Journal of the
Military Operations Research
Society of Korea, Vol. 7, No. 2
December, 1981

Point Availability of Multi-Component System When Each Component Has a Finite Number of Spares

Jee, Man Won*

ABSTRACT

Computational expressions for point availability and average availability of a system of components each of which is subject to random failures and has random restoration times are determined. Each component is assumed to have a fixed number of spares such that where all spares are exhausted no restoration can take place. These expressions are useful in deciding PL and ASL in the military logistic applications.

Given a fixed length of mission duration and finite number of spares, a system may not be available at the end of a mission due to lack of spares. The probability distribution of system down time due to lack of spares is determined as a function of number of spares and mission duration.

I. INTRODUCTION

A key concern of military planners is the material readiness of their weapon systems. Many attempts have been made in the past to develop measurable indicators of material readiness. Among these are reliability, fill rate, down time, maintenance man-hours, supply response time, and availability. Out of these attempts to measure

* Korea Institute of Defense Analyses

readiness, operational availability has become the most widely accepted indicator.

Many people talk and write about operational availability, but not all have the same notion as to what is meant by the expression. Operational availability is generally considered to be a measure of the likelihood that a system, when used under stated conditions in the actual operational environment, shall operate satisfactorily. It is clear that component reliabilities, maintenance policy, spare parts support, system configuration, repair times, and supply response times all have impact on availability. However, this notion of operational availability can give rise to more than one workable mathematical expression. To illustrate, consider a ship on a 60-day deployment. Let A(t) be the probability that a given weapon system is operational at time t. One might use any of the following as reasonable measures of availability:

a) A(60) (end of deployment point availability)

b) $\frac{1}{60}\int_0^{60} A(t)dt$ (average availability)

c) The probability that the system shall operate satisfactorily when called upon

d) $\frac{\text{Mean time to failure (MTBF)}}{\text{MTBF + Mean time to replace (MTTR)}}$

The first expression is the likelihood that the equipment will be operational at the end of the deployment; the second represents an average availability over the duration of the deployment; the third considers only the likelihood that the system is operational at those times when needed (thus introducing the mission duty cycle as a factor to consider); and the last expression is the "definition" of availability specified by Ref. (1), and used most frequently in practice. Under certain very restrictive conditions the above expressions may yield essentially the same values. However, for most real-world cases the expressions are not equivalent; they can be substantially different. The following specific cases illustrate some differences.

Case 1 : The equipment is a single component which has exponential life time with failure rate λ. There are unlimited spares aboard ship. The distribution of the sum of supply response time (time required to get the replacement part to the equipment) and repair time (hereafter called replacement time) is exponential with rate η. The system is operational at the start of the deployment.

Define the random process $\{X(t): t \geq 0\}$ as follows:

$$X(t) = \begin{cases} 1 & \text{if component is operational at } t \\ 0 & \text{otherwise} \end{cases}$$

Under the conditions above, X(t) is a renewal process and A(t) = P(X(t) = 1) is easily derived from renewal theory to be

$$A(t) = \frac{\eta}{\lambda+\eta} + \frac{\lambda}{\lambda+\eta} \exp\{-(\lambda+\eta)t\} \qquad (1\text{-}1)$$

or, since MTBF $= \frac{1}{\lambda}$ and MSRT + MTTR $= \frac{1}{\eta}$ we can write (1-1) as

$$A(t) = \frac{\text{MTBF}}{\text{MTBF} + (\text{MSRT} + \text{MTTR})} + \frac{(\text{MSRT} + \text{MTTR})}{\text{MTBF} + (\text{MSRT} + \text{MTTR})} \exp\{-(\lambda+\eta)t\} \qquad (1\text{-}2)$$

To allow numerical comparisons of the four interpretations a), b), c), and d) of availability, let $\lambda = 0.01$ and $\eta = 0.10$. Then

a) $A(60) = 0.90921$

b) $\frac{1}{60}\int_0^{60} A(t)dt = 0.92285$

d) $\frac{MTBF}{MTBF + (MSRT + MTTR)} = 0.90909$

There are only minor differences between a) and d) but a bit larger difference between b) and the others. The calculation for interpretation c) yields exactly the same result as b) if the times that the component is needed are uniform on the interval (0,60]. Interpretation d) provides the most conservative estimate of availability. From equation 1-2 one can easily see that

$$\lim_{t=\infty} A(t) = \frac{MTBF}{MTBF + (MSRT + MTTR)}$$

Thus d) is simply the limiting availability. This, however, is not always the case as is shown later.

A plot of A(t) vs. t demonstrates how point availability vaies as a function of time (length of deployment). See figure 1.1.

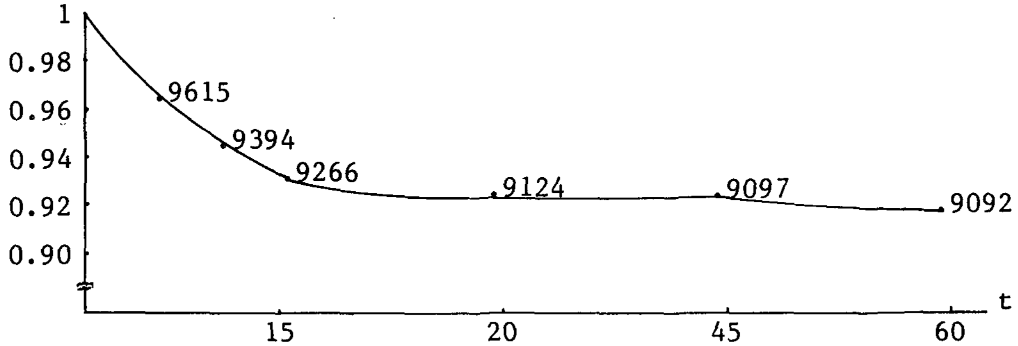


Fig.1.1 Availability over time

Consider now a case that is more operationally realistic. Significant differences will be seen in the estimates yielded by the different expressions.

Case 2 : The equipment is a single component which has exponential life time with rate λ. There is a single spare aboard ship and no chance for resupply until the deployment is over. The replacement time is exponential with rate η. The system is operational at time 0.

The random process $\{X(t), t > 0\}$ which describes the up/down status of the equipment is not now a renewal process. A sample path for X(t) is shown in figure 1.2.

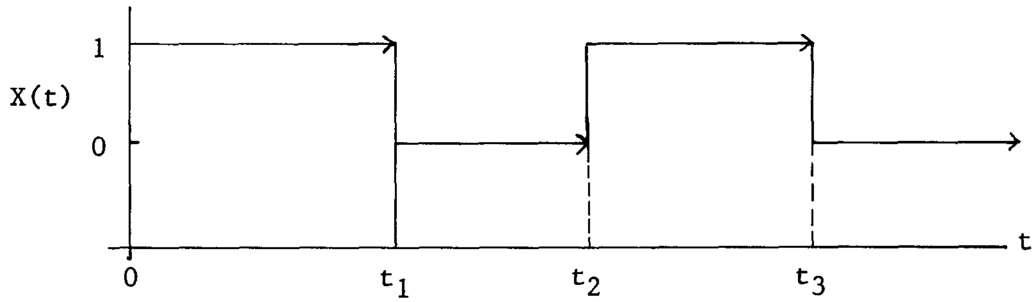


Fig.1.2 Sample path for X(t) when only one spare is stocked

The important thing to note is that the component will remain in a down status after time t_3 since there is no spare for replacement. In the next chapter an expression is derived for the point availability from which the following values are determined (λ = 0.01 and η = 0.10):

a) $A(60) = 0.8472$

b) $\frac{1}{60}\int_0^{60} A(t)dt = 0.9054$

d) $\frac{MTBF}{MTBF + MTTR} = 0.9091$

Greater differences among the availability estimates are observed when a single spare is stocked. The differences can be made quite significant by appropriate choice of the parameters λ and η. For example, if $\lambda = 0.05$ and $\eta = 0.50$,

a) $A(60) = 0.2096$

b) $\frac{1}{60}\int_0^{60} A(t)dt = 0.578361$

d) $\frac{MTBF}{MTBF + MTTR} = 0.9091$

The above examples show that there can be significant differences in the values obtained from the different definitions of availability. Each expression has its supporters. For logistics planning purposes, military planners want to determine the likelihood that a deployed unit will complete a mission with a given system operational. Therefore, they need to know the point availability at time τ where τ is the length of the deployment period. Furthermore, the average availability and the limiting availability are both functions of the point availability. Therefore, we focus primarily on point availability in this study. Examples of calculations of average availability and the limiting availability are provided to illustrate their calculation. The definition of availability based on the ratio of mean time to failure to the sum of mean time to failure and mean time to replace leaves much to be desired. The definition assumes that the up and down states of the system satisfy an alternating renewal process. This implicitly assumes infinitely many spares making the expression useless for determining the number of spares required to support a system. Furthermore, the ratio expression is a limiting result, whereas, the major interest is in the availability at specified points in time or over specified intervals. Finally, the expression does not lend itself for calculating the system availability as a function of the availabilities of its components.

Let us now look at the point availabilities of some simple systems. Consider first the case in which the system is composed of two components connected in series each with infinitely many spares. We assume that the components operate independently. It is easy to see from first principles that the system availability is the product of the component availabilities. That is,

$$A_{sys}(t) = A_1(t) \cdot A_2(t)$$

Similarly, if the system is composed of two components in parallel, the system availability can easily be shown to be

$$A_{sys}(t) = 1 - (1 - A_1(t))\ (1 - A_2(t)).$$

The reader familiar with reliability theory will recognize the above calculations as identical to those used to calculate reliabilities of systems composed of two components in seires and parallel, respectively. These results are easily extended to any finite number of components. The formulae are also valid when each component has only finitely many spares for support. Thus, the system availability of any series/parallel/mixed system can be determined easily from the availabilities of its components provided the components operate independently.

II. AVAILABILITY OF ONE COMPONENT SYSTEM WITH FINITE NUMBER OF SPARES

A. Point Availability

In this section we determine mathematical formulae for the point availability of a single component system having n spares and no repair capability. Since there are only finitely many spares and no possibility for repair, the system will alternate between up and down states until all spares are exhausted and will remain in a down state whenever the last spare fails.

Let us introduce some notation that will be used. Let T_i be the lifetime and R_i the replacement time of the ith unit. We assume that $\{T_i\}_{i=1}^{n}$ are independent and identically distributed (iid) with distribution function F(t) and probability density f(t). Similarly, the replacement times $\{R_i\}_{i=1}^{n}$ are iid with distribution G(t) and density g(t). Furthermore, the replacement times are independent of the lifetimes. We use f*g to indicate the convolution of f and g and $f^{(k)}$ to represent the k-fold convolution. Finally, let $P_k(t)$ be the probability that the kth unit will be in operation at time t.

The first result we derive is the general expression for the point availability of the single-component system when there are n spares, $A^{(n)}(t)$.

The system will be operational at time t if and only if the kth unit (k = 1, 2, ..., n+1) is in operation at time t. Thus,

$$A^{(n)}(t) = \sum_{k=1}^{n+1} P_k(t)$$

Now,

$$P_k(t) = P\left[\sum_{i=1}^{k-1} (T_i + R_i) \le t \text{ and } T_k + \sum_{i=1}^{k-1} (T_i + R_i) > t\right]$$

$$= \int_0^t (f*g)^{(k-1)}_{(s)} \bar{F}(t-s)\, ds$$

$$= [(f*g)^{(k-1)} * \bar{F}]_{(t)}$$

where

$$\bar{F}(t) = P(T_i > t).$$

We have,

$$A^{(n)}(t) = \sum_{k=1}^{n+1} [(f*g)^{(k-1)} * \bar{F}]_{(t)}$$

or

$$A^{(n)}(t) = \bar{F}(t) + \sum_{k=1}^{n} [(f*g)^{(k)} * \bar{F}]_{(t)} \quad (2\text{-}1)$$

This result can be rewritten recursively as

$$A^{(n)}(t) = A^{(n-1)}(t) + [(f*g)^{(n)} * \bar{F}]_{(t)} \quad (2\text{-}2)$$

and

$$A^{(n)}(t) = \bar{F}(t) + [A^{(n-1)} * f * g]_{(t)}$$

Each expression has some usefulness. Equations (2-1) and (2-2) are probably preferable computationally. Equation (2-2) provides a simple expression for the marginal contribution that the nth spare provides to system availability.

$$\triangle^{(n)}(t) = A^{(n)}(t) - A^{(n-1)}(t) = [(f*g)^{(n)} * F]_{(t)}$$

For the special case in which $f(t) = \lambda e^{-\lambda t}$ and $g(t) = \eta e^{-\eta t}$ the convolution expression is found using Laplace transforms to be

$$[f^{(k)} * g^{(k)} * \bar{F}]_{(t)} = [\frac{\theta}{\delta}]^k \cdot [\frac{t^k}{k!} + \sum_{r=1}^{k} (-1)^r \frac{(k+r-1)^P r}{r!\ \delta^r} \cdot \frac{t^{(k-r)}}{(k-r)!}] e^{-\lambda t}$$

$$+ (-1)^{k+1} \cdot [\frac{\theta^k}{\delta^{k+1}}] \cdot [\frac{t^{(k-1)}}{(k-1)!} + \sum_{\ell=1}^{k-1} \frac{(k+\ell)^P \ell}{\ell!\ \delta^\ell} \cdot \frac{t^{(k-\ell-1)}}{(k-\ell-1)!}] e^{-\eta t} \qquad (2\text{-}3)$$

where

$${}_n^P k = \frac{n!}{(n-k)!}, \quad \theta = \lambda\eta, \quad \text{and} \quad \delta = \eta - \lambda > 0.$$

As an example, if $\lambda = 1/30$, $\eta = 1/5$ and $t = 90$, we have

$$[(f*g)^{(7)} * \bar{F}]_{(t)} = [\frac{\theta}{\delta}]^7 [\frac{t^7}{7!} - \frac{7}{\delta} \cdot \frac{t^6}{6!} + \frac{7 \cdot 8}{2!\ \delta^2} \cdot \frac{t^5}{5!} - \frac{7 \cdot 8 \cdot 9}{3!\ \delta^3} \cdot \frac{t^4}{4!}$$

$$+ \frac{7 \cdot 8 \cdot 9 \cdot 10}{4!\ \delta^4} \cdot \frac{t^3}{3!} - \frac{7 \cdot 8 \cdot 9 \cdot 10 \cdot 11}{5!\ \delta^5} \cdot \frac{t^2}{2!} + \frac{7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12}{6!\ \delta^6} \cdot \frac{t}{1!}$$

$$- \frac{7\cdot 8\cdots 12\cdot 13}{7!\,\delta^{7}} e^{-\lambda t} + [\frac{\theta^{7}}{\delta 8}]\cdot[\frac{t^{6}}{6!}+\frac{8}{\delta}\cdot\frac{t^{5}}{5!}+\frac{8\cdot 9}{2!\,\delta^{2}}\cdot\frac{t^{4}}{4!}+\frac{8\cdot 9\cdot 10}{3!\,\delta^{3}}\cdot\frac{t^{3}}{3!}$$

$$+\frac{8\cdot 9\cdot 10\cdot 11}{4!\,\delta^{4}}\cdot\frac{t^{2}}{2!}+\frac{8\cdot 9\cdots 11\cdot 12}{5!\,\delta^{5}}\cdot t+\frac{8\cdot 9\cdots 12\cdot 13}{6!\,\delta^{6}}]e^{-\eta t} = 0.0033$$

B . Average Availability

Let $A_{av}^{(n)}(t)$ be the average availability, i.e.,

Then, from eq. (2-1), (2-2), (2-3) and the identity

$$\int \tau^{m} e^{a\tau} d\tau = e^{a\tau} \cdot \sum_{\rho=0}^{m} (-1)^{\rho} \cdot \frac{m!}{(m-\rho)!} \cdot \frac{\tau^{m-\rho}}{a^{\rho+1}}$$

we have

$$A_{av}^{(n)}(t) = \frac{1}{\lambda t}[1-e^{-\lambda t}]+\frac{1}{t}\sum_{k=1}^{n}\int_{0}^{t}[\frac{\theta}{\delta}]^{k}[\frac{\tau^{k}}{k!}$$

$$+\sum_{r=1}^{k}(-1)^{r}\cdot\frac{(k+r-1)^{P}r}{r!\,\delta^{r}}\cdot\frac{\tau^{(k-r)}}{(k-r)!}]e^{-\lambda\tau}\,d\tau$$

$$+\frac{1}{t}\sum_{k=1}^{n}\int_{0}^{t}(-1)^{(k+1)}[\frac{\theta^{k}}{\delta^{k+1}}][\frac{\tau^{k-1}}{(k-1)!}$$

$$+\sum_{\ell=1}^{(k-1)}\frac{(k+\ell)^{P}\ell}{\ell!\,\gamma\ell}\cdot\frac{\tau^{(k-\ell-1)}}{(k-\ell-1)!}\quad e^{-\eta\tau}\,d\tau$$

$$=\frac{1}{\lambda t}(1-e^{-\lambda t})+\frac{1}{t}\sum_{k=1}^{n}[\frac{\theta}{\delta}]^{k}\{[\frac{1}{\lambda^{k+1}}$$

$$+\sum_{\rho=0}^{k}(-1)^{\rho}\cdot\frac{e^{-\lambda t}}{(k-\rho)!}\cdot\frac{t^{(k-\rho)}}{(-\lambda)^{\rho+1}}+\sum_{r=1}^{k}(-1)^{r}\cdot\frac{(k+r-1)^{P}r}{r!\,\delta^{r}}$$

$$[\frac{1}{\lambda^{k-r+1}} + \sum_{\rho=0}^{k-r} (-1)^{\rho} \cdot \frac{e^{-\lambda t}}{(k-r-\rho)!} \cdot \frac{t^{k-r-\ell}}{(-\lambda)^{\rho+1}}]\}$$

$$+ \frac{1}{t} \sum_{k=1}^{n} (-1)^{k+1} [\frac{\theta^k}{\delta^{k+1}}]\{[\frac{1}{\eta^k} + \sum_{\rho=0}^{(k-1)} (-1)^{\rho} \cdot \frac{e^{-\eta t}}{(k-1-\ell)!}$$

$$\cdot \frac{t^{(k-1-\rho)}}{(-\eta)^{\rho+1}}] + \sum_{\ell=1}^{(k-1)} \frac{(k+\ell)^{P_\ell}}{\ell!\,\delta^{\ell}} [\frac{1}{\eta^{k-\ell}}$$

$$+ \sum_{\rho=0}^{k-\ell-1} (-1)^{\rho} \frac{e^{-\eta t}}{(k-\ell-1-\rho)!} \cdot \frac{t^{(k-\ell-1-\rho)}}{(-\eta)^{\rho+1}}]\}. \qquad (2\text{-}4)$$

For a numerical example, let $\lambda = \frac{1}{30}$, $n = \frac{1}{5}$ and $t = 90$. From egs. (2-1), (2-2), (2-3), (2-4), we determine the following values:

Table 2.1 : Point Availability vs . Average Availability

	$A^{(n)}(t)$	$A_{av}^{(n)}(t)$
n = 0	0.0498	0.316737644
1	0.2170845	0.57439463
2	0.4622845	0.739965467
3	0.6721845	0.81423
4	0.791011967	0.844388
5	0.8388	0.8534416
6	0.8531	0.8555475
7	0.856412	0.856957
	0.859	0.8639455

C. Tradeoff of Replacement vs. Repair

A high level of operational availability can be achieved in different ways. One can increase the reliability of the system components, one can provide generous spares support, one can build in system redundancy, and one can provide a maintenance capability. After an equipment is put into operation there is little chance to do anything about component reliability or system configuration. However, one can still consider logistics tradeoffs between providing spares support and a repair capability which consumes only piece-parts support.

Suppose that a system can be repaired with repair rate η' and assume that infinitely many repairs can be made. The repair rate η' required to provide a specified level of availability, A(t), can be determined by solving for η' in:

$$A_{\eta'}(t) = \frac{\eta'}{\lambda+\eta'} + \frac{\lambda}{\lambda+\eta'} \exp-(\lambda+\eta')t \qquad (2\text{-}5)$$

In evaluating the tradeoff between repair and spares support it is useful to compare the repair rate η' with the number of spares, n, required to achieve the same point availability. That is, we compare η' with the value of n which satisfies

$$A^{n}(t) = \exp(-\lambda t) + \sum_{k=1}^{n} [(f*g)^{(k)}*\bar{F}]_{(t)} \qquad (2\text{-}6)$$

Certainly, there are other factors to consider in making a decision about repair vs. replacement. For example, one would need to consider manpower requirements, training, piece-parts support needs, and space considerations. However, the tradeoff discussed above is useful as an indicator of whether or not one should even consider repair.

As a numerical example let $\lambda = \frac{1}{30}$, $\eta = \frac{1}{5}$, and t = 90.

$$A^{(1)}(t) = e^{-\lambda t} + (f*g*\bar{F})_{(t)} = 0.2170845$$

To achieve this level of availability we find that the maintenance rate η' should be at least 0.008233.

For n = 2,

$$A^{(2)}(t) = e^{-\lambda t} + \sum_{k=1}^{2} [(f*g)^{(k)}*\bar{F}]_{(t)} = 0.4622845.$$

On solving (2-5) for η', with $A_{\eta'}(t)$ set to 0.4622845, we find that η' = 0.028423.

Tables 2.2 shows the comparisons between n and η' for n = 1 to 7 (for values of n larger than 7 there is very little increase in $A^{(n)}(t)$).

Table 2.2 : Repair Rate as a Function of n(for a given $\{\lambda,\eta\}$)

η	A(t) Availability (λ,η)	η'
n = 1	0.2170845	0.008233 (η = 24.3 η')
2	0.4622845	0.028423 (η = 7.036 η')
3	0.6721845	0.0683353 (η = 2.98 η')
4	0.791012	0.126167 (η = 1.585 η')
5	0.8388	0.173449 (η = 1.1531 η')
6	0.8531	0.193578 (η = 1.033 η')
7	0.856412	0.198812 (η = 1.006 η')

Recall that η (the replacement rate in the finite spares case) is 0.20. The repair rate η' required to achieve the same level of system availability as achieved in the finite spares case is always less than η, but must converge to η as n gets large. The ratio between η and η' is shown in the last column of Table 2.2.

D. Distribution of Down Time Due to Lack of Parts

In a mission of a fixed duration, the contribution of the nth spare part cannot be determined solely by looking at the point availability. This is because, even with generous spares support, the system will alternate between up and down states as the system fails and is replaced. One indicator of the contribution of the nth spare part is the decrease in downtime that results from the inclusion of the spare part. In this section we address this problem by deriving the distribution of downtime due to the lack of spare parts.

Suppose we have n-1 spares available for a component for a mission of duration (0,t). As before let $\{T_i\}_{i=1}^{\infty}$ be i.i.d. random variables (exponential) representing "up" times and $\{R_i\}_{i=1}^{\infty}$ be i.i.d. random vairables representing replacement times.

Let

$$X_1 = \sum_{i=1}^{n} T_i + \sum_{i=1}^{n-1} R_i.$$

X_1 is the random variable representing the duration from time 0 to the point in time at which the component fails for the nth time.

If an nth spare (n+1 parts) is added to support this component then the nth spare will begin to function after a replacement time of length R_n. Let $X_2 = X_1 + R_n$, then the actual contribution of the nth spare to the availability is made after time X_2.

Let $Y = \max\{0, t-X_2\}$. Then Y is a random variable representing the duration of "down" time due to the lack of more than n-1 spare parts. Let $k^{(n)}(t)$ be the pdf of the random variable X_2. Then

$$k^{(n)}(t) = [(f*g)^{(n)}](t)$$

and the distribution function is given by

$$L_Y(x) = P\{Y \leq x\} = P\{X_2 \geq t-x\} = \int_{t-x}^{\infty} k^{(n)}(\tau)d\tau .$$

For example, when n = 2,

$$k^{(2)}(\tau) = [(f*g)^{(2)}](\tau) = [\frac{\theta}{\delta}]^2 [(\tau - \frac{2}{\delta})e^{-\lambda\tau} + (\tau + \frac{2}{\delta})e^{-\eta\tau}]$$

$$\ell^{(2)}(x) = \frac{dL(x)}{dx} = k^{(2)}(t-x)$$

$$= [\frac{\theta}{\delta}]^2 [(t-x-\frac{2}{\delta})e^{-\lambda(t-x)} + (t-x+\frac{2}{\delta})e^{-\eta(t-x)}]$$

for $0 < x \leq t$, and

$$\ell^{(2)}(0) = \int_t^{\infty} k^{(2)}(\tau)d\tau = [\frac{\theta}{\delta}]^2 [\int_t^{\infty} (\tau - \frac{2}{\delta})e^{-\lambda\tau} d\tau$$

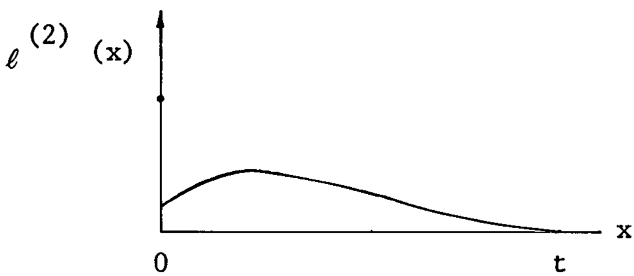
$$+ \int_t^{\infty} (\tau + \frac{2}{\delta})e^{-\eta\tau} d\tau]$$

since Y = 0 when $X_2 \geq t$.

Thus, the p.d.f. of Y when n = 2 is

$$\ell^{(2)}(x) = \begin{cases} [\frac{\theta}{\delta}]^2 [(\frac{t}{\lambda} + \frac{1}{\lambda^2} - \frac{2}{\lambda\delta})e^{-\lambda t} + (\frac{t}{\eta} + \frac{1}{\eta^2} + \frac{2}{\eta\delta})e^{-\eta t}] & \text{for } X = 0 \\ \text{and} & \\ [\frac{\theta}{\delta}]^2 [(t-x-\frac{2}{\delta})e^{-\lambda(t-x)} + (t-x+\frac{2}{\delta})e^{-\eta(t-x)}] & \text{for } 0 < x \leq t \end{cases}$$

Graphically, (2-7)



The closed form expression for the distribution function of Y for arbitrary n is:

$$L^{(n)}(x) = [\frac{\theta}{\delta}]^n \{ \frac{e^{-\lambda(t-x)}}{(n-1)!} \sum_{i=0}^{n-1} (-1)^i \frac{(n-1)!}{(n-i-1)!} \cdot \frac{(t-x)^{n-i-1}}{(-\lambda)^{i+1}}$$

$$+ \sum_{r=1}^{n-1} (-1)^r \frac{(n+r-1)^{P}{}_{r}}{r!\,\delta^r} \cdot \frac{e^{-\lambda(t-x)}}{(n-r-1)!} [\sum_{j=0}^{n-r-1} (-1)^j \frac{(n-r-1)!}{(n-r-j-1)!} \cdot$$

$$\cdot \frac{(t-x)^{n-r-j-1}}{(-\lambda)^{j+1}}]\} + (-1)^n [\frac{\theta}{\delta}]^n \{ \frac{e^{-\eta(t-x)}}{(n-1)!} \sum_{i=1}^{n-1} (-1)^i \cdot$$

$$\cdot \frac{(n-1)!}{(n-i-1)!} \frac{(t-x)^{n-i-1}}{(-\lambda)^{i+1}} + \sum_{r=1}^{n-1} \frac{(n+r-1)^{P}{}_{r}}{r!\,\delta^r} \cdot \frac{e^{-\eta(t-x)}}{(n-r-1)!} \cdot$$

$$\cdot [\sum_{j=0}^{n-r-1} (-1)^j \cdot \frac{(n-r-1)!}{(n-r-j-1)!} \cdot \frac{(t-x)^{n-r-j-1}}{(-\lambda)^{j+1}} . \quad (2\text{-}8)$$

If we define $\bar{L}^{(n)}(x)$ to be $1 - L^{(n)}(x)$, then the expected down time in the interval $(0,t]$ when there are n spares is given by

$$\int_0^t \bar{L}^{(n)}(x)\,dx,$$

and the expected amount of down time prevented by adding the nth spare part is

$$\int_0^t (\bar{L}^{(n-1)}(x) - \bar{L}^{(n)}(x))\,dx.$$

III. SUMMARY AND CONCLUSIONS

Operational availability is widely understood to be a measure of the likelihood that a system will function successfully when called upon. However, there are differences in the way that operational availability is calculated. U.S. Military documents[2] specify that operational availability be calculated by taking the ratio between the meantime to failure and the sum of the mean time to failure and the mean time to replace a failed unit. This definition ignores many of the critical factors of interest and it is often not mathematically correct.

The weapon system implied in this study is composed of components connected in series and parallel. Each component is subject to random failures and random restoration times until all replacement spares are exhausted. The failure times of each component are assumed to be independent and identically distributed with exponential distributions.

Similarly, the restoration times for a component are exponential, iid random variables. We have considered a case where the components operate independently of each other. For this system, the system availability can be determined easily after the point availability of each component has been determined as a function of the number of spares available. The formulae derived in this study can usefully applied to military logistic applications in calculating number of spare parts to each unit(PL/ASL) to achieve a desired level of availability of any weapon system.

Exact expressions for the point availability of a one component system and for the marginal contribution to availability of the nth spare are determined as a function of the failure and repair parameters and the number of spares. For large values of n, normal approximation are obtained to provide simpler expressions.

The exact expression for the average availability is determined from the point availability. The tradeoff between providing a maintenance capability (implicitly equivalent to an assumption of infinitely many spares and larger replacement times) and providing modular replacement with finitely many spares is discussed. The maintenance repair rate η' that achieves the same availability provided by the replacement policy with n spares is determined. When n is small, the replacement rate η must be very large compared to the repair rate η'; however, when n gets large the ratio of η' to η approaches one.

Given a fixed length of mission duration and a finite number of spares a system may not be available at the end of a mission due to a lack of spares. The probability distribution of this downtime is determined as a function of the number of spares and mission duration. This probability distribution could be used to generate other measures of effectiveness for determining the number of spares that should be allocated to a system.

For systems containing two or more components, the system availability can be determined easily by calculations identical to those used to calculate system reliability from component reliabilities whenever the components operate independently. All that is needed is to replace component reliabilities with component availabilities.

REFERENCES

1. MAVMAT Instruction P-4000, 1978.

2. OPNAV Instruction 4441. 12A, "Supply Support of the Operating Forces," 9 August, 1973.