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A Model of Organizational Decision Process

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ABSTRACT

The generalized goal decomposition model proposed by Ruefli as a single period decision model is presented for the purpose of a review and extended to make a multiple period planning model. The multiple period planning model in the three level organization is formulated with linear goal deviations by introducing the goal programming method. Dynamic formulation using the generalized goal decomposition model for each single period problem is also presented. An iterative search algorithm is presented as an appropriate solution method of the dynamic formulation of the multiple period planning model.

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I. INTRODUCTION

Major issues relating to the choice of resource allocation mechanisms appear in the decision making process because of the huge size and complexity of most organizations. Decentralization will be or should be undertaken due to conditions of uncertainty and lack of information. An organization is rarely presented with a clear-cut objective function. It's objective functions do not remain constant but grow out of its experiences and out of the changing external environment. Decentralization almost inevitably involves some conflict of point of view between the central authority and lower decision making levels. The conflict, however, may lead to better results than that occurring in apparently clear-cut centralized processes.

Ruefli has proposed the generalized goal decomposition model with a three level organization as a single period model. The organization consists of a central unit, management units and operating units. Each level is vertically interrelated by a specific information flow. Levels of organization are assumed to be horizontally independent.

In fact, most of the organizations are concerned with a planning horizon that is more than a single period in length. If the planning horizon is restricted to a single period, then it is actually difficult to make an appropriate evaluation of the alternatives because there can be a lack of relationship and information between planning periods and between levels of the organization.

In this paper, the generalized goal decomposition model is extended to the multiple period planning model of an organization in order to get more realistic alternatives into the decision making process.

Ⅱ. GENERALIZED GOAL DECOMPOSITION MODEL

The generalized goal decomposition model proposed by Ruefli is a goal programming model of an organization where the solutions depend upon the structure of the organization.

Refli formulates the problem for the whole organization in terms of the problems of the organizational subunits and does so in such a manner that it is impossible to speak of the global objective except in terms of the objectives of the organizational subunits. This is primarily because goal progromming deals with vector optimizations.

The model involves a three level organization. The central unit represents the top level of the organization is responsible for setting goals and allocating global resources to the rest of the organization. The management units from the middle level of the organization allocate the local resources under their control within the bounds established by the central unit. The operating units are the lowest organizational units and they are responsible for generating project proposals for the superordinate management units.

In terms of a complex organization, this is a gross simplification, but it will enhance the clarity of the presentation and may, in fact, be a reasonable model of the organization of a highly

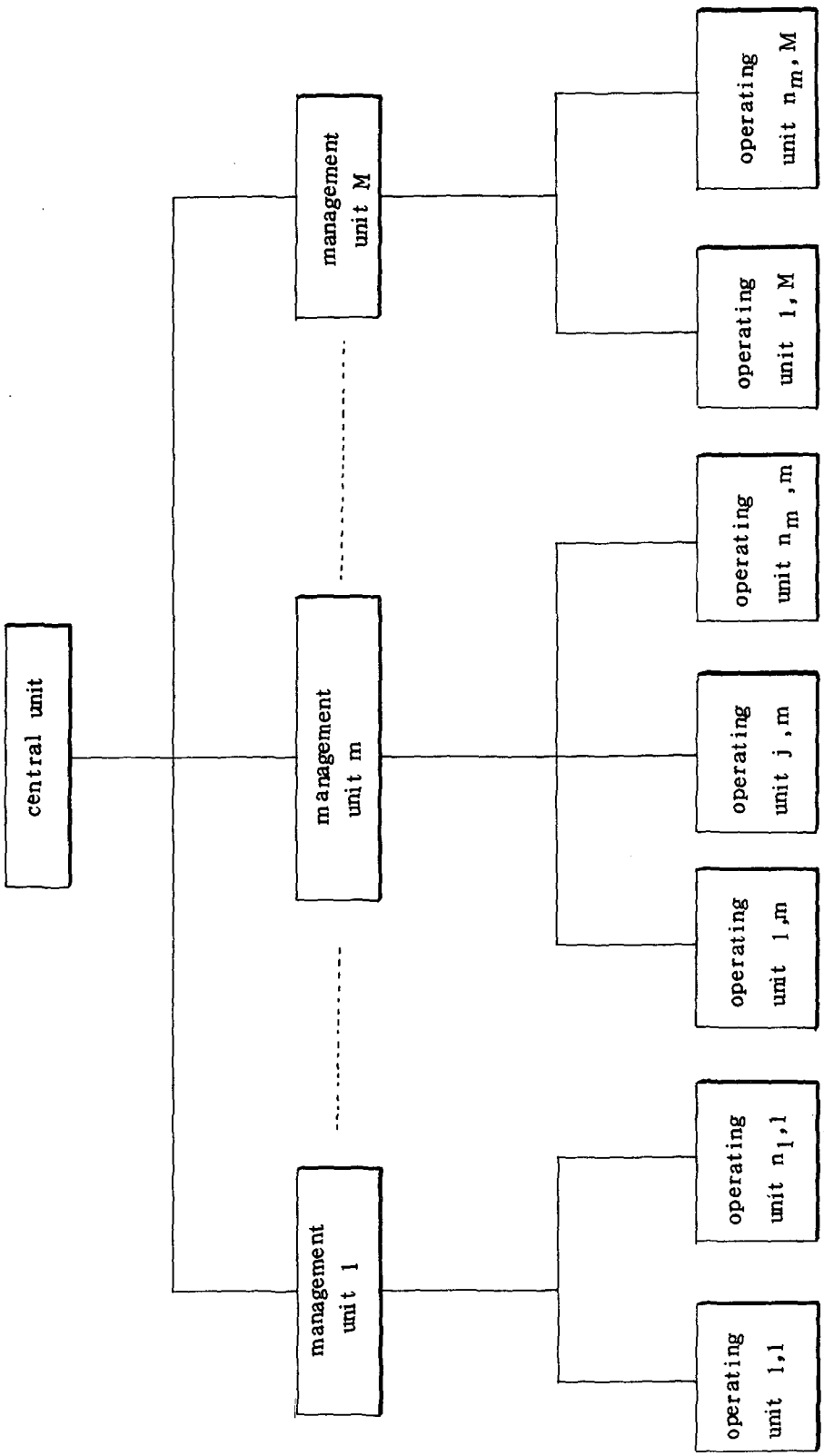
central unit
management unit 1
management unit m
management unit M
operating unit 1,1
operating unit n1,1
operating unit 1,m
operating unit j,m
operating unit nm,m
operating unit 1,M
operating unit nm,M

Fig. 1

aggregated level. The structure of the organization under consideration can be mapped onto the structure presented in Fig. 1.

The problem statement of the central unit is as follows:

$$\max \sum_{m=1}^{M} \left[\pi_m \right] \left[G_m \right]$$

$$\text{subject} \sum_{m=1}^{M} \left[P_m \right] \left[G_m \right] \lesseqgtr \left[G_o \right] \qquad (1)$$

$$G_m \geq 0$$

where,

π_m is a vector of shadow prices generated by the m^{th} management unit,

G_m is a vector of goal levels assigned to the m^{th} management unit,

P_m is a matrix of the m^{th} management unit's joint utilization of organizational resources,

G is a vector of global resources and requirements.

The central unit generates goals that maximize the inputed values of goals as determined by all management units subject to resource and requirement constraints.

The problem statement of the m^{th} management unit is presented in the following formulas:

$$\min \left[W_m^+ \right] \left[Y_m^+ \right] + \left[W_m^- \right] \left[Y_m^- \right] \qquad (2)$$

subject

$$\begin{bmatrix} A_m \end{bmatrix} \begin{bmatrix} X_m \end{bmatrix} - \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} Y_m^+ \end{bmatrix} + \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} Y_m^- \end{bmatrix} = \begin{bmatrix} G_m \end{bmatrix}$$

$$0 \leq X_m \leq 1 \ , \quad Y_m^+ \ , \quad Y_m^- \geq 0$$

where,

W_m^+ , W_m^- are vectors of weights determined a priori for positive and negative deviations from goals,

Y_m^+ , Y_m^- are vectors of positive and negative deviations from goal vector G_m,

A_m is a matrix of attributes of project proposals for all n_m subordinate operating units,

X_m is a vector of activity levels for project proposals,

I is an identity matrix.

The problem statement indicates that the objective of the m^{th} management unit is to minimize the weighted sum of the deviations from the goals subject to the technology of achieving the management unit's goals. This technology describes how the n_m operating units interact to achieve management unit's goals. The assumption is made that the weights on the goal deviations are derived from management policy and are assigned a priori by the central unit.

The problem statement of the jm^{th} operating unit is illustrated by the following formulation:

$$\min \begin{bmatrix} \pi_m \end{bmatrix} \begin{bmatrix} A_{jm} \end{bmatrix} \qquad (3)$$

$$\text{Subject} \quad \begin{bmatrix} D_{jm} \end{bmatrix} \begin{bmatrix} A_{jm} \end{bmatrix} \geq \begin{bmatrix} F_{jm} \end{bmatrix}$$

$$A_{jm} \geq 0$$

where,

A_{jm} is a vector of variables representing activity levels of jm^{th} operating unit,

D_{jm} is a matrix of technological coefficients,

F_{jm} is a vector of minimum output levels of project.

The problem of the jm^{th} operating unit is to minimize the inputed cost of their project proposals subject to the physical technology of that production.

The complete model of planning for the organization is summarized in the following problem statement.

Unit	Formulation
central unit	(1)
management units	(2) $m = 1, ---, M$
operating units	(3) $m = 1, ---, M$ $j=1, ---, n_m$

These $i + M + \sum_{m=1}^{M} n_m$ problem statements comprise the generalized goal decomposition model as a single period model.

The solution procedure for the generalized goal decomposition model is based on an iterative process. This process commences with the central unit setting initial goal levels $G_m(0)$ for the M management units. This goal vector contains resource and requirement goals. The initial goal vector may reflect the current operating conditions or mature judgment in forecasting goal levels.

Each of the management units, with a previous technological coefficient matrix and a set of goals, seeks to minimize the weighted deviations from the respective goals. At optimality, the dual variables to this problem solution are the shadow prices. Each management unit responds to the central unit with a proposal of shadow prices. This vector of shadow prices is provided to each of the operating units of the respective management unit and to the central unit.

Having received a vector of shadow prices for this iteration, the operating units seek to minimize the inputed cost of their proposals. This optimal solution yields a new proposal for their management unit. This new proposal is sent to their respective superordinate management unit for the next iteration.

After receiving the shadow prices from the subordinate management units, the central unit uses the shadow prices to generate new sets of goal levels G_m. These goal vectors are transmitted to the management units again.

Provided with a new technological coefficient matrix by their respective operating units and goal levels by their central unit, each of the management units optimizes the revised program generating a new vector of shadow prices. It is important to remember that shadow prices are variables for the management units while they are considered fixed by the central unit and operating units.

This process continues until the deviations from the management unit goals are within prescribed tolerance limits or at a minimum

and no adjustment of goal levels on the part of the central unit or adjustment of goal levels on the part of the central unit or modification of proposals on the part of the operating units will yield a net decrease in the deviations from the goal levels for the organization as a whole. The iterative solution procedure is described in Fig. 2.

Ruefli indicates that there are three types of externalities possible in the generalized goal decomposition model. The first type involves interdependence among operating units subordinate to the same management unit. The model assumes that these interdependencies are expressed in the constraints of the relevant management unit's problem statement. The second type of externality is present when the goal levels of the m^{th} management unit are dependent on the goal levels of one or more of the other management units. The constraints of the central unit are assumed to express these relationships. The third type of externality arises when levels or project characteristics are interrelated for operating units with different superordinate management units. The central unit then passes down upper limits on goal levels in the initial conditions in order to rectify this problem.

If there are no technological and goal dependencies, then the entire process will reach an optimum in a finite number of iterations.

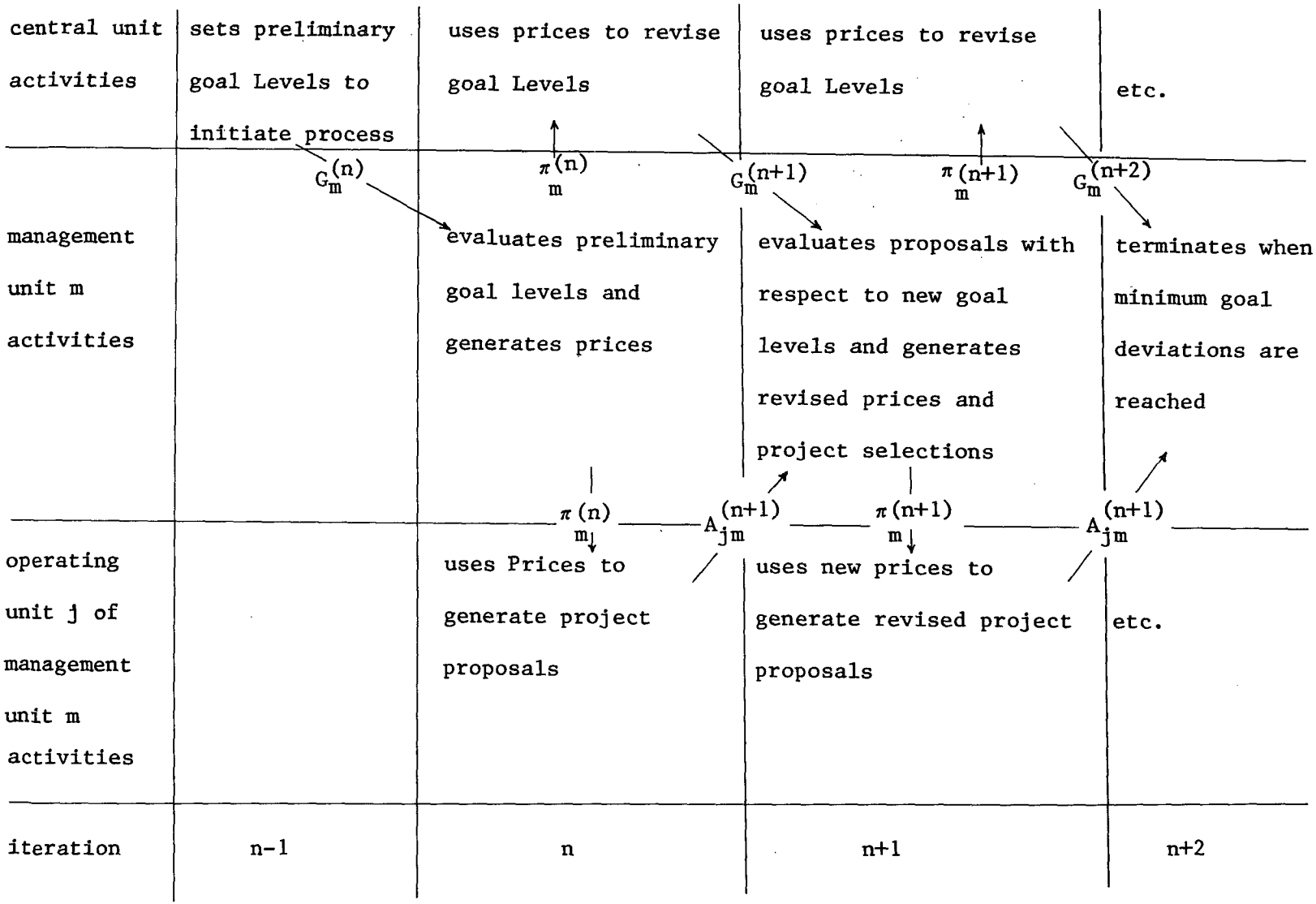
central unit activities
sets preliminary goal Levels to initiate process
uses prices to revise goal Levels
uses prices to revise goal Levels
etc.
$G_m^{(n)}$
$\pi_m^{(n)}$
$G_m^{(n+1)}$
$\pi_m^{(n+1)}$
$G_m^{(n+2)}$
management unit m activities
evaluates preliminary goal levels and generates prices
evaluates proposals with respect to new goal levels and generates revised prices and project selections
terminates when minimum goal deviations are reached
$\pi_m^{(n)}$
$A_{jm}^{(n+1)}$
$\pi_m^{(n+1)}$
$A_{jm}^{(n+1)}$
operating unit j of management unit m activities
uses Prices to generate project proposals
uses new prices to generate revised project proposals
etc.
iteration
n-1
n
n+1
n+2

Fig. 2

Ⅲ. MULTIPLE PERIOD PLANNING APPROACH OF GENERALIZED GOAL DECOMPOSITION MODEL

In the previous chapter, Ruefli's generalized goal decomposition model has been presented as a single period planning model. Many organizations do exist over long periods rather than in just a single period. Consequently, in the real world a lot of goals and objectives of an organization are concerned with a planning horizon that encompasses more than a single period.

If the decision maker's horizon is extended to the next period, he can do little more than make incremental adjustments in the existing plan. The more later planning periods are considered, the more adjustments can be made. Also, this feature of multiple period planning can provide some perspectives on the future direction that are anticipated.

This section will be concerned with the extension of the generalized goal decomposition model to do multiple period planning of the organizations.

A. LINEAR FORMULATION OF MULTIPLE PERIOD PLANNING MODEL

The solution of the generalized goal decomposition model is dependent on the structure of the organization. The change of organizational structure may be considered as a variable over the planning periods. Therefore, it is necessary to assume that the structure of the organization does not change over the time periods and to assume that both central unit and management units have control over the whole planning periods in the same assumption of the generalized goal

decomposition model, and further to assume that the objective functions and constraints of all Levels of organization are linear functions because of the computation algorithm. From these assumptions, the problem statements of the central unit can be presented as follows:

$$\max \sum_{t=1}^{T} \sum_{m=1}^{M} \left(\pi_{mt} \right) \left[G_{mt} \right] \tag{1-1}$$

s.t. (cost goal)

$$\sum_{t=1}^{T} \sum_{m=1}^{M} \left(p^{i}_{mt} \right) \left[G_{mt} \right] \leq g \tag{1-2}$$

s.t. (non-cost goal)

$$\sum_{m=1}^{M} \left[p_{mt} \right] \left[G_{mt} \right] \lessgtr \left[G \right] \quad t=1, \text{----}, T \tag{1-3}$$

$$\left[G_{mt} \right] + \left[\delta_{mt} \right] = \left[G_{mt+1} \right] \quad t=1, \text{----}, T-1 \tag{1-4}$$

$$G_{mt} \geq 0$$

where,

t represents the planning period,

m represents the management unit,

π_{mt} is a vector of shadow prices generated by the m^{th} management unit in the t^{th} planning period,

G_{mt} is a vector of goal levels assigned to the m^{th} management unit in the t^{th} planning period,

P_{mt} is a matrix of the m^{th} management unit's joint utilization of the organizational goals in the t^{th} planning period,

P^{i}_{mt} is the i^{th} row of the matrix P_{mt} and represents the cost per unit resources and requirements in the vector G_{mt},

δ_{mt} is a vector of the m^{th} management unit's goal attrition and acquisition levels in the t^{th} planning period,

G is the global goal vector,

g is the global cost goal.

The objective function of the central unit problem is just the sum of all planning periods' objective functions from the generalized goal decomposition model.

The cost goal has the constraint of Equation 1-2 because the total cost goal is the same as the sum of all period's cost goal. The non-cost goals have the constraints of Equations 1-3 and 1-4 because the current goal levels should be the sum of the previous period's goal levels and the levels of attrition and acquisition.

In the central unit's problem, we have assumed that the central unit has information about the global goal vector G and has set all of the goal levels G_{11}, ----, G_{mt} for the M management units in each planning period, in light of their relation ship to each other and the global goals, and in light of the information about their needs supplied by the management units. The central unit's problem is to maximize the inputed values of all of the management units' output subject to the global goals.

The problem statements of the m^{th} management unit is presented

in the following equations:

$$\min \sum_{t=1}^{T} \left(\begin{bmatrix} W_{mt}^{+} \end{bmatrix} \begin{bmatrix} Y_{mt}^{+} \end{bmatrix} + \begin{bmatrix} W_{mt}^{-} \end{bmatrix} \begin{bmatrix} Y_{mt}^{-} \end{bmatrix} \right)$$

s.t.

$$\begin{bmatrix} A_{m1} & & & 0 \\ & A_{m2} & & \\ & & \ddots & \\ 0 & & & A_{mr} \end{bmatrix} \begin{bmatrix} X_{m1} \\ X_{m2} \\ \vdots \\ X_{mr} \end{bmatrix} - \begin{bmatrix} I & & & \\ & I & & \\ & & \ddots & \\ & & & I \end{bmatrix} \begin{bmatrix} Y_{m1}^{+} \\ Y_{m2}^{+} \\ \vdots \\ Y_{mr}^{+} \end{bmatrix}$$

$$+ \begin{bmatrix} I & & & \\ & I & & \\ & & \ddots & \\ & & & I \end{bmatrix} \begin{bmatrix} Y_{m1}^{-} \\ Y_{m2}^{-} \\ \vdots \\ Y_{mr}^{-} \end{bmatrix} = \begin{bmatrix} G_{m1} \\ G_{m2} \\ \vdots \\ G_{mT} \end{bmatrix}$$

$$0 \leq X_{mt} \leq 1 , \quad Y_{mt}^{+}, \quad Y_{mt}^{-} \geq 0$$

where

W_{mt}^{+}, W_{mt}^{-} are vectors of weights assigned to the positive and negative goal deviations in the t^{th} planning period,

Y_{mt}^{+}, Y_{mt}^{-} are vectors of positive and negative deviations from the goal vector G_{mt},

A_{mt} is a matrix of attributes of project proposals for all n_m subordinate operating units in the t^{th} planning period,

X_{mt} is a vector of activity levels for the project proposals in the t^{th} planning period,

I is an identity matrix.

The objective function of the m^{th} management unit is the sum of all single planning period's weighted goal deviations from their respective goal levels. The large matrix equation of the constraints is composed of the constraints of each single period. The management unit's problem is to minimize its aggregate weighted goal deviations from each planning period's goals which are set by the central unit, subject to the technology of achieving the management unit's goal.

The formulation of the jm^{th} operating unit's problem can be described as follows:

$$\min \sum_{t=1}^{T} \begin{bmatrix} \pi_{mt} \end{bmatrix} \begin{bmatrix} A_{mjt} \end{bmatrix}$$

s.t.

$$\begin{bmatrix} D_{jm1} & & \\ & D_{jm2} & 0 \\ 0 & & \ddots \\ & & D_{jmT} \end{bmatrix} \begin{bmatrix} A_{mj1} \\ A_{jm2} \\ \vdots \\ A_{jmT} \end{bmatrix} \geq \begin{bmatrix} F_{jm1} \\ F_{jm2} \\ \vdots \\ F_{jmT} \end{bmatrix}$$

$$A_{jmT} \geq 0$$

where,

A_{jmt} is a vector of variables representing activity levels for jm^{th} operating unit in the t^{th} planning period,

D_{jmt} is a matrix of technological coefficients in the t^{th} planning period,

F_{jmt} is a vector of minimum output levels in the t^{th} planning period.

The objective function of the jm^{th} operating unit is the sum of each single period's generalized goal decomposition model over all planning periods and the number of constraints are increased by the number of those periods.

The jm^{th} operating unit's problem is to minimize the inputed cost of its production subject to the physical technology of that production. Having considered the formulations of each organization level's problem statement, the multiple period planning model is formulated by a slight modification of Ruefli's model.

B. DYNAMIC FORMULATION OF MULTIPLE PERIOD PLANNING MODEL

In the multiple period planning model, the objective of the management unit's problem is the minimum weighted goal deviations from the assigned goals as a whole. The global cost goal is distributed over all planning periods and then distributed over all management units. The non-cost goals are distributed over all management units and these goal levels are available in every period without changing.

From this point of view, the multiple period planning model can be described in the following simplified formula by using the planning period's subscript t:

$$\min \sum_{t=1}^{T} r_t \quad (G_t, \ g_t) \tag{2-1}$$

s.t. (cost goal)

$$\sum_{t=1}^{T} g_t \leq g \tag{2-2}$$

s.t. (non-cost goal)

$$G_t \lesseqgtr G \qquad (2\text{-}3)$$

where,

t represents planning periods,

G_t is a non-cost goal vector assigned for central unit in the t^{th} planning period,

G is the global non-cost goal vector,

g_t is the cost goal levels for the central unit in the t^{th} planning period.

g is the global cost goal levels.

$r_t(G_t, g_t)$ is a function of G_t and g_t, i.e., the weighted goal deviations from the management units' goal vector.

In the t^{th} planning period, the goal vector G_t and the cost goal g_t assigned by the central unit vary, the weighted goal deviations $r_t(G_t, g_t)$ will have different values. Furthermore, the objective function for all the periods is an additive form of each single sub-period. Since the optimization problem can be decomposed into recursive equations, then the system can be described in terms of a number of state variables. That is, separability and monotonicity conditions are satisfied because of the additivity of the objective function.

If we pick up planning periods for stages, then the stage diagram can be presented in Fig. 3.

where,

stage T corresponds to the first planning period,

stage 1 corresponds to the last planning period,

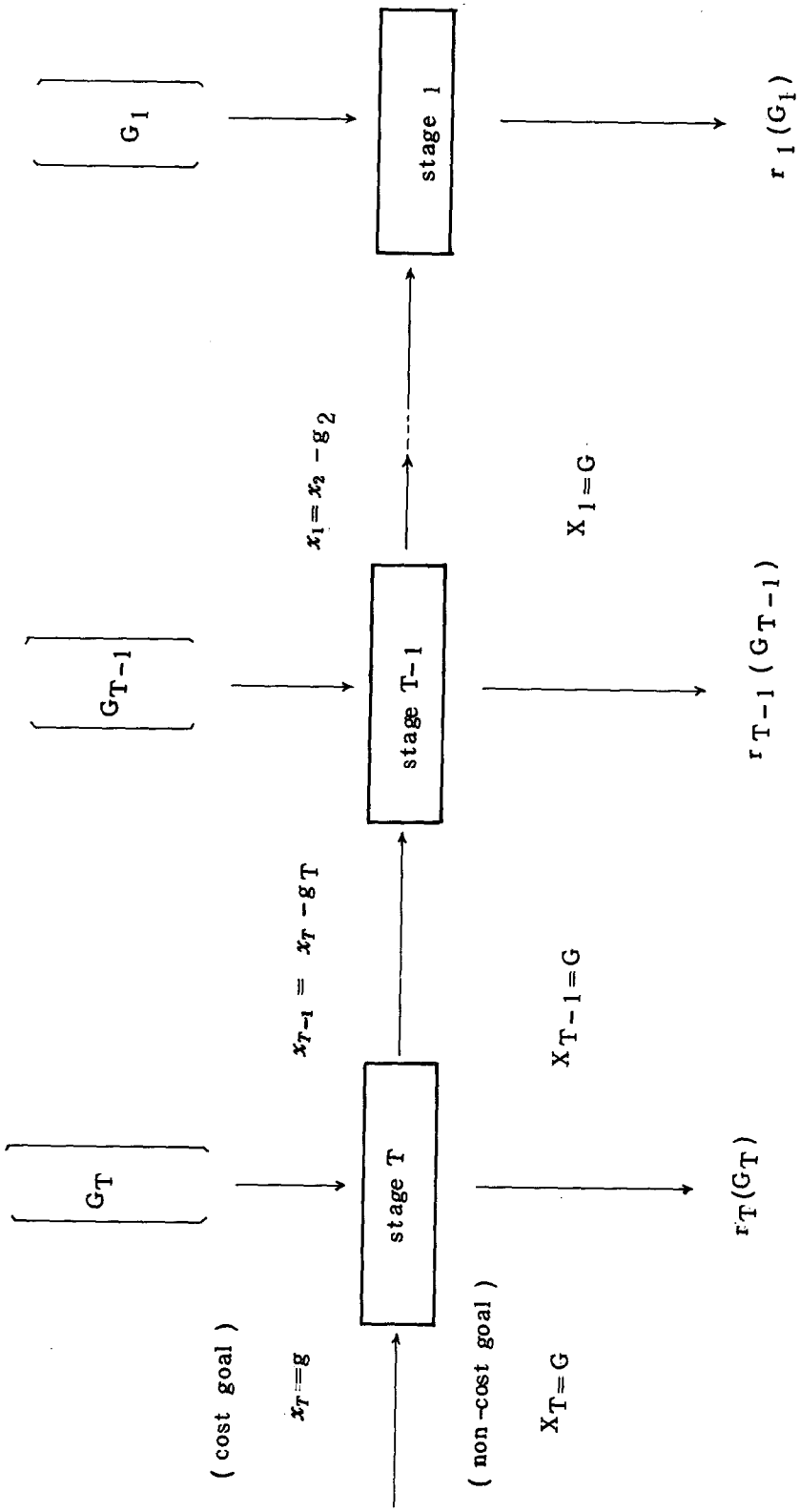

Fig. 3 Stage Diagram

X_t is a state variable vector representing the amount of goals left to be allocated to stages t, t-1, ----, 1,

G_t is a decision variable vector representing the amount of goals to be allocated in stage t,

x_t is a state variable relating to the cost goal in stage t,

g is the global cost goal to be allocated over all planning periods,

G is a global non-cost goal vector,

$r_t(G_t)$ is immediate return function in stage t,

g_t is the decision variable relating to the cost goal in stage t.

All of the global goal levels are available to use in stage T (the first planning period), therefore the state variable vector X_T is the global goal vector G and the state variable x_T of the cost goal is the global cost goal g.

The state variable vector X_{T-1} of the non-cost goals is just the global goal vector G because the amount of goal levels left in stage T-1 is still the global goal vector. But the state variable x_{T-1} of the cost goal is $x_T - g_T$. The amount of cost goals allocated in stage T should be subtracted from the state variable x_T because the state variable x_{T-1} is the amount of cost goals left to be allocated to the stages T-1, ----, 1. The state variables in the other stages can be produced from the formulas in the stage diagram.

In each stage, the state variable vector X_t and the decision variable vector G_t are the input variables and the return function

$r_t(G_t)$ is the immediate output. The decision variables are presented in the following equation derived from the central unit's problem in the Ruefli model.

$$\begin{bmatrix} G_t \end{bmatrix} = \sum_{m=1}^{M} \begin{bmatrix} P_{mt} \end{bmatrix} \begin{bmatrix} G_{mt} \end{bmatrix}$$

where,

P_{mt} is a matrix of the m^{th} management unit's joint utilization of the organizational goals in stage t,

G_{mt} is a vector of goal levels assigned to the m^{th} management unit in stage t.

The objective of the multiple period planning model is to minimize the goal deviations from each management unit's goal levels in each period. In this sense, the return function can be presented in the following equation which is the sum of all management unit's objective functions in the generalized goal decomposition model.

$$r_t\ (G_t) = \sum_{m=1}^{M} \left(\begin{bmatrix} W_{mt}^{+} \end{bmatrix} \begin{bmatrix} Y_{mt}^{+} \end{bmatrix} + \begin{bmatrix} W_{mt}^{-} \end{bmatrix} \begin{bmatrix} Y_{mt}^{-} \end{bmatrix} \right)$$

where

W_{mt}^{+}, W_{mt}^{-} are vectors of weights for positive and negative deviations from goals in stage t,

Y_{mt}^{+}, Y_{mt}^{-} are vectors of positive and negative deviations from goal vector in stage t.

The corresponding recursion equation of multiple period planning model can be presented as follows:

$$f_t\ (X_t) = \min \left[r_t\ (G_t) + f_{t-1}\ (X_{t-1}) \right]$$

$0 \leq G_t \leq X_t$, where the inequalities of Equations (2-2) and (2-3) are $\leq$.

$G_t > X_t$, where the inequalities of Equations (2-2) and (2-3) are $>$.

where

$r_t(G_t)$ is the immediate return function,

$f_{t-1}(X_{t-1})$ is optimal return function for remaining stages t-1, ----, 1 with the remaining goal levels X_{t-1}.

In the recursion equation defined above, the central unit has to choose the goal vector G_t to minimize the sum of immediate return and the optimal return, which is the best we can do for the remaining stages t-1, ---, 1 with the remaining goals X_{t-1}. Since some parts of the goals have been used already in the previous stages, the rest of the goals are X_t.

The important feature of the recursion is that we do not have to think about the decisions G_{t-1}, ----, G_1 which give $f_{t-1}(X_{t-1})$, thus we have a single period optimization over G_t. Due to Nemhauser, an optimal set of decisions must be optimal with respect to the outcome which results from the first decision.

The standard solution process has the following steps.

1. Compute $f_1(X_1)$ for all possible values of X_1 by using the generalized goal decomposition model and store the results.

$$f_1(X_1) = \min r_1(G_1)$$

$$0 \leq G_1 \leq X_1 \quad \text{or} \quad G_1 > X_1$$

2. Compute $r_2(G_2)$ for all possible values of G_2 by using Ruefli's single period model in stage 2 and then $f_2(X_2)$ for all values of X_2 by using the following recursion equation in stage 2 and store the results.

$$f_2(X_2) = \min \left[r_2(G_2) + f_1(X_1) \right]$$

$$0 \leq G_2 \leq X_2 \text{ or } G_2 > X_2$$

3. Continue recursion computations $f_3(X_3)$, ----, $f_T(X_T)$.

4. Compute the decision variables by the backwards track of the optimal solutions.

C. SOLUTION METHOD OF MULTIPLE PERIOD PLANNING MODEL

The linear and dynamic formulations are presented in the previous sections. The linear formulation cannot be solved simultaneously by the simplex method of linear programming because of a lack of information, and too many variables and constraints in the problem statements of each level of the organization. Also, the standard dynamic programming approach is not available for the solution method of the multiple period planning model due to many decision variables in each stage. The optimization is to minimize the weighted goal deviations from the assigned goal levels. If the goal levels are too large or small, the weighted goal deviations are large. Conversely, if the goal levels are appropriate in each stage, then the goal deviations will reach a minimum.

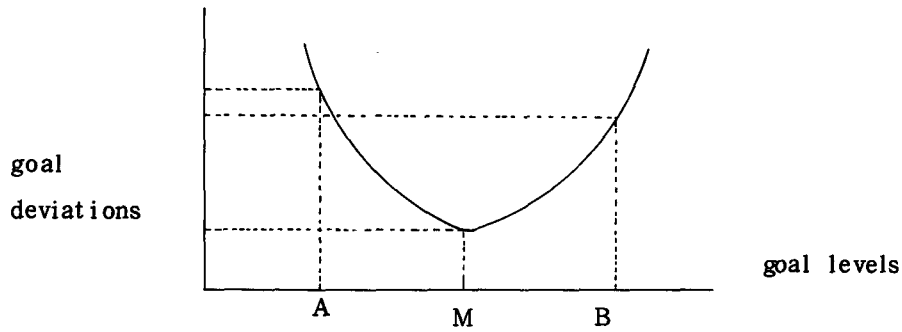

Fig. 4

In Fig. 4, the goal deviations of point M are minimum and smaller than those of A and B. From this feature of the model, we can reasonably assume that the weighted goal deviations of each stage problem are convex functions of the goal levels. Therefore, the following search algorithm can be used instead of the previous standard method. The search points of each stage are presented in Figs. 5-1 and 5-2, where g_1 and g_2 are the elements of the global goal vector G to be allocated over all stages.

G_t^1 is the t^{th} stage's initial iteration point assigned by the central unit with the existing information, G_t^2 is the t^{th} stage's second iteration point determined by the search algorithm, etc.

If the initial points G_T^1, $G^1{}_{T-1}$, ----, G_1^1 are the optimal solution points, then moving to any direction from these points increases the goal deviations. If the goal deviations of stage T are large because of the extra amount of goal levels, the goal deviations of stage T-1 are also large due to small amount of goal levels, and further those of all remaining stages are optimal points, then the minimum goal deviations are produced to the decreasing direction of goal levels in stage T and the increasing direction in stage T-1.

Two elements of the global goal vector

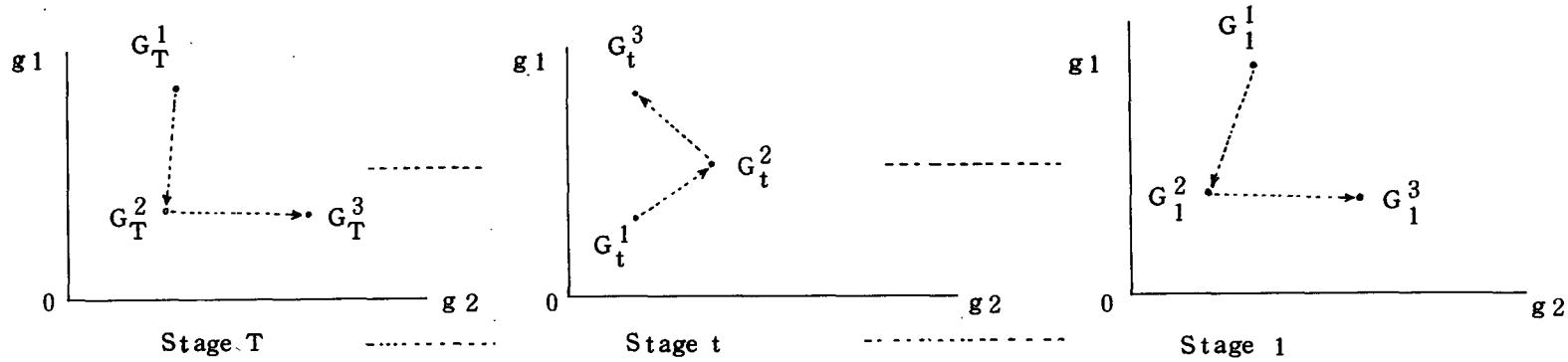

Fig. 5-1

n elements of the global goal vector

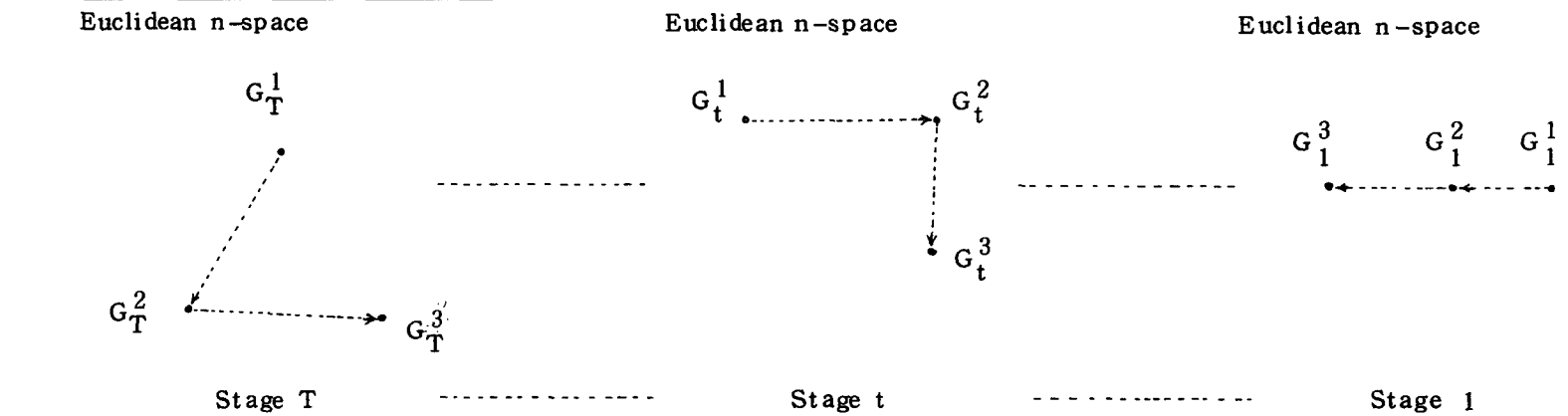

Fig. 5-2

This feature of the model can be used to find the successive iteration points without computation of all feasible points.

The central unit determines the initial points G_T^1, ----, G_1^1 with the existing information. These points are the central unit's global goal levels of the generalized goal decomposition model; therefore Ruefli's model exists in each stage. The optimization problem can be solved by using the generalized goal decomposition model for stage T through stage 1. The management units produce the weighted goal deviations and the positive and negative goal deviation vectors of all stages at the last iteration of each stage problem.

The goal deviation vectors are sent to the central unit and the management unit's cost goal weighting factors are also transmitted to the central unit. The central unit has to determine the second iteration points G_T^2, ----, G_1^2 to reduce the management units' weighted goal deviations. Therefore, the central unit uses this information transmitted by the management units to choose the second iteration points. The central unit's process choosing the second iteration points is presented in the following pages.

To determined the second iteration points, it is needed to compute each stage's positive and negative deviation vectors, the difference vector of the positive and negative deviations, the total positive and negative cost goal deviations and the differences of the total positive and negative cost goal deviations.

$$\begin{bmatrix} Y_t^+ \end{bmatrix} = \sum_{m=1}^{M} \begin{bmatrix} P_{mt} \end{bmatrix} \begin{bmatrix} Y_{mt}^+ \end{bmatrix} \quad t=1, \text{----}, T \qquad (3\text{-}1)$$

$$\begin{bmatrix} Y_t^- \end{bmatrix} = \sum_{m=1}^{M} \begin{bmatrix} P_{mt} \end{bmatrix} \begin{bmatrix} Y_{mt}^- \end{bmatrix} \qquad t=1, \text{----}, T \qquad (3\text{-}2)$$

$$\begin{bmatrix} Y_t^D \end{bmatrix} = \begin{bmatrix} Y_t^- \end{bmatrix} - \begin{bmatrix} Y_t^+ \end{bmatrix} \qquad t=1, \text{----}, T \qquad (3\text{-}3)$$

(non-cost goal)

$$Y^+ = \sum_{t=1}^{T} Y_t^+ \qquad \text{(cost goal)} \qquad (3\text{-}4)$$

$$Y^- = \sum_{t=1}^{T} Y_t^- \qquad \text{(cost goal)} \qquad (3\text{-}5)$$

$$Y^D = Y^- - Y^+ \qquad (3\text{-}6)$$

where,

Y_t^+, Y_t^- are the positive and negative goal deviation vectors including the cost goal deviations in stage t,

Y_{mt}^+, Y_{mt}^- are the m^{th} management unit's positive and negative goal deviation vectors in stage t,

P_{mt} is the m^{th} management unit's joint utilization matrix in stage t,

Y_t^D is the difference goal deviation vector in stage t,

Y_t, Y_t are the total positive and negative cost goal deviations over all management units and over all stages,

Y^D is the differences of the total positive and negative cost goal deviations.

The positive and negative goal deviation vectors Y_t^+, Y_t^- in each stage are computed from Equations (3-1) and (3-2), and then each stage's difference goal deviation vector is acquired from equation (3-3).

The total positive and negative cost goal deviations Y^+, Y^- are obtained from Equations (3-4) and (3-5) and the total differences of the cost goal deviations are produced from Equation (3-6).

The element of the difference goal deviation vectors can be positive, negative or zero. The positive element represents that the goal levels are too large, the negative element means that the goal levels are too small and the zero values of the element indicate that there is no shortage or surplus of the goal levels.

In the case of the positive elements of the vector Y_t^D, there is no need to modify the elements of the vector Y_t^+, but the element of the vector Y_t^- should be examined whether the goal deviations of each element can be reduced in the next iteration. On the contrary, in the case of the negative elements of the vector Y_t^D, no restrictions exist in the elements of the vector Y_t^- while the elements of the Y_t^+ should be considered whether the adjustment of each element is needed or not. If the elements of the vector Y_t^D are positive and the corresponding inequalities of the Equations (2-2) and (2-3) are $\leq$ (less than), or the elements of the vector Y_t^D are negative and the corresponding inequalities are $>$ (greater than), then the elements of the vector Y_t^- is the former case and the elements of the vector Y_t^+ in the latter case do not need any adjustments because the elements of the global goal vector G represent the maximum levels in the case of less than inequalities and the minimum levels in the case of greater than inequalities.

The central unit has each stage's positive and negative deviation

vectors, the difference vector, the total positive and negative cost goal deviations and the differences of the total goal deviations. But the central unit also needs the actual goal levels left in the first iteration. Each stage's non-cost goal levels and the total cost goal levels left can be computed from Equations (3-7) and (3-8).

$$\begin{bmatrix} G_t^R \end{bmatrix} = \begin{bmatrix} G \end{bmatrix} - \begin{bmatrix} G_t^1 \end{bmatrix} \qquad T=1, \text{----}, T \qquad (3\text{-}7)$$

$$g^R = g - \sum_{t=1}^{T} g_t^1 \quad \text{(cost goal)} \qquad (3\text{-}8)$$

where,

g is the global cost goal,

G_t^R is the remaining goal vector in stage t,

g^R is the total remaining cost goals,

g_t^1 is the t^{th} stage's cost goal assigned by the central unit for the first iteration,

G_t^1 is the t^{th} stage's non-cost goals for the first iteration.

The elements of the vector G_t^R can be positive, negative or zero. The positive element of the vector G_t^R means that the goals are underdistributed in stage t, the negative element indicates that the goals are overdistributed and the zero values represent that there are no surpluses or shortages.

If the elements of the vector Y_t^D are positive and the corresponding inequalities of the Equations 2-2 and 2-3 are $>$ (greater than), then the values of the elements of the vector Y_t^D and G_t^R are substracted from the elements of the vector Y_t^- to meet the global goal constraints at least because the elements of the vector G are negative.

If the elements of the vector Y_t^D are negative and the corresponding inequalities of the Equations 2-2 and 2-3 are $\leq$ (less than), then the values of the elements of the vector Y_t^D and G_t^R are added to the elements of the vector Y_t^+ to meet the global goal constraints at most because the elements of the vector G_t^R are positive.

This modification of the vectors Y_T^+, ---, Y_1^+, Y_T^-, ---, Y_1^-, produces the new vectors Y_T^{N+}, Y_T^{N-}, ---, Y_1^{N-} of the positive and negative goal deviations in each stage. The new elements Y^{N+} and Y^{N-} of the total positive and negative cost goal deviations are produced from the same process using the total remaining cost goals g^R and the total difference goal deviations Y^D.

The elements of the vector Y_t^{N+} represent that these positive goal deviations can be reduced by adding the additional goals. The elements of the vector Y_t^{N-} indicate that these negative goal deviations can be subtracted from the goals distributed over stage t in the previous iteration.

If it is assumed that the global goal vector G has K elements, then the following Equations 3-9 through 3-12 can be used to produce the positive and negative ratios.

$$r_{ti}^+ = \frac{Y_{ti}^{N+}}{Y_{ti}^+} \qquad \begin{matrix} i=1, \text{----}, K-1 \\ t=1, \text{----}, T \end{matrix} \qquad \text{(non-cost goal)} \qquad (3\text{-}9)$$

$$r_{ti}^- = \frac{Y_{ti}^{N-}}{Y_{ti}^-} \qquad \begin{matrix} i=1, \text{----}, K-1 \\ t=1, \text{----}, T \end{matrix} \qquad \text{(non-cost goal)} \qquad (3\text{-}10)$$

$$r^{+} = \frac{Y^{N+}}{Y^{+}} \qquad \text{(cost goal)} \qquad (3\text{-}11)$$

$$r^{-} = \frac{Y^{N-}}{Y^{-}} \qquad \text{(cost goal)} \qquad (3\text{-}12)$$

where r_{ti}^{+} is the i^{th} element's ratio of the vectors Y_t^{N+} and Y_t^{+}, and r_{ti}^{-} is the i^{th} element's ratio of the vector Y_t^{N-} and Y_t^{-}. The ratios of the total positive and negative cost goal deviations can be used for all stages; therefore each stage's vectors R_T^{+}, ----, R_1^{+}, R_T^{-}, ---, R_1^{-}, where the vector R_t^{+} consists of r_{ti}^{+} for all i, are produced from Equations 3-9 through 3-12.

If the i^{th} elements of the vectors Y_T^{+}, ----, Y_1^{+}, Y_T^{-}, ---, Y_1^{-} are zero, then the i^{th} positive and negative ratios are zero because the zero cannot be used as a denominator. These zero values mean that there is no goal deviations at all.

The central unit has to consider the zero values of the cost goal ratios r^{+}, r^{-}. The zero values are produced for the following two reasons:

1. The numerator and denominator are both zero.
2. The numerator is zero, but the denominator is positive.

In Case 1, there is no need to adjust the existing cost goal levels because the positive and negative deviations are zero. In case 2, the central unit cannot distribute more cost goals over the stages and subtract the extra goals from the stages with surplus goal levels, but the central unit can transfer the goal levels from the stages with large weighting factors to the other stages with small

weighting factors to reduce the weighted goal deviations of the management units. Therefore, the ratios of the cost goal in Case 2 should be modified by using the weighting factors:

$$W^{+} = \frac{1}{M} \sum_{m=1}^{M} W_{mt}^{+} \qquad t=1, \text{----}, T \qquad (3\text{-}13)$$

$$W^{-} = \frac{1}{M} \sum_{m=1}^{M} W_{mt}^{-} \qquad t=1, \text{----}, T \qquad (3\text{-}14)$$

where W_t^+, W_t^- are the t^{th} stage's positive and negative weighting factors about the cost goal and W_{mt}^+, W_{mt}^- are the m^{th} management unit's positive and negative weighting factors in stage t. Each stage's positive and negative weighting factors about the cost goal are the average of all management units weighting factors. These weighting factors obtained from equations 3-13 and 3-14 can be different from stage to stage in spite of the same cost goal.

If the positive ratio r^+ is the Case 2, then it is needed to choose the positive cost goal deviation with the largest weighting factor among Y_T^+, ---, Y_1^+ and then to select the corresponding element of the weighting factor among W_T^+, ---, W_1^+. It is also needed to select the smallest element of the weighting factor. If the W_T^+ is selected for the former and the W_t^+ is for the latter, then the positive cost goal ratio of the vector R_t^+ is replaced with 1 and the positive cost goal ratio of the vector R_t^+ is replaced with -1, and also the positive cost goal deviation Y_t^+ is replaced with the positive cost goal deviation Y_t^+ to transfer the cost goal levels from stage T to stage t.

The positive cost goal deviations in stage T can be eliminated, while the positive cost goal deviations in stage t will be increased by this replacement of the positive ratios and deviations. However, the total weighted goal deviations of all stages should be decreased. This replacement is performed in the negative cost goal ratios and deviations by using the same process. If the case 2 does not exist, then the positive ratios r_T^+, ----, r_1^t are all the same and the negative ratios r_T^-, ----, r_1^- have also the same values.

$$\begin{bmatrix} Y_+^{M+} \end{bmatrix} = \begin{bmatrix} Y_t^+ \end{bmatrix} \cdot \begin{bmatrix} R_t^+ \end{bmatrix} \quad t=1, \text{----}, T \qquad (3\text{-}15)$$

$$\begin{bmatrix} Y_t^{M-} \end{bmatrix} = \begin{bmatrix} Y_t^- \end{bmatrix} \cdot \begin{bmatrix} R_t^- \end{bmatrix} \quad t=1, \text{----}, T \qquad (3\text{-}16)$$

$$\begin{bmatrix} Y_t^{MD} \end{bmatrix} = \begin{bmatrix} Y_t^{M+} \end{bmatrix} - \begin{bmatrix} Y_t^{M-} \end{bmatrix} \quad t=1, \text{----}, T \qquad (3\text{-}17)$$

$$\begin{bmatrix} G_t^2 \end{bmatrix} = \begin{bmatrix} G_t^1 \end{bmatrix} + \begin{bmatrix} Y_t^{MD} \end{bmatrix} \quad t=1, \text{----}, T \qquad (3\text{-}18)$$

where

Y_t^{M+}, Y_t^{M-} are the modified positive and negative goal deviation vectors of the stage t,

Y_t^{MD} is the modified difference vector in stage t,

G_t^1, G_t^2 are the vectors representing the first and second iteration points in stage t.

The modified positive and negative goal deviation vectors in each stage are computed from the Equations 3-15 and 3-16 and the modified difference vector is computed from Equation 3-17. These modified vectors Y_T^{MD}, ----, Y_1^{MD} indicate the intervals between the initial

points G_T^1, ----, G_1^1 and the second iteration points G_T^2, ----, G_1^2. Therefore, the second iteration points are obtained from Equation 3-18.

The moving from the initial points to the second iteration points reduces the management units' weighted goal deviations as a whole. The central unit supplies more goal levels to the stages with the positive elements of the modified difference vectors and, on the contrary, the central unit reduces the existing goals of the stages with the negative elements. If all elements of the vector Y_t^{MD} are zero, then the t^{th} stage's first iteration point G_t^1 is the same as the second iteration point G_t^2.

If the second iteration points G_T^2, ----, G_1^2 are determined, then each stage problem can be optimized by the feedback process of the generalized goal decomposition model. When the optimal solutions about the second iteration points are obtained, the management units send the positive and negative deviation vectors and the positive and negative weighting factors about the cost goal to the central unit in each stage. The central unit uses these results to choose the next iteration points.

This process continues until all elements of the modified difference vectors Y_T^{MD}, ----, Y_1^{MD} are zero. This means that there is no movement of the iteration points in the next iteration and also the current solution is optimal. If the convexity assumption of each stage problem is correct, then the management units' weighted goal deviations become smaller and smaller in accordance with iteration.

Eventually the minimum weighted goal deviations should be achieved by this iterative search algorithm in a finite number of iterations.

The iterative search algorithm produces the multiple period plan based on the existing information at the beginning of the first planning period.

In the real world, the information in the far planning periods are unclear and do not remain constant, but grow out of the experiences and external environment in accordance with the moving from period to period. The information in the near planning periods is more clear and unchanging than the far planning periods. Therefore, the plan is updated by excluding the first planning period at the beginning of the second planning period while bringing in the $T+1^{st}$ planning period. This modification of the plan can be made at the beginning of every planning period if it is needed.

Ⅳ. SUMMARY

In this paper Ruefli's generalized goal decomposition model has been extended to make a more realistic evaluation of the alternatives in the decision making process of an organization from the long run point of view. The model formulated with linear goal deviations by introducing the goal programming method used in Ruefli's model. The global goals are distributed over all planning periods and then over all management units. The management units' minimum weighted goal deviations are obtained from the optimal distributions of these goals. Dynamic formulation using the generalized goal decomposition model for each single period problem is also presented.

The linear formulation cannot be directly used to obtain the optimal solution due to lack of information and too many variables and constraints in the problem statements of each level of the organization. The dynamic formulation also cannot be directly used because of too many decision variables in each stage.

An iterative search algorithm is presented as an appropriate solution method of the dynamic formulation of the multiple period planning model. In the iterative search algorithm, the generalized goal decomposition model is used to solve the problem of each planning period in every iteration.

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