Analysis of Money Supply Data by the Box-Jenkins Techniques A Case Study of Seasonal Time Series[1]

K.M Kim*
Y,H, Kim*
C.J. Park**
H.P. Shin*

ABSTRACT

This paper illustrates the application of the Box-Jenkins techniques for analyzing the monthly data of money supply from January 1969 to December 1977 of Korea.
Keywords: Box-Jenkins methods, Seasonal time series.

1. Introduction

The purpose of this paper is to illustrate the Box-Jenkins approach to time series analysis through a "case study". The techniques developed by Box and Jenkins (1) are applied to construct models for money supply (2) using the monthly data from January 1969 to December 1977. The Box-Jenkins method depends heavily on using a "good" package of computer program and we used the time series programs recently implemented at the Korea University Computer Center to illustrate a step-by-step application of the Box-Jenkins approach to seasonal time series. Throughout this paper we will assume that the reader is familiar with Box and Jenkins book (1).

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*: Professors at the Korea University, Seoul, Korea
**: Professor at the San Diego State University, San Diego, California, U.S.A.

There are numerous time series related articles published in recent years and annotated bibliography is presented.

2. The Data

The first step in time series analysis is to plot the observations against time. This will often show up important features such as trend (deterministic or stochastic), seasonality, outliers, and possible interventions if present in the series. The data are given in Table 1 and plotted in Fig. 1. The series shows a marked upward (deterministic) trend and a mild seasonal pattern. The size of seasonal pattern increases with mean level, indicating that the seasonal effect is multiplicative. Assuming that the error is also multiplicative, we took natural logarithms of the original series in order to fit a model with additive seasonal effect. Fig. 2 shows the transformed data. The size of the seasonal effect now appears to be roughly constant.

Table 1 **Money Supply in Korea. January 1969-December 1977**

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1969	1537	1555	1575	1610	1635	1648	1969	1756	1923	2010	2164	2179
1970	2310	2468	2365	2447	2562	2509	2593	2668	2750	2797	2972	3065
1971	3133	2997	2968	3193	3156	3001	3098	3199	3426	3365	3555	3608
1972	3602	3627	3573	3595	3651	3805	3924	4199	4638	4663	4978	5094
1973	5787	5790	5684	5854	5788	5977	6185	6267	6537	6802	7131	7303
1974	7689	7670	7517	7219	7105	7307	7497	7570	8850	8283	8841	9557
1975	9630	9421	9158	9060	9156	9284	9895	9948	10727	10499	11368	11817
1976	12000	12302	12144	12209	12221	12583	12869	13080	13275	13970	14374	15400
1977	15620	16382	16175	16395	16547	17357	17968	18081	20006	20403	20773	21726

(in 100 million won)

We examined the stability of the additive seasonal effect in the transformed series by plotting the successive January money supplies, successive February money supplies and so on. The trend lines for the 12 different months turn out to be roughly linear and parallel indicating that the seasonal pattern is

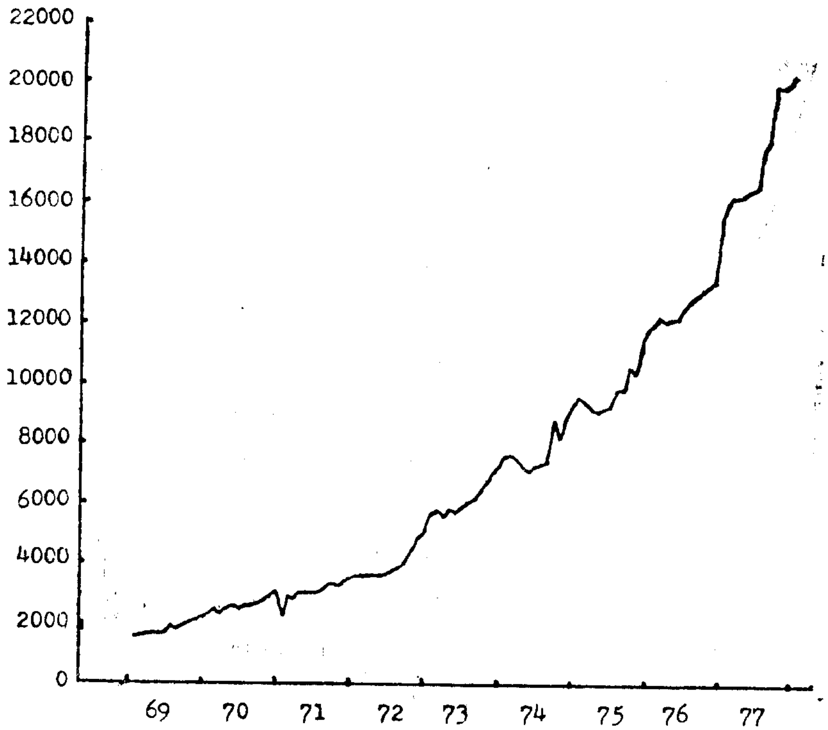

Figure 1. Money Supdly in Korea, Jan, 1969-Dec. 1977

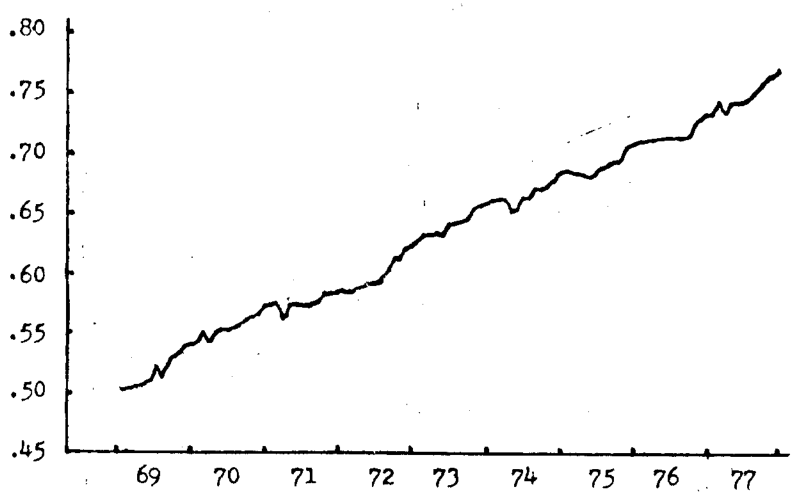

Figure 2. Logarithms of Money Supply in Korea

reasonably stable.

3. The Box-Jenkins procedure

Once the intial exploratory step is over, we come to the main part of our analysis the setting up of a Box-Jenkins forecasting model. The Box-Jenkins iterative procedure has three main stages;

(a) Model Indentification. Keep differencing the series until the SACF (Sample Autocorrelation Function) of a differenced series appears to have ACF(theorretical autocorrelation function) of a stationary series. Overdifferencing is to be avoided keeping with the principle of model parsimony. Next, examine the SACF and the SPACF(sample partial autocorrelation function) of the differenced series to see which model in the class of ARIMA processes appears to be the most appropriate. Again in determining the orders of an ARIMA model, model redundency should be carefully avoided.

(b) Estimation. Estimate the parameters of the chosen model by least squares. The SACF provides the initial values of the iteration algorithm.

(c) Diagnostic Checking. Examine the estimated residuals from the fitted model to see if it is adequate. We can use Portmanteau x^2-test of fit for this purpose. If the model appears inadequate for some reason, then other ARIMA models may be repeating these three stages until a satisfactory model is obtained. In some cases, one may have to trace all the way back to the exploratory stage to see if any other transformation of the data would have been more sensible.

4. Differeneing and Identifieation

Let the obsered money supplies at time t be denoted by x_t, and put $z_t=\log x_t$. The first stage of the Box-Jenkins procedure is to difference the series

$\{z_t\}$ until a stationary series, $\{w_t\}$ say. is obtained. As the series has a trend and a seasonal pattern, we have tried combinations of differencing of the form $(1-B)^d$ $(1-B^{12})^D$ z_t where B denotes the backshift operator. Normally the values of d and D do not need to exceed one.

The SACF and SPACF for each of the series $\{z^t\}$, $\{(1-B)z_t\}$, $\{(1-B^{12})z_t\}$, and $\{(1-B)(1-B^{12})z_t\}$ are computed and are given in Table 2. The series contain 108, 107, 96, and 95 observations respectively. We note that the autocorrelations for $\{z_t\}$ and $\{(1-B^{12})z_t\}$ are large and fail to die out at higher lags, $\{(1-B)z_t\}$ has reduced autocorrelations in general but still shows large correlations at lags 12 and 24, and $(1-B)(1-B^{12})$ markedly reduces autocorrelations throughout. We would have guessed this result beforehand, for it can be easily shown that $(1-B)(1-B^{12})$ removes a linear trend and a stable seasonal pattern.

The standard error of each autocorrelation coefficient would be $1/\sqrt{95}\cong$. .1 if the series $\{(1-B)(1-B^{12})z_t\}$ were random. Thus, "significant" values occur only at lags 1, 2, and 10. This is typical of most ARIMA processes

Table 2 Sample autocorrelation function of $(1-B)^d(1-B^{12})^D z_t$

Series	Lags	Autocorrelations								
z_t	1∼ 9	.97	.94	.91	.87	.84	.81	.79	.76	.73
	10∼18	.70	.68	.65	.62	.60	.57	.55	.52	.50
	19∼27	.48	.45	.43	.41	.38	.36	.34	.32	.29
	28∼36	.27	.25	.22	.20	.18	.16	.14	.11	.09
$(1-B)z_t$	1∼ 9	−.27	.13	.02	.01	−.11	−.18	.01	−.12	−.08
	10∼18	.16	.01	.27	−.05	.14	−.11	−.12	−.01	−.16
	19∼27	−.12	−.14	−.02	.15	−.11	.23	.03	.10	−.04
	28∼36	−.06	−.03	−.15	−.06	−.09	−.01	.11	.00	.25
$(1-B^{12})z_t$	1∼ 9	.85	.77	.73	.62	.48	.37	.29	.15	.02
	10∼18	−.07	−.17	−.33	−.39	−.43	−.49	−.52	−.53	−.54
	19∼27	−.54	−.52	−.51	−.48	−.46	−.39	−.29	−.24	−.15
	28∼36	−.07	−.01	−.05	−.10	.14	.19	.24	.27	.29
$(1-B)(1-B^{12})z_t$	1∼ 9	−.28	−.09	.19	.18	−.15	.02	.11	.02	−.13
	10∼18	.04	.22	−.34	−.03	−.08	−.15	−.10	.08	−.02
	19∼27	−.07	−.05	−.00	.02	−.16	−.07	.12	−.15	.12
	28∼36	.00	−.01	.00	.05	−.07	−.01	.08	.03	.07

Sample partial autocrrelation function of $(1-B)^d(1-B^{12})^D z_t$

Series	Lags	Partial Autocorrelations								
z_t	1~ 9	.97	−.00	−.03	−.02	.01	−.02	.05	−.05	.01
	10~18	−.01	−.01	−.01	−.01	.02	−.04	.00	−.00	−.03
	19~27	−.00	.00	−.02	−.02	−.02	−.01	−.02	−.03	−.01
	28~36	−.01	−.00	−.04	.01	−.01	.00	−.04	−.02	−.04
$(1-B)z_t$	1~ 9	−.27	.06	.07	.02	−.12	−.27	−.10	−.09	−.12
	10~18	.13	.07	.28	.06	.02	−.14	−.23	−.05	−.03
	19~27	−.08	−.17	−.18	.03	−.12	−.02	.02	.06	.06
	28~36	−.05	−.10	−.08	−.05	−.04	−.01	.01	−.04	.06
$(1-B^{12})z_t$	1~ 9	.85	.16	.13	−.17	−.30	−.07	.03	−.14	−.14
	10~18	−.06	−.07	−.31	.08	.08	−.01	.05	−.09	−.15
	19~27	−.04	−.09	−.10	.01	−.08	−.02	.17	−.04	.07
	28~36	−.05	−.04	−.10	−.05	−.14	.02	.10	−.09	−.07
$(1-B)(1-B^{12})z_t$	1~ 9	−.28	−.19	.12	.29	.04	−.02	.00	.04	−.08
	10~18	−.07	.20	−.23	−.18	−.13	−.19	−.02	.07	.06
	19~27	.04	−.04	−.13	−.04	−.03	−.21	−.03	−.14	.09
	28~36	.04	.04	.05	.03	−.07	−.19	.03	.03	.06

obtained through the differencing $(1-B)(1-B^{12})$. Note that further differencing will only reduce the number of terms in the series, thus increasing the standard error, without substantially improving the autocorrelations.

once apparent stationarity is achieved by proper differencing, the next step is model identification on the basis of the SACF and the SPACF. The general class of seasonal ARIMA models may be written

$$\phi p(B)\Phi_P(B^s)w_t=\theta_o+\theta_q(B)\Theta_Q(B)^s)a_t, \qquad (1)$$

where $w_t=(1-B)^d(1-B^s)^D z_t$ is the differenced series, $\phi_p, \Phi_p, \theta_q, \Theta_Q$ are polynomials of order P, p, q, Q respectively, and a_t are independent random variables with mean zero and variance $\sigma_a{}^2$. The integer s is the seasonal period and the constant θ_0 allows for polynomial deterministic trend of order $d+D$. The seasonal ARIMA model (1) with $\theta_0=0$ is simply denoted by $(p,d,q)\times(P,D,Q)_S$. We have identified the values of s, d, D, already and $\theta_0=0$ from assumed stationarity of w_t. Our problem now is to identify the orders p,q,P,Q, Keeping in mind that they should be chosen as small as possible so long as they adequately fit the series.

To find the values of P and q the first several values of the SACF and the SPACF are examined. Relatively high values of the first four or five coefficients in both the SACF and the SPACF suggest that large values of P and q will not guarantee substantial improvement in fit. Hence the values of p and q are safely restricted to 0 or 1. But looking at the autocorrelation coefficients, the possibllity of $p=1$ is also ruled out, because then the negative first autocorrelation coefficient should have been followed by a positive second coefficient of almost square of the first coefficient. Whatuer $q=0$ or i is hard to tell just from the coefficients and our choice $q=1$ is admitetdly subjective in this case. Similar resoning is applied to the coefficients at lags 12, 24, and 36 suggesting again the first order MA Process for seasonal averages ($p=0$, $q=1$). Thus we arrive at the trial model $(0, 1, 1)\times(0, 1, 1)_{12}$: $(1-B)(1-B^{12})z_t=(1-\theta B)(1-\Theta B^{12})a_t$.

4. Estimation and Diagnostie Cheeking

Having identified the tentative model, the next step is to estimate the parameters θ and Θ by finding the least-squares solution of $\underset{\theta,\Theta}{\text{Min}} \sum a_t^2$ where

$$a_t=a_t(\theta,\Theta)=w_t+\theta a_{t-1}+\Theta a_{t-12}-\theta\Theta a_{t-13}. \qquad (3)$$

We have set the initial values a_t, $t\leqq 0$, to zero and the a_t are calculated by using (3) recursively for any given pair of values (θ,Θ).

A nonlinear least-squares approach recommended by Box-Jenkins is used in our program which finds the solution by numerical iteration. Noting that the theoretical values of the first and the 12th autocorrelation coefficients of our trial model (2) are given by $\rho_1=\frac{-\theta}{1+\theta^2}$ and $\rho_{12}=\frac{-\Theta}{1+\Theta^2}$, the preliminary values θ_0 and Θ_0 for the iteration are computed from these identities when ρ_1 and ρ_{12} are replaced by the corresponding sample coefficients. The program gave estimates $\hat{\theta}=.34$ and $\hat{\Theta}=.525$. Our fitted model is now given by

$$w_t=(1-.34\mathrm{B})(1-.525\mathrm{B}^{12})a_t. \qquad (4)$$

The last stage in the Box-Jenkins iterative procedure is diagnostic check.

The most important diagnostic check is provided by the ACF of the residuals listed in Table 3. We have only one 'significant' value (i.e. a value greater than twice the standard error(1) at lag 4 out of 24 coefficients.

The overall adequacy of the model is checked by the Portmanteau lack of fit test. The test statistic $Q=95\sum_{1}^{24} r_k^2=21.71$ is computed where r_k is the k^{th} autocorrelation coefficient of the residuals. If the model is adequate, Q should have approximately chi-square distribution with $22=24-1-1$ degrees of freedom. The computed value of the statistic is not significant indicating that the model is not inadequate. Hence the fitted model (4) appears to give a reasonable fit to the data despite the high value at lag 4.

5. Foreeasting

Once the three stage iterative process is completed, the forecasting is the next step in the Box-Jenkins procedure. We have calculated forecasts and 95% tolerance limits based on the fitted model and they are presented in Table 4. The forecasts are made at two different time origins up to 12 months ahead. The tolerance limits are calculated by $1\times\sqrt{1+\sum_{1}^{l-1}\Psi_j^2\times\hat{\sigma}_a^2}$ where $\hat{\sigma}_a^2$ is the estimated resimated variance $\sum a_t^2(\hat{\theta},\hat{\Theta})/95$ and the Ψ_j are the weights for the process $(1-B)(1-B^{12})z_t=(1-.34B)(1-.525B^{12})a_t$ when the process is rewritten in the form $z_t=(1+\Psi_1B+\Psi_2B^2+\ldots)a_t$ as an infinite moving average process. The computed weights are .66 for Ψ_1 through Ψ_{11} and 1.135 for Ψ_{12}.

Table 3 **Autocorrelation function of the residuals frcm the model $(1-B)(1-B^{12})z_t=(1-.34B)(1-.525^{12})a_t$**

Lags	Autococrrelaticns					
1~ 6	−.03	−.03	.18	.24	−.05	.03
7~12	.06	.07	−.06	.02	.02	.02
13~18	−.16	−.09	−.10	−.06	−.00	−.04
19~24	−.10	−.11	−.05	.00	−.20	−.08

Table 4 Forecasts of z_t made at Dec. 1975 for the following 12 months

Periods ahead	Lower limit	Forecast	Upper limit	Actual
1	7.037	7.116	7.194	7.090
2	7.010	7.105	7.199	7.114
3	6.974	7.082	7.189	7.102
4	6.955	7.075	7.194	7.107
5	6.946	7.076	7.206	7.108
6	6.954	7.095	7.235	7.137
7	6.994	7.143	7.293	7.159
8	6.996	7.154	7.312	7.176
9	7.080	7.246	7.413	7.191
10	7.051	7.226	7.400	7.242
11	7.113	7.295	7.477	7.270
12	7.147	7.336	7.525	7.339

Forecasts of z_t made at Dec. 1976 for the following 18 months

Periods ahead	Lower limt	Forecast	Upper limit	Actual, if known
1	7.283	7.361	7.440	7.353
2	7.273	7.368	7.462	7.401
3	7.242	7.350	7.457	7.388
4	7.229	7.348	7.468	7.402
5	7.219	7.350	7.480	7.411
6	7.233	7.373	7.513	7.459
7	7.260	7.409	7.559	7.493
8	7.264	7.423	7.581	7.509
9	7.312	7.478	7.645	7.601
10	7.317	7.492	7.666	7.620
11	7.360	7.542	7.724	7.633
12	7.407	7.596	7.785	7.683
13	7.412	7.622	7.831	
14	7.405	7.628	7.851	
15	7.374	7.610	7.846	
16	7.361	7.609	7.856	
17	7.350	7.610	7.869	
18	7.363	7.633	7.904	

What we really want fo hnow in the forecart of z_t mode at Dec. 1977 for the following 12 manths.

6. Conelusion

We believe that there is no such thing as a true model for the money supply series. Rather the model selected for the data will be an 'approximation' to the truth. The Box-Jenkins procedure is based on a notion of '"Let the data speak for itself". Thus what we have done in this paper is an illustration of how the statisticians communicate with the data filtered by the Box-Jenkins procedure. We leave one last note of warning to the future users of the Box-Jenkins program: that the subjective element in the Box-Jenkins procedure which allows us to choose from a wide class of models can be both the strengh and the weakness of the procedure. Not being a fully automatic forecasting procedure unlike other simpler forecasting procedures such as Holt-Winters, the user's subjective judgement is crucial at times and hence considerable experience with the program and thorough understanding of the Box-Jenkins theory are a must before the stregth of the procedure is realized. For example, we have tried this program to other seasonal data such as monthly industrial production index data and monthly coal production data in Korea, and encountered serious difficulties in enforcing the Box-Jenkins 'recipes' in the program straightforwardly. There was no one model with superior fit, and even on the basis of forecasting ability, the verdict was inconclusive with different models faring differently depending on the origin of forecast and the lag time. The same type of difficulty was reported by Chatfield and Prothero(1973) with monthly sales data of a company.

With this warning notwithstanding, we urge prospective users to try their hands on this program to find out for themselves the type of data suitable for the Box-Jenkins and to gain the expertise necessary for the analysis to be effective.

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Key to notation: each reference has been annotated by the following letters to indicate the major topic and the level of the reference.

A-Application

B-Bayesian Model

E-Exponential Smoothing, Forecasting

F-Adaptive, Filtering

G-General interest or Survey

P-Periodic, Seasonal

T-Theoretical

X-Estimation

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