Mechanism of Longitudinal Dispersion in Open Channel Flow with Secondary Flow Effects

by
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要 旨

本 論文에 있어서 汚染物質의 分散現象에 關한 解析은 乱流의 剪断効果에만 依存해온 過去의 研究와는 달리, fingering 現象과 2次流의 特性과의 關係로 考察하는 것이 本 研究의 目的이다. fingering 現象의 発達過程과 一般的인 諸因子들을 決定하기위해서, 横方向速度分布와 速度変動은 taft 法과 水素気泡法에 依해서 測定 하였고, 特히 濃度의 流下状態는 laser－beam System을 使用해서 明確하게 測定 하였다. 이 現象은 流動모델에 依한 2次流를 考慮 하므로써 明白하게 説明 할수 있었다. 그 結果, 分散係数는 乱流構造와 함께 断面内 2次流에 依해서도 強한 影響을 받는 것이 究明되었다. 또, 分散係数와 2次流와의 關係는 試行的으로 (5)式을 提示 하였고, 한편 実用的으로는 流速, 水深, 水路幅, 粗度等의 諸水理量을 考慮한 実験式 (8)을 提案 하였다.

I. Introduction

Understanding on the stream ability of dispersing pollutants is essentially important to an effective abatement of pollution in streams. An important parameter characterizing the ability is the longitudinal dispersion coefficient (hereafter, simply referred to the "dispersion coefficient", D.). The coefficient combines the effect of diffusion with the dispersion, making it possible to determine in a simple method the spread of pollutants or nonpollutants over long distances along streams. An application of D is pollution forecast or prediction. Following an accidential discharge of radioactive materrial etc. in a river, whatever the cause of the accident, it is important to providewater users in downstream with an immediate information on the expected pollutant concentration in river at various times and locations. Another application of D is the prediction of water temperature variation in downstream which is needed not only in predicting the salt intrusion into tidal estuaries, but also in river water quality models etc., In practice, the numberical value of D is obtained from an experimental tracer tests. Taylor[1], the first researcher to attempt such a prediction, assumed that the over-all mixing described by the dispersion coefficient D was the sum of that caused by tuebulent velocity fluctuations and the differences in time-average velocity over the cross-section. He carried out his analysis for an empirical velocity distribution in a circular pipe and assumed that turbulent eddy diffusivities were isotropic and could be defined by the Reynolds analogy, which states that mass transport equation and obtained. Elder[2] used Taylor's mathematical approach and applied a logarithmic velocity distribution for two-dimensional open channel flows and proposed simple formula. This result has been widely cited and examined in literatures. It is now well known that even for truly two-dimensional open channel flows, the value of (dimensionless dispersion coefficient) could differ considerably from 5.93. This is due to the fact that the logarithmic velocitydistribution is not exactly correct, and that small variation of velocity distribution can cause large difference in the magmitude of.

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Fischer[3] simplified the general expression of Aris, and showed that the general method used by Elder was also applicable to lateral velocity gradients. He derived the equation for dispersion dominated by the lateral variation of velocity. He was also applicable to a time scale for the dispersion. On the other side, the main mechanism causing the diffusive movement of tracer cloud in laboratory channel flows has been shown by A. Murota[4)5)] to be the motion of plural logitudinal streak in the convective phenomenon, (hereafter, referred simply to the fingering phenomenon). Fingering phenomenon observed is that the dye cloud mixing in water devides in several branches and looks as if it has several fingers with longitudinal streak. This phenomenon can be attributed to the fact that is special phenomenon in the laboratory open channel, and if this is related to the secondary flow, it can give effective knowledge for understanding in the mechanism of longitudinal dispersion.

The analysis of dispersion in this paper is different from past works which is dependent upon the effects of shear flows in the turbulent. The objective of this work is to investigate dispersion mechanism in relation to fingering phenomenon and the characteristics of secondary flow. The results will be clearly reported in the form of the empirical correlations and the method of multiple regression from known hydraulic parameters with the characteristics of the secondary flow. All correlations are based on the hydraulic parameters defined by a manner described in the following section.

2. Experiment and Results

The laboratory flume used is 20 m long, 0.5 m wide, and 0.32 cm high, with an acylic flume. It is mounted on a tilting truss and is provided with a recirculating system. Values of experimental results are shown in Table-1 with the hydraulic data, where Sn, H, and Ua represent respectively the number of secondary circulation, water depth, and mean velocity. The discharge and the flume slope are within 1.7-7.0 l/sec and (0.2-1.5)x10^{-4}, respectively. Dispersion measurements were made using methyle violet (blue) solution as a tracer.

The methyle violet solution is mixed with fresh water, and the concentration of methyle violet in the tracer solution is 200 ppm and 0.3 liter in a time. The solution is discharged instataneously as a line-source into the flume. The concentration of tracer were measured at mid-depth using a laser-beam system, and it is recorded on a tape recorder and chart recorder. Flows-direction with the laser-beam system are made in 10 m (section I) and 15 m (section II) at two downstream points (from the injection point (x=0). In the case of uniform steady flow, the distribution of velocity measurements is performed by a method which take serially the photograph of the ranks of hydrogen bubble that advanced in cross-section 15 m from uperstream end (x=0 by 16 mma and 32 mm camera. Then the flow structure and thus the velocity components can be defined from the photographs in the lateral direction at space of 2 cm by the following characteristics; u_i (y, z) is momentory velocity components in the flow -direction (x-direction) every 0.3 second. U(y, z) and u (y, z) are respectively an ensemble average velocity and a turbulence intensity in each measuring points, in which y and z are respectively an axis of vertical and lateral direction. As shown in Col. (5)-(12) of Table -1, Uo, ΔU/Uo, u/Ua, D, D/u*H, D/ΔUL, and L represent respectively the mean velocity-deviation (hereafter referred for convenience as the "intensity of secondary flow"), the turbulence intensity, the dimensionless quantities of shear velocity, the dispersion coefficient, the dimensionless dis-

Table 1 Summary of hydraulic data and results

Case Sn-H-Ua	u* (1)	Fr. (2)	Re. (3)	fx10^{-3} (4)	Uo (5)	ΔU/Uo (6)	u/Uo (7)	u*/Ua (8)	D (9)	D/u*H (10)	D/ΔUL (11)	L (12)
5-10-14	0.802	0.14	13903	5.02	14.63	0.063	0.102	0.057	94.4	13.5	2.59	32
6- 8-14	0.697	0.16	11538	4.96	14.94	0.056	0.101	0.052	90.4	15.6	2.70	31
7- 7-14	0.897	0.16	10310	8.83	16.15	0.063	0.072	0.064	90.8	14.5	2.61	30
7- 7-10	0.660	0.18	7630	9.07	12.46	0.066	0.073	0.066	93.6	20.0	3.43	30
7- 7- 5	0.327	0.06	3820	8.39	7.50	0.063	0.075	0.065	65.0	28.8	3.74	31
10- 5-14	0.759	0.20	6951	5.88	15.08	0.059	0.106	0.054	80.0	21.1	3.00	30

** $u^*=(gRI)^{1/2}$; $Fr.=Ua/(gH)^{1/2}$; $Re. = 9UaH/\mu$; $f=2(u^*/Ua)^2$**

persion coefficient, the dimensionless dispersion coefficient by u*H, the dimensionless dispersion coefficient by LΔU, and character-length for mixing.

And then all the velocitys used in this paper are defined as followings (1) The mean velocity in a cross section; Ua=Q/A. (2) Uo(y) is the average velocity in the lateral section, (mean velocity at the depth of y/H=0.86). (3) The average time velocity in the voluntary points, $U(y, z) = 1/N\sum_{i=1}^{N} ui(y, z)$. (4) The average turbulent velocity fluctuations, $u(y, z) = [1/N\sum_{i=1}^{N} (ui(y,z) - U(y,z)^2]^{1/2}$. (5) The deviation of velocity in a cross-section, CU = Up (the velocity of high-speed streak) - U_L (the velocity of low-speed streak). Where Q, A, and N represent respectively the discharge, the area, the number of sample (N=25 in this paper).

3. Determination and Estimation of Characteristics for secondary Flow

For the fingering phenomena, it can be seen that the number of fingers or the number of longitudinal streak N_f produced by a liquidity model of the secondary flow versus aspect ratio B/H appears to be much more affected than the other factor, N_f can be express ed as,

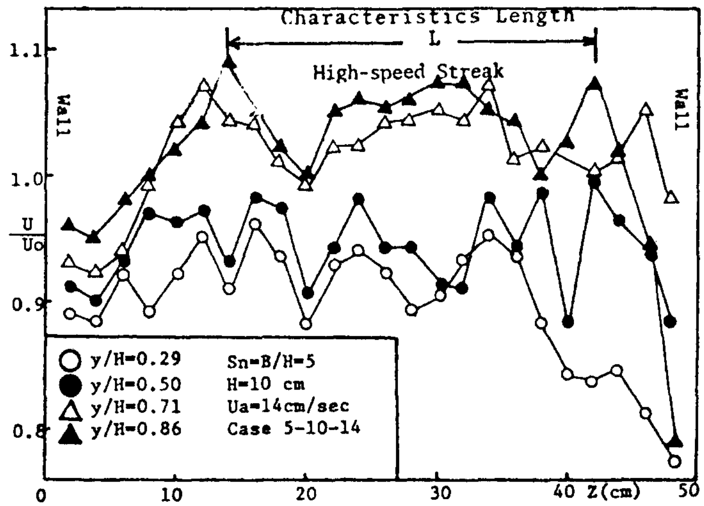

Fig. 1. Determination of the characteristics length L

$$N_f = \frac{1}{2} \left(\frac{B}{H}\right)$$

For the stability of fingering phenomena, a relationship to aspect ratio in the laboratory channel is examined. When the number of B/H is even, the number of circulation become stable, but when the number of B/H is odd, N_f become unstable. For example, in the case of B/H=5, the number of N_f become 2 or 3 pairs. Namely, it can be considered that occuring condition of the secondary flow becomes stable for an even number of B/H, but becomes unstable for an odd number of B/H.

3-1. Determination of Characteristics of Secondary Flow

A lateral distribution of the main velocity component U/Uo is shown in Fig. 1 for determination of the characteristics of the secondary flow. This distribution of U/Uo indicates a nonuniformity in cross-section. In this case, it is assumed that the velocity distribution of nonuniformity has an important effect upon the dispersion phenomena of the tracer in the open channel flow. All the characteristics of the secondary flow is described in the following several quantities. First of all, the deviation of velocity ΔU from an averaged value of main stream in used to index for the intensity of secondary flow, and if Uo is chosen as a denominator, then the value of ensemble averaging ΔU can be nondimensionally given as ΔU/Uo. This value for experimental data is summarized in Col. (6) of Table-1 in the case the values of y/H=0.5

Secondly, the method specially used to determine the characteristic-length of the lateral direction L is assumed in the form to be seems in Fig. 1, i.e. L is an average distance from occuring point of high-speed streak to the most distant wall with four points in the y/H (y/H=0.29, 0.50, 0.71, and 0.86 points). The results are listed in Col. (12) of Table-1. Of course, a lot of methods to determine L has been represented. For example, by using a typical Fischer's prediction, the distance between an occuring point of low-speed streak, and the distance from the point of maximum high-speed streak to the most of distant wall etc., In this paper, however L is obtained by the method as in Fig. 1. In Table-1, dimensionless velocities, ΔU/Uo, u/Uo, and u*/Ua have approximately a same order of magnitude.

As shown in Col. (12) of Table-1, values of L for these streams range is between 30 cm and 35 cm, which is 3.9-6.0 times the depth and 0.6 times the width.

3-2. Estimation of characteristics

Characteristic-quantities of secondary flow give a define as following; (1) The aspect ratio (width-to-depth) is a characteristic for an occurence of the secondary circulation in a cross-section. (2) ΔU/Uo represents an intensity of the secondary flow in a cross-section. (3) L is a characteristic-length for a mixing scale. (4) B/H, ΔU/Uo, and L are the characteristics for an occurence, intensity and mixing scale, respectively.

Fig. 2 shows ΔU/Uo versus dimensionless friction velocity u*Ua.

According to Fig. 2, it can be seen that a correlation between these parameters is closely connected with the increase in ΔU/Uo as u*/Ua. This result also suggests that ΔU/Uo is sufficiently included universality with the dimensionless friction velocity u*/Ua.

Fig. 3 is a plot of ΔU/Uo versus the aspect ratio of ΔU/Uo versus the aspect ratio B/H using all the data.

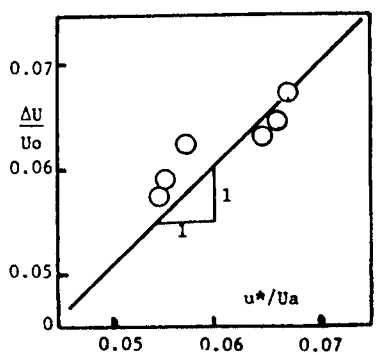

Fig. 2. Relation between ΔU/Ua and u*Ua

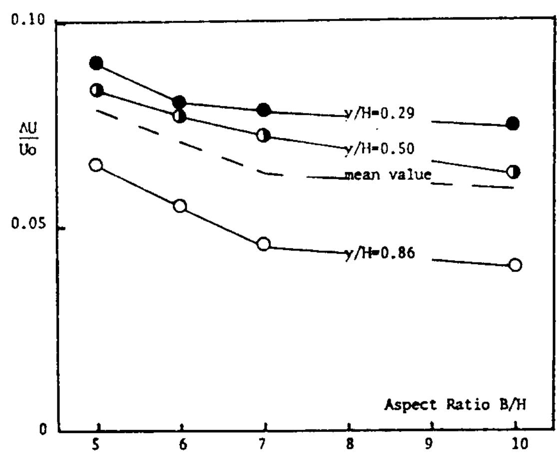

Fig. 3. Relation between intensity of secondary flow (ΔU/Uo) and aspect ratio(B/H) at constant velocity, roughness and width (changing depth)

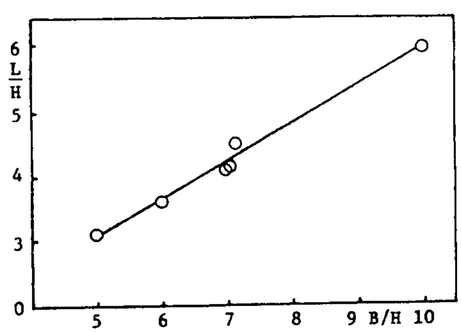

Fig. 4. Relation between L/H and B/H

The value of y/H is written on mid-way each data line.

It can be seen that ΔU/Uo also increased as the width was reduced in the trend. As compared with water depth, an occuring source of the secondary flow can be found along the channel bed. A dotted line in Fig. 3 is a plot averaged value in the all y/H. Fig. 4 shows a relation between the dimensionless characteristic-length L/H versus aspect ratio B/H, considering the secondary flow for the estimation of characteristic-length L. Fig. 4 shows that the correlation is very high. From the above-mentioned discussion, it can be concluded that the intensity of the secondary flow ΔU/Uo versus dimensionless friction velocity u*Ua may play an important role for an analysis of the longitudinal dispersion mechanism.

4. Characteristics of Secondary Flow and Dispersion Coefficient

The dispersion coefficient is calculated by the change of moment method from a measured concentration-time curve (simply, C-t curve).

The results are listed in Col. (9) of Table-1. The relationship between the characteristics and the dimensionless dispersion coefficient is analysed as follows;

Fig. 5 is a plot of the dimensionless dispersion ceofficient D/u*H versus aspect ratio B/H using the friction

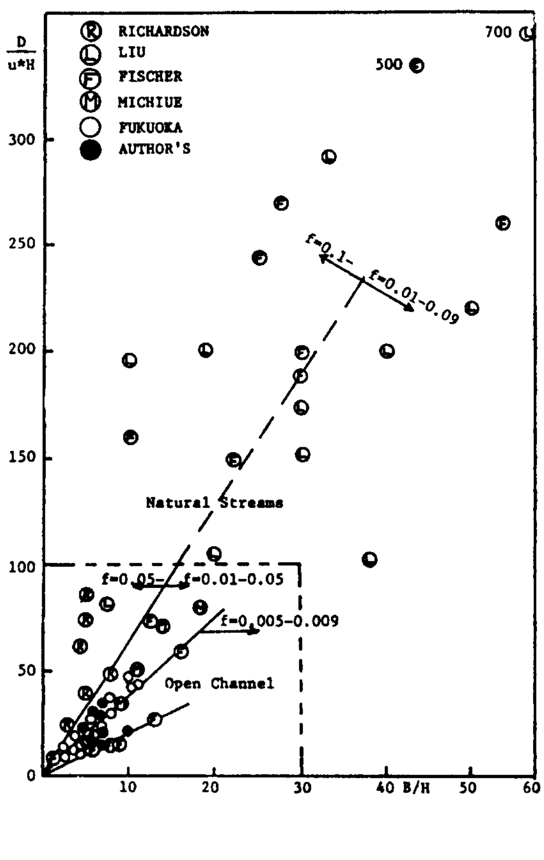

Fig. 5. Published data of dispersion coefficients D/u*H versus aspect ratio B/H for open channel and natural streams (values of friction factor f are written straight line)

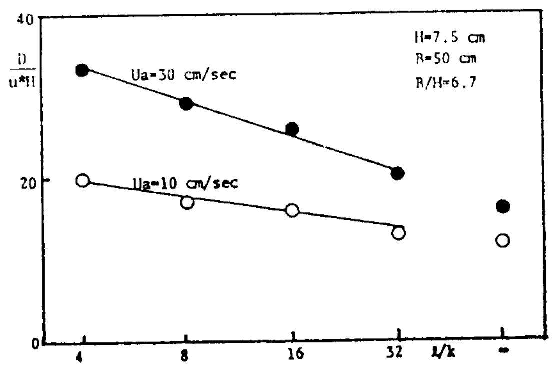

Fig. 6. Normalized dispersion coefficients versus strip roughness at constant width and depth

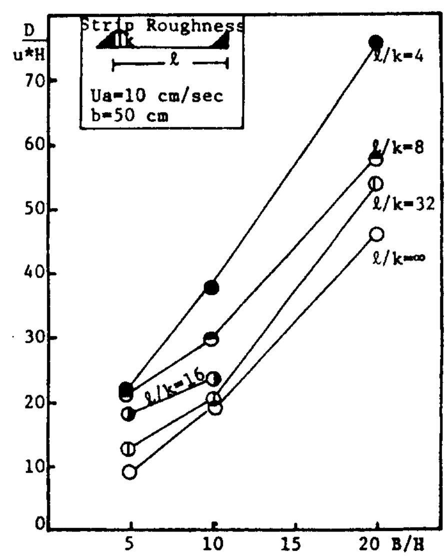

Fig. 7. Variation of D/u*H with B/H and l/K

coefficients of for previously published data and writers' data. And Fig. 5 plotted to compare the dispersion coefficient with the natural streams and laboratory channel in the wide range. As shown in the figure, D/u*H is propotion to B/H with increasing f, although as larger values of f, the trends shown become to steep gradient. The correlation with D/u*H, f, and B/H in laboratory channel is scattered smaller than natural streams in Fig. 5. However, the values of friction factor in natural streams is as giving a larger spreading range. The distributional state of D/u*H is scattering from the shown f-line, can be thought as tendency to ambiguity of an determined method for f.

On the other side, one for the purpose of getting a fitting estimation in Fig. 5, where the value of dispersion coefficient D are nondimensionalized. As considering that the secondary flow appears to be the more important factor governing shear effects in a cross-section, it seems more appropriate to use the channel width as chasection, it seems more appropriate to use the channel width as characteristic-length scale. In this case, D/u*HB would be the dimensionless dispersion coefficient to use. This shows the plot of D/u*B versus B/H with friction factor as the third parameters, and it is clearly recognized from the figure that D/u*B is not constant even for in the experiments of straight open channels. On the other hand, the work which tried to account for the effects of parameter is the relationship between the transverse diffusion coefficients E_z and the aspect ratio B/H, i.e. it is a plot E_z/u*B versus B/H A plot of E_z/u*B versus B/H shows clearly relationship of a hyperbola, with decrease of Ez/u*B and increase of B/H. Eventually, the transverse diffusion coefficients is E_z =Mu*H, but it can be though that the dispersion coefficient D is related to the channel width B. Fig. 6 shows a few cases in which the aspect ratio is same, but the mean velocity and bed roughness (l/k) is changed. In the case, l/k is the ratio of the space-to-high of strip-roughness and, the roughness coefficient and friction factor also increase as l/k is reduced. It can be seen that D/u*H increases with the mean velocity but also decreases as l/k increases. For the purpose of getting a fitting relationship between the secondary flow and Fig. 6 objectively, Fig. 7 shows a few case in which the width and velocity is constant, but the flow depth and bed roughness is changed. It is a plot of correlation with D/u*H, B/H, and l/k. As shown in the figure, D/u*H increases as f and B/H decrease.

From Fig. 6 and Fig. 7, when B/H is constant, the effects of the secondary flow aresame, and the increase of D/u*H be effected by roughness, the intensity of secondary flow will be increased.

This is on the base of relationship between occuring source of the secondary flow and the coefficent of roughness as above-mentioned. And, the more B/H or mean velocity increase the more D/u*H will be. In this fact, the secondary flow be accelerated to mixing pollutant, and same time, the mixing poluutant will be convected by mean velocity.

5. Prediction of Dispersion Coefficient

5-1. Dimensional Analysis

In the former of a theoretical analysis for the dispersion coefficient, it ought to be useful to perform a dimensional analysis of the relationship between the secondary flow and dispersion and examine the variables which may be important. In the straight rectanggular open channel, the parameters affecting the dispersion coefficient are mean velocity Ua, depth H, width B, shear velocity u*, density p and viscosity μ. Therefore, the dispersion coefficient D is represented by the following equation.

$$D = F\ (Ua,\ H,\ B,\ u^*,\ p,\ \mu,) \tag{2}$$

If, H u* and p are chosen as the basic parameters, then, Eq. (2) can be nondimensionalized and given as E. (3).

$$\frac{E}{u^*H} = F\ (\frac{u^*}{Ua},\ \frac{B}{H},\ \frac{pu^*H}{\mu}) \tag{3}$$

The effect of Re can be neglected by the empirical method (for highly turbulent flows with roughly boundaries, the effect of viscosity can be neglected). And one rewrite as Eq. (4).

$$\frac{D}{u^*H} = F\left(\frac{u^*}{Ua}, \frac{B}{H}\right) \quad \text{or} \quad \frac{D}{u^*H} = F\left(\frac{\Delta U}{Uo}, \frac{B}{H}\right) \tag{4}$$

Therefore, it can be seen that the usual dimensionless dispersion coefficient is a function of both, the dimensionless dispersion coefficient is a function of both, the dimensionless shear velocity (or intesity of secondary flow) and the aspect ratio.

Next, estimation of the dispersion coefficient is tried to directly account for using characteristics of the secondary flow (L and ΔU). In the case, it is in need of two ways for fundamental thought as follows; (1) In order to clarify therelationship between the characteristics of secondary flow and the dispersion coefficient it is necessary to be considered of a concetion of scale.

(2) The dispersion coefficient, in which the characteristics of the secondary flow in entroduced, is evaluated in the same manner to Elder's method. And, from the results the characteristics of both and properties of the secondary flow effectingon the dispersion phenomenon can be estimated. From (1) and (2) above," when on open channels the secondary flow is considered, the dispersion coefficient can be expressed by the characteristic-length α of the secondary flow multiplied by the deviation of velocity in a cross-section ΔU".

$$D = \alpha_s L \ \Delta U \tag{5}$$

in which M_s are given in Col. (11) of Table-1, its average value is 2.98. Considering from Fig. 2 to Fig. 7 the Eq. (5) can be expressed as Eq. (6).

$$D = \alpha \ u^* H \tag{6}$$

The value α in Eq. (6) is 100 times larger than the proportional constant α_E of the transverse diffusion coefficients expressed in the section 4, and αs is 0.16 α. As shown in Table-1, the value of α_s will increase proportionally with the value of B/H, but Elder's α values in open channel have no relation with the aspect ratio, and keeping constant the values of α in general.

Consequently, if Eq. (5) will be introduced into same Eq. (5) differed fundamentally with Eq. (6). Because u* in the Elder's is a representative member of the vertical velocity distribution, and the characteristic-length L related with mixing length are based on the water depth but from estimating LΔU, ΔU are based on the lateral velocity distribution which shows the effects of shear stress in the lateral, and it should be remembered that the effect of width on L has been assumed to have larger than depth. From the above-mentioned discussions, the relation of D to LΔU will be same as Elder's equation, but from the phenomena estimations, the characteristics of secondary flow used, can be nearing the Fischer's idea.

5-2. Determination of Experimental Formula and estimation

The section 5-1 is the results which is the more emphatically a conception of the scale than partical view, but determination of the experimental formula in this section is tried to considering practical use the base on the results of the section 4. As mentioned in the section 4 practical parameters (velocity, depth, width, and roughness etc.) versus the dimensionless dispersion coefficient show all the case same trend of straight line in the variation.

For the purpose of getting a fitting estimation of the experimental formula, correlation of dimensionless parameters, which is refered to the multiple regression analysis, be proposed as following Eq. (7) from Eq. (4).

$$\frac{D}{u^*H} = \beta o + \beta_1 \frac{u^*}{Ua} + \beta_2 \frac{B}{H}$$

Table 2 The results of multiple regression analysis for longitudinal dispersion coefficient

Reference	Coe. of Reg.	Coe. of Simple Corr.	Coe. of Partial Corr.	Coe. of Multiple Corr.	Degrees of Freedom	Unbiasedness Variance	Variance RAte
y=D6u*H	β_0 = - 9.577	R_{12} = 0.342	R_{12}·y=-0.569	R=0.949	f_R = 2 =3070.593	$V_R = S_R/f_R$ =103.930	$Fo=V_R/V_e$
x_1=u*/Ua	β_1 = 122.617	R_1y = 0.615	$R_1y_{\cdot 2}$= 0.724	R^2=0.9006 = 90%	f_e=N-3=23	$V_e=S_e/f$ = 29.545	
x_2=B/H	β_2 = 2.458	$R_2y_{\cdot 1}$=0.889	$R_2y_{\cdot 1}$=-0.916		F_T=N-1=25		

***Coe. =Coefficient, Reg. =Regression, Corr. =Correlation

Table-2 is the results in the respective correlative coefficients of Eq. (7). With the use of Eq. (7) and the results of Table-2, one can rewrite as follows;

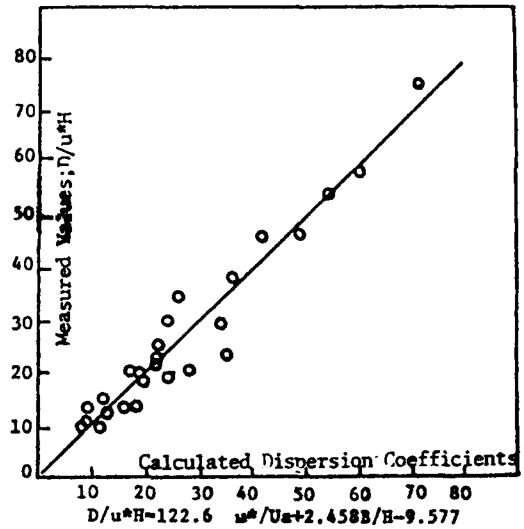

Fig. 8 Estimation of experimental formura for longitudinal dispersion coefficient in open channel flow

$$\frac{D}{u^*H} = 122.6\frac{u^*}{Ua} + 2.458\ \frac{B}{H} - 9.577 \qquad (8)$$

For the purpose of getting a estimation to Eq. (8), when D/u*H of experimental formula is plotted versus D/u*H of measurement on a graph paper, the data also appeared to have very good effect on the 45°-straight line. Based on an examination of existing data on dispersion in open channel considering the secondary flow effects, Eq. (8) will be proposed to the experimental formula for practical use.

Further new and improvement of the accuracy in predicting the dispersion coefficient as Eq. (8) is expected by using a geometerical factor, along with the hydraulics factor.

6. Conclution

In this paper, the dispersion mechanism is pointed out to be strongly effected by the secondary flow with the turbulence mechanism. The relation between the dispersion coefficient and the secondary flow is proposed by formula (5). On the other hand, the experimental formula (8) is presented for the practical use considering the velocity, water depth, width, roughness, etc.

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