As long as grains do not coagulate each other, this assumption must hold true. Further revison to the equation would be necessary for very dense parts of molecular clouds where coagulations seem to happen.

Equation (2) with $A_V = 2.5$ mag gives us the total hydrogen column density $N_H = 3.8 \times 10^{21}$ cm$^{-2}$, which in turn yields $n = 400$ hydrogens per cm$^3$ with a path-length of 3 pc. The cloud extends over 1.5 degrees of arc, which corresponds to about 4 pc at the distance of 165 pc (Garrison, 1967). However, the morphology of the cloud suggests that the thickness along the line of sight should be less than the width at least by a factor of two. This raises the mean number density to about 800 cm$^{-3}$.

One may derive the mean number density of hydrogen from observations of microwave molecular lines. Molecules like $H_2CO$, $NH_3$, CS and SO (Greenberg, Minn and Tielens, 1979; Myers and Ho, 1975; Morris et al., 1973; Penzias et al., 1972) yield a few to ten times $10^3$ hydrogen molecules per cm$^3$ for the mean density. These are likely to be overestimates for mean densities in the region where stars are observed, because the very existence of such molecules itself implies a significant amount of ultraviolet shielding. On the other hand, one obtains about $10^3$ hydrogen molecules per cm$^3$ from the necessary collision rates to account for the observations of CO line(Encrenaz,1974; Penzias et al., 1972). Considering strong density contrast between inner and outer parts of the cloud (Bok, 1956), we may interprete the radio observations as giving 500 hydrogens per cm$^3$ for a representative of the mean density in the region where optical extinction and polarization measurements are available. Thus, our analysis based on the interstellar extinction measures gives a fair agreement with the radio observations.

## III. MAGNETIC FIELD STRENGTH AND DENSITY

a) Davis-Greenstein Grain Alignment

The consistency we have obtained in the previous section lends a support for our further use of the grain model in the determination of magnetic field strength from observations of optical polarization. Recently, Hong and Green-

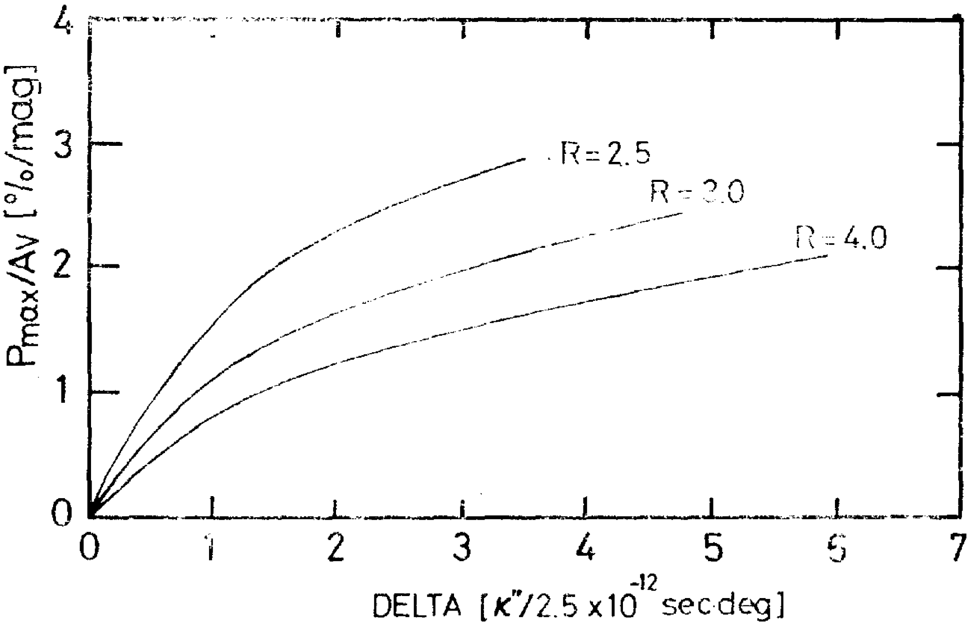

**Fig. 3.** – The ratio of expected maximum polarization to total extinction at the visual wavelength, $P_{max}/A_V$, is plotted against the ratio of collisional disalignment time scale to magnetic alignment time scale evaluated for the mean sized grain.

berg (1980) performed an extensive model calculation for interstellar polarization with the Davis-Greenstein mechanism for grain alignment. The most important parameter in the alignment mechanism is the ratio of the collisional disalignment time scale to the magnetic alignment time scale:

$$\delta=\left[\frac{\pi}{2m_H k}\right]^{1/2}\kappa''\frac{B^2}{na}\frac{1}{T_d}\frac{1}{\sqrt{T}},\quad(3)$$

where $T_d$ is the dust temperature, $T$ the gas kinetic temperature, and $\kappa''$ is related to the imaginary part of the magnetic susceptibility of the grain material with the other symbols having their usual meanings (Davis and Greenstein, 1951; Spitzer, 1978). According to their model calculations, the maximum polarization to the visual extinction ratio, $P_{max}/A_V$, produced by an ensemble of partially aligned particles is directly related to $\langle\delta\rangle$ evaluated for the mean grain size $\langle a\rangle$ in the given size distribution. And the mean particle size determines the ratio of total to selective extinction. Thus, as shown in Fig. 3, one may represent the ratio of maximium polarization to total extinction as a function of $\langle\delta\rangle$ for a number of different $R$ values. The ordinate represents $P_{max}/A_V$ in units of percent per magnitude, and the abscissa represents $\langle\delta\rangle$ in units of $\kappa''/2.5\times10^{-12}$ sec deg. The use of this unit for the abscissa is due to uncertainties in the magnetic property of grain material. In recent years, there are a growing number of doubts on the earlier estimate of $\kappa''=2.5\times10^{-12}$ sec deg (Purcell, 1979 and references therein). However, this uncertainty would not affect our final result, because we take a ratio of $\langle\delta\rangle$'s for two different regions.

b) Polarization to Extinction Ratios

There are stars with $P_{max}/A_V$ even up to 6 %/mag in diffuse interstellar clouds. However, these stars of extremely high polarization may either have some intrinsic polarizations or incorrect extinction measures. A very strict upper limit can be set at about 3%/mag for the ratio, and most frequently observed value for the ratio is 1.5%/mag (Serkowski, Mathewson and Ford, 1975). Thus, we will characterize the diffuse interstellar clouds with $P_{max}/A_V\simeq$ 1.5%/mag and $R\simeq3$. Putting these values in Fig. 3, we have $\langle\delta\rangle\simeq1.8$ for diffuse clouds.

On the other hand, the same ratios for stars in the $\rho$ Oph cloud are distributed mostly between 1 and 1.5%/mag, which can be seen from Fig. 4. Thus, $\langle\delta\rangle$ for grains in the cloud ranges from 1.2 and 3. The lower bound is more appropriate to the dense part of the cloud where $R$ is about 4.

c) Field Strength and Density Relation

From equation (3) we now have

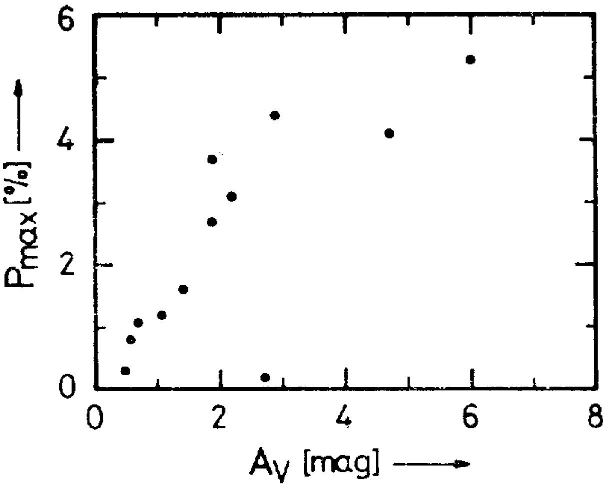

**Fig.** 4. - The observed ratio of $P_{max}/A_V$ is plotted against $A_V$ for 12 stars in the $\rho$ Oph cloud region. The data are taken from Carrasco, Strom and Strom (1973).

$$\frac{\langle\delta\rangle}{\langle\delta'\rangle}=\left[\frac{B}{B'}\right]^2\left[\frac{n'a'}{n\ a}\right]\left[\frac{T_d'}{T_d}\right]\left[\frac{T'}{T}\right]^{1/2}, \quad (4)$$

where primed variables represent quantities in diffuse cloud. From now on all the variables should be understood as appropriately averaged values. Introducing $B\propto n^x$ into equation (4), we obtain an equation for the exponent $x$:

$$\left[\frac{n}{n'}\right]^{2x-1}=\frac{a}{a'}\ \frac{T_d}{T_d'}\left[\frac{T}{T'}\right]^{1/2}\ \frac{\langle\delta\rangle}{\langle\delta'\rangle}. \quad (5)$$

Since the efficiency factor for emission cross-section is proportional to $a/\lambda$ for grains of sub-micron sizes at the far infrared wavelength, the cooling rate should be proportional to $aT_d^5$. Therefore, the dust temperature is proportional to $a^{-1/5}$ for a given heating rate. Dusts inside the $\rho$ Oph cloud itself shield out diffuse stellar radiation from outside the cloud. The reduction in heating rate by this self-shielding can be approximated by $\exp[-A_V]$. We thus have

$$\frac{T_d}{T_d'}=\left[\frac{a}{a'}\right]^{-1/5}\exp[-A_V/5]. \quad (6)$$

One further simplification is possible: Dense molecular clouds are generally in contact with diffuse clouds. Hence, they are in approximate pressure equilibrium. This can be easily inferred from Fig. 1, which shows a smooth change of $R$ from 3 for less reddened stars to 4 for heavily reddened ones. The same can be said from Fig. 4, since the ratio of maximum polarization to total extinction for less reddened stars in the $\rho$ Oph cloud is same as its corresponding value for the diffuse cloud. Therefore, we may use the pressure equilibrium condition

$$nT = 2n'T' \quad (7)$$

to eliminate the kinetic temperature ratio ($T/T'$) from equation (5). Here, the factor 2 has been introduced in order to take into account for the difference in mean molecular weight between the two cloud phases.

Combining equations (6), (7) and (5) with $a/a'=1.15$ and $A_V=2.5$, we finally obtain

$$\left[\frac{n}{n'}\right]^{2x-1/2}=0.96\frac{\langle\delta\rangle}{\langle\delta'\rangle}. \quad (8)$$

Taking logarithms to both sides of equation(8) yields an approximate solution for $x$

$$x\simeq\frac{1}{4}+\frac{1}{2}\frac{\log(\langle\delta\rangle/\langle\delta'\rangle)}{\log(n/n')}, \quad (9)$$

where we have ignored the term due to 0.96 in equation(8). Without substituting numerical values for $\langle\delta\rangle/\langle\delta'\rangle$ and $n/n'$, we can say that most $x$ is not far from 1/4, because the second term in the right hand side of equation (9) is generally smaller than the first term. The hydrogen number density $n'$ in diffuse interstellar cloud is certainly less than 100 $cm^{-3}$, but it is not less than 10 $cm^{-3}$ (Spitzer, 1978). With $n=$ 500 $cm^{-3}$ and $n'=10$ $cm^{-3}$, equation (9) yields $x\simeq0.20$ for $\langle\delta\rangle/\langle\delta'\rangle=1.2/1.8$ and $x\simeq0.32$ for $\langle\delta\rangle/\langle\delta'\rangle=3/1.8$. We have put a quite strict constraint on the exponent $x$ in the $B\propto n^x$ relation as

$$\frac{1}{5}\lesssim x\lesssim\frac{1}{3} \quad (10)$$

with $x=1/4$ being the most likely value for the $\rho$ Oph cloud.

## IV. DISCUSSION

The principal conclusion of the present study is that the magnetic field does not increase as fast as $n^{2/3}$ in moderately dense molecular clouds. Uncertainties in parameters characterizing the $\rho$ Oph cloud will modify the constraints on the exponent; however, it is definitely smaller than the value for isotropic contraction with the field being completely coupled with the matter.

This result yields some implications on theory of star formation and role of magnetic field in collapsing clouds. The magnetic field is, after all, not so strong a deterrent for fragmentation and may give ways to form condensations at early phase of cloud collapse. The slow increase of magnetic field also explains some of the negative results in attempts to observe Zeeman split from OH masers, and density estimates for OH masers should be modified from the view point of slow field increase.

Recently Mouschovias(1978) treated the self-gravitating magnetized clouds with a series of equilibrium configurations. He showed that the flow following down field lines reduces the exponent considerably below 2/3. His constraints on $x$ lie between 1/3 and 1/2, while Scalo's (1977) empirical upper limit for $x$ is 0.55. Comparison of our result with Mouschovias' and Scalo's suggests that diffusion of field lines seems to occur as the matter preferentially flows down the field lines. Morphology of molecular clouds with respect to magnetic field configurations should bear witness to the dynamic nature of magnetized cloud-collapse.

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