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On the Hydrodynamic Forces of Oscillating Cylinders in the Presence of a Free Surface

by

J.H. Hwang* · K.P. Rhee* · S.W. Hong**

Abstract

The integral equation method to solve the boundary-value problem of a 2-dimensional body oscillating in the presence of a free surface generally breaks down at and near irregular frequencies due to the hypothetical flow inside the body.

In this paper singularity distributions were extended to an inner free surface to remove the irregular frequency as Ohmatsu's work in 1978, and the solution for the above problem was found by using stream function.

For various bodies including Lewis form cylinders, the hydrodynamic forces were calculated numerically at various wave numbers.

From the results we concluded that the irregular frequencies can be removed even for the Lewis form cylinder as Ohmatsu done for circular cylinders, and calculated hydrodynamic forces by the present method are little higher than those of Ohmatsu's when the singularities are put on the inner free surface of the body.

We specially point out that the solution for heaving motion converges in an oscillatory manner but not for swaying and rolling motions.

Nomenclature

$\bar{A}_H$	:heave amplitude ratio
$\bar{A}_R$	:roll amplitude ratio
$\bar{A}_S$	:sway amplitude ratio
B	:beam of the cylinder
C_1, C_2	:arbitrary constants
$E(iKz)$	:Exponential Integral
$G(P,Q)$	:Green function
$\bar{G}(P,Q)$	:Complex conjugate of $G(P,Q)$
$H(K)$	:Kochin function
$Im\{\ \}$	:imaginary part of a complex variable
$R_e\{\ \}$	:real part of a complex variable
K	:wave number
K_H	:heave added mass coefficient
K_R	:roll added moment of inertia coefficient
K_S	:sway added mass coefficient
NB	:number of line sources on the half of f_i
$P(x,y)$, $P_i(x,y)$	:field points
$Q(x',y')$	:source point

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* Member, Seoul National University
** Member, Korea Research Institute of Ships

S_0	: outer domain of the problem
S_i	: inner domain of the problem
T	: draft of the cylinder
$\vec{V}$	: cylinder velocity vector
V_x, V_y	: x- and y- omponents of $\vec{V}$
b	: half beam of the cylinder
e^x	: exponent of x
f_i	: inner free surface of the cylinder
g	: gravity constant
h	: contour of the cylinder surface
i	: imaginary unit, $\sqrt{-1}$
n	: unit normal vector on the cylinder surface
r_1	: $\{(x-x')^2+(y-y')^2\}^{\frac{1}{2}}$
r_2	: $\{(x-x')^2+(y+y')^2\}^{\frac{1}{2}}$
$\bar{v}(x)$	: arbitrary function of x
y_c	: y coordinate of the roll center
z	: $\|x-x'\| +i(y+y')$
Φ	: time-dependent velocity potential
Θ	: amplitude of roll motion
ψ	: stream function
ϕ	: velocity potential
ϕ_i, ϕ_0	: velocity potentials in S_i and S_0
ϕ_y	: $\frac{\partial\phi}{\partial y}$
ϕ_n	: $\frac{\partial\phi}{\partial n}$
ω	: circular frequency of oscillation
θ_1	: $\tan^{-1}\{(y-y')/(x-x')\}$
θ_2	: $\tan^{-1}\{(y+y')/(x-x')\}$
σ_h	: source strength on h
σ_f	: source strength on f_i
σ^*	: area coefficient of the Lewis form section
∞	: infinity

I. Introduction

Currently, the strip theory is widely used for the prediction of seakeeping qualities of a ship in waves. For using the strip theory it is essential to solve the 2-dimensional boundary-value problem of each ship section

This 2-D problem was first solved by Ursell [1] for the a circular cylinder by the multipole expansion method in 1949. Subsequently the Ursell's method has been modified and extended. For example, by Tasai [2] and by Porter [3] it was modified for the cylinders of Lewis form. And it was extended to the case of finite-depth waters by C.H. Kim [4].

But these method cannot be applied easily for arbitrary cylinders. MacCamy [5] solved the 2-D problem for shallow draft cylinders by singularity distribution method in 1961. After that, Frank [6] applied this method to arbitrary cylinders in 1967. Maeda [7] also solved the same problem as Frank's by adopting stream function in stead of velocity potential. And Rhee [8] extended Maeda's method to the case of finite-depth waters. But these singularity distribution methods cannot give a unique solution at or near irregular frequencies. This shortcoming was first pointed out by John [9].

Subsequently Potash [10] and Kan [11] reported this phenomenon.

To remove this shortcoming, Palling and Wood suggested that the singularity distribution should be extended to the inner free surface to supress the resonance of hypothetical flow inside the cylinder. Through numerical calculations Ohmatsu [12] confirmed this suggestion reasonable.

Meanwhile Ogilvie and Shin[13] proposed another method to remove this phenomenon.

In this paper, we extend the singularity distribution to the inner free surface and apply rigid wall boundary condition there like Ohmatsu's work and solve the problem by introducing stream function.

II. Boundary value problem

A long cylinder with uniform cross-section is oscillating harmonically in the free surface of semi-infinite fluid domain. The fluid is assumed to be inviscid, incompressible and fluid motion be irrotational.

A two-dimensional coordinate system is used with the y-axis pointing downward and the x-axis located in the undisturbed free surface, and let us denote the fluid domain and the boundary contour as shown in Fig. 1.

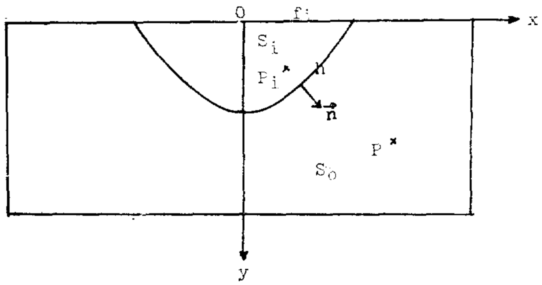

Fig.1. Coordinate system and fluid domain

The problem can be treated as boundary value problem to obtain a velocity potential Φ which can be written as follow,

$$\Phi(x,y,t)=Re\{\phi(x,y)e^{i\omega t}\} \qquad (1)$$

where ω is the circular frequency of oscillation. Then the velocity potential $\phi(x,y)$ must satisfy the following equations.

$$\nabla^2\phi=0 \text{ in fluid domain} \qquad (2)$$

$$\phi_y+K\phi=0 \text{ at } y=0, \text{ with } K=\omega^2/g \qquad (3)$$

$$\phi_y=0 \text{ as } y\to\infty \qquad (4)$$

$$\phi_n=-\vec{V}\cdot n \text{ on the cylinder surface} \qquad (5)$$

$$\phi=iH^{\pm}(K)e^{-Ky-iK|x|} \text{ as } x\to\pm\infty \qquad (6)$$

Where $\vec{V}$ means the cylinder velocity vector and n the outward unit normal vector on the cylinder surface. $H^{\pm}(K)$ stands for the Kochin function.

Ⅲ. Velocity potentials in the inner and outer fluid domains

It is well known that the Green's function $G(P,Q)$ which satisfies conditions (2), (3), (4) and (6) can be written as

$$G(P,Q)=\log r_1-\log r_2-2Re\{e^{iKz}E(iKz)\}+2\pi i\,\text{Exp}\{-K(y+y')-iK|x-x'|\} \qquad (7)$$

Where

$$P=(x,y),\ Q=(x',y')$$

$$r_1=\{(x-x')^2+(y-y')^2\}^{\frac{1}{2}} \qquad (8)$$

$$r_1=\{(x-x')^2+(y+y'^2)\}^{\frac{1}{2}} \qquad (9)$$

$$z=|x-x'|+i\,(y+y') \qquad (10)$$

$E(iKz)$ is the exponential integral

By applying Green's theorem to the outer demain S_0 shown in Fig. 1, the velocity potential becomes

$$\phi(P)=\frac{1}{2\pi}\int_h\left\{\frac{\partial\phi(Q)}{\partial n}G(P,Q)-\phi(Q)\frac{\partial}{\partial n}G(P,Q)\right\}dc(Q) \qquad (11)$$

Let us introduce a velocity potential ϕ_i which satisfies condition (2) in the inner domain S_i and again apply Green's theorem.
We get

$$0=\frac{1}{2\pi}\int_h\left\{\frac{\partial\phi_i(Q)}{\partial n}G(P,Q)-\phi_i(Q)\frac{\partial}{\partial n}G(P,Q)\right\}dc(Q)+\frac{1}{2\pi}\int_{f_i}\left\{K\phi_i(Q)+\frac{\partial\phi_i(Q)}{\partial y'}\right\}G(P,Q)\,dc(Q) \qquad (12)$$

From equations (11) and (12), $\phi(P)$ can be expressed in the form

$$\phi(P)=\frac{1}{2\pi}\int_h\left\{\left(\frac{\partial\phi}{\partial n}-\frac{\partial\phi_i}{\partial n}\right)G(P,Q)-(\phi-\phi_i)\frac{\partial G(P,Q)}{\partial n}\right\}dc(Q)+\frac{1}{2\pi}\int_{f_i}\left\{-K\phi_i(Q)-\frac{\partial\phi_i(Q)}{\partial y'}\right\}G(P,Q)\,dc(Q) \qquad (13)$$

By putting the condition $\phi=\phi_i$ on h in equation (13), we finally get

$$\phi(P)=\frac{1}{2\pi}\int_h\sigma_h(Q)G(P,Q)dc(Q)+\frac{1}{2\pi}\int_{f_i}\sigma_f(Q)G(P,Q)dc(Q) \qquad (14)$$

where

$$\sigma_h(Q)=\frac{\partial\phi}{\partial n}-\frac{\partial\phi_i}{\partial n} \qquad (15)$$

$$\sigma_f(Q)=-K\phi_i(Q)-\frac{\partial\phi_i(Q)}{\partial y'} \qquad (16)$$

Similarly we can express the velocity potential ϕ_i as

$$\phi_i(P_i)=\frac{1}{2\pi}\int_h\sigma_h(Q)G(P_i,Q)dc(Q)+\frac{1}{2\pi}\int_{f_i}\sigma_f(Q)G(P_i,Q)dc(Q) \qquad (17)$$

Equations (14) and (17) show us the velocity potential ϕ and ϕ_i can be represented by source distributions on h and f_i.

Ⅳ. Integral equation

Instead of the velocity potential ϕ, let us introduce a stream function ψ.

Then equations (14) and (17) can be written as follows by utilizing the relation between ϕ and ψ.

$$\psi(P)=\frac{1}{2\pi}\int_{f_i}\sigma_f(Q)\bar{G}(P,Q)dc(Q)+\frac{1}{2\pi}\int_h\sigma_h(Q)\bar{G}(P,Q)dc(Q) \tag{18}$$

$$\psi_i(P_i)=\frac{1}{2\pi}\int_{f_i}\sigma_f(Q)\bar{G}(P_i,Q)dc(Q)+\frac{1}{2\pi}\int_h\sigma_h(Q)\bar{G}(P_i,Q)dc(Q) \tag{19}$$

Where $\bar{G}$ is the complex conjugate of G and it is expressed as

$$\bar{G}(P,Q)=\theta_1-\theta_2-2\frac{x-x'}{|x-x'|}\ I_m\{e^{iKz}\ E(iKz)\}-2\pi e^{-K(y-y)-iK(x-x')} \tag{20}$$

where

$$\theta_1=\tan^{-1}\{(y-y')/(x-x')\} \tag{21}$$

$$\theta_2=\tan^{-1}\{(y+y')/(x-x')\} \tag{22}$$

Equations (18) and (19) are the Fredholm's integral equations of the first kind for a Dirichlet boundary-value problem. Now the boundary condition on h and on f_i must be specified. Let cylinder velocity $\vec{V}$ be

$$\vec{V}=V_x\vec{i}+V_y\vec{j} \tag{23}$$

From equation (5) and Cauchy-Riemann relation, we can deduce the boundary condition on h as

$$\psi=V_y\cdot x \text{ for heave motion} \tag{24}$$

$$\psi=-V_x\cdot y+C_1 \text{ for sway motion,} \tag{25}$$

where C_1 is an integral constant.

Suppose Θ be the angular velocity of roll motion and the poins $(0,y_c)$ be the center of rotation, we get the boundary condition on h for roll motion as

$$\psi=\tfrac{1}{2}\Theta\ (x^2+(y-y_c)^2)+C_2 \tag{26}$$

for roll motion, where C_2 is an integral constant.

On the other hand the boundary condition on f_i can be obtained by following Ohmatsu's work[12]

$$\frac{\partial\phi_i}{\partial y}=v(x) \text{ on } f_i \tag{27}$$

where $v(x)$ is an arbitrary function, but it is an even function for heave motion and an odd function for sway and roll motions.

Equation (27) can be changed in terms of ψ as

$$\psi=\bar{v}(x) \text{ on } f_i \tag{28}$$

where $\bar{v}(x)$ is an odd function for heave motion and an even function for sway and roll motions. We employ the following functions for $\bar{v}(x)$

$$\left.\begin{aligned}&\bar{v}(x)=V_x\cdot b+C_1 \text{ for sway motion}\\&\bar{v}(x)=V_y\cdot x \text{ for heave motion}\\&\bar{v}(x)=\tfrac{1}{2}\Theta\ (x^2+(y-y_c)^2)+C_2 \text{ for roll motion}\end{aligned}\right\} \tag{29}$$

where b is the half beam of the cylinder.

V. Numerical Calculations

Next, we proceed numerical calculations of the integral equation given by substituting equations

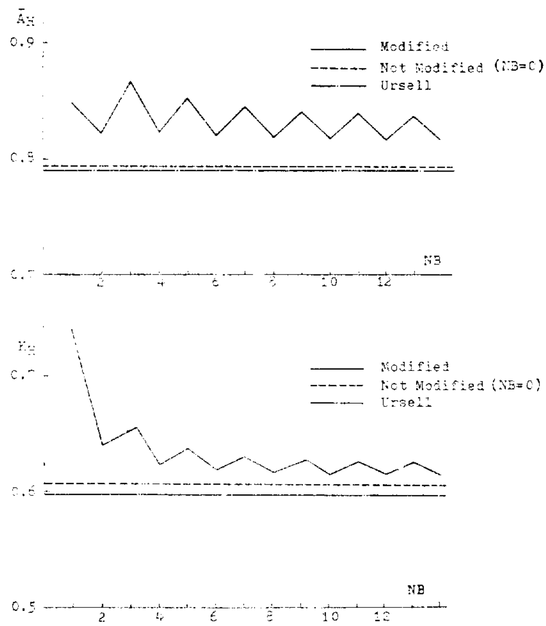

Fig.2. Convergence test for heaving motion of a circle KT=1.0

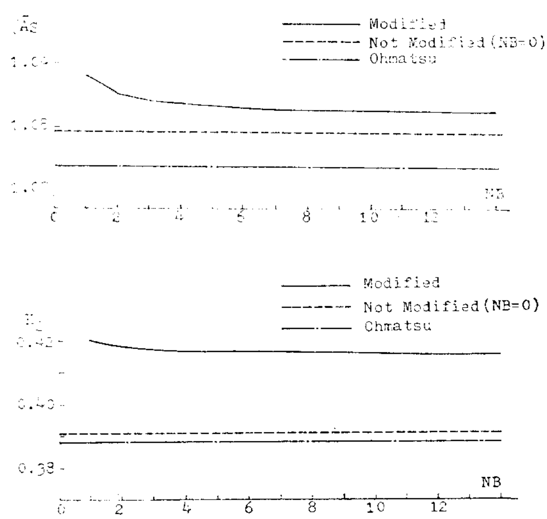

Fig.3. Convergence test for swaying motion of a circle KT=1.0

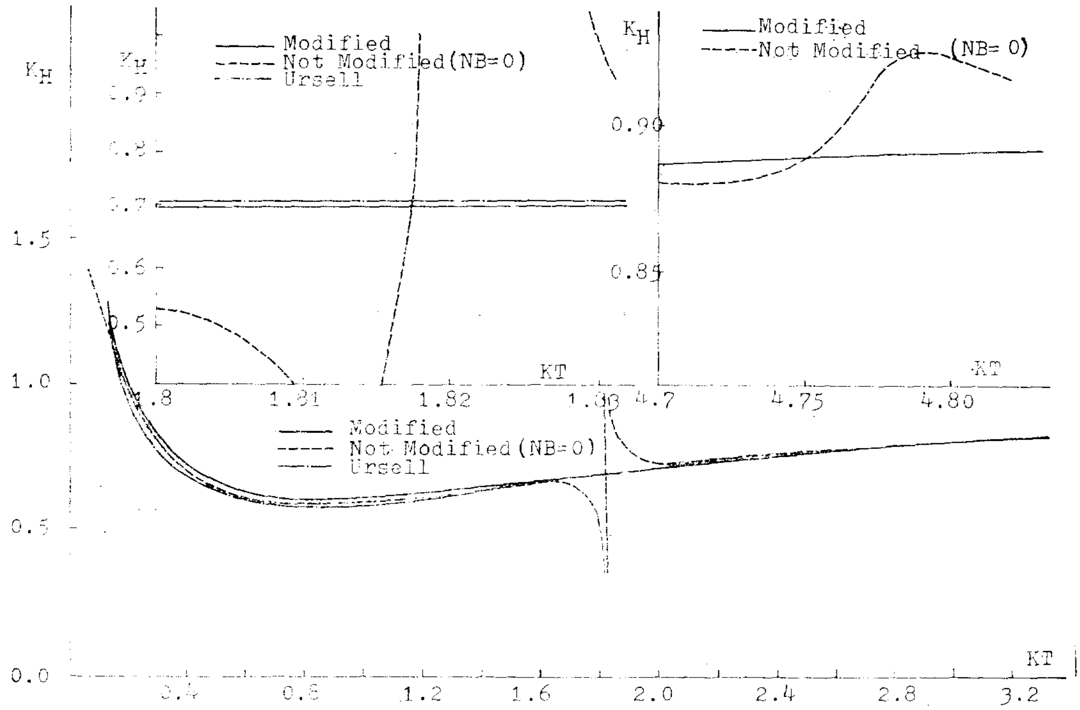

Fig.4. Heave added mass coefficient for a circular cylinder

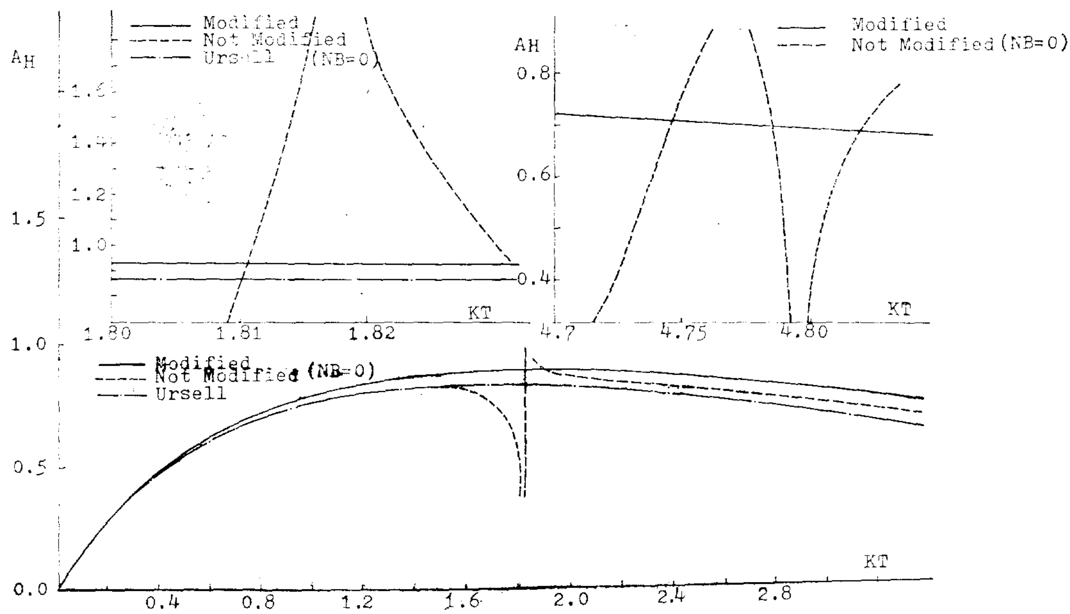

Fig. 5. Heave damping coefficient for a circular cylinder

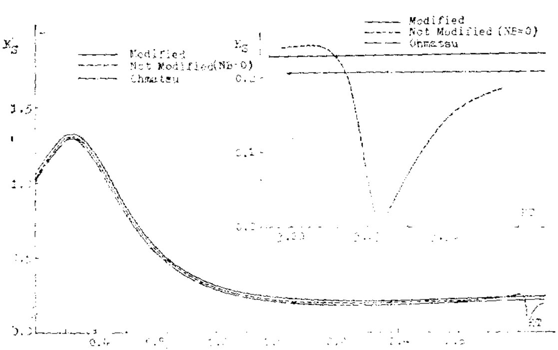

Fig. 6. Sway added mass of a circular cylinder

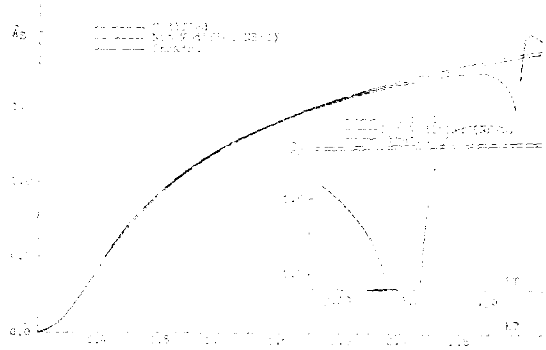

Fig. 7. Sway damping coefficient for a circular cylinder

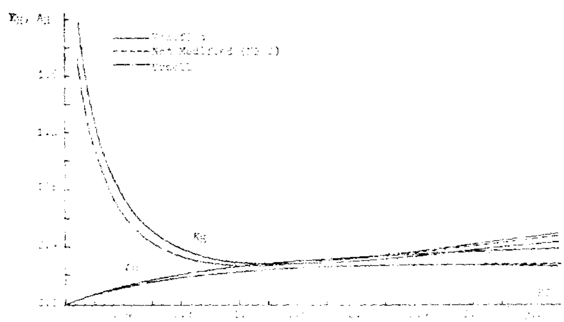

Fig. 8. Heave added mass and damping coefficient for a Lewis form B/2T=0.2, $\sigma^*=0.6$

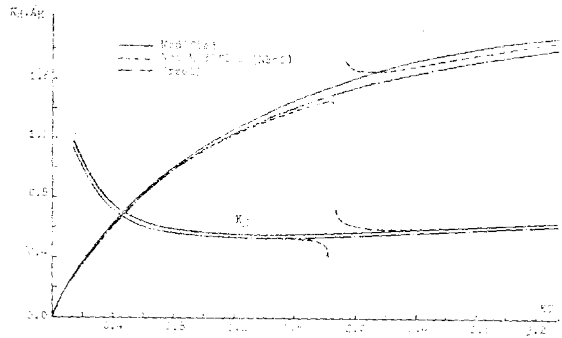

Fig. 9. Heave added mass and damping coefficient for a Lewis form B/2T=1.2, $\sigma^*=0.6$

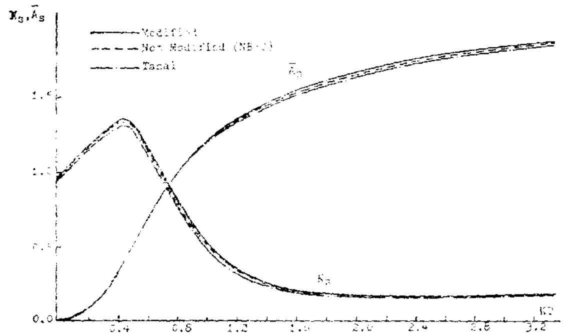

Fig. 10. Sway added mass and damping coefficient for a Lewis form B/2T=0.2, $\sigma^*=0.6$

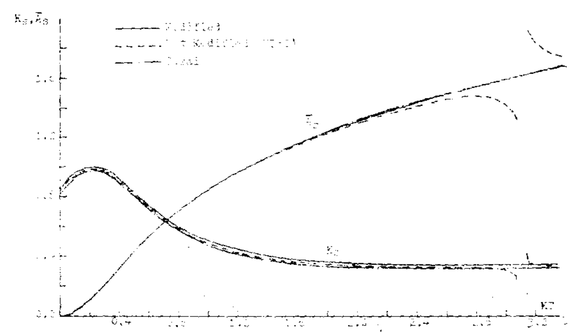

Fig. 11. Sway added mass and damping coefficient for a Lewis form B/2T=1.2, $\sigma^*=0.6$

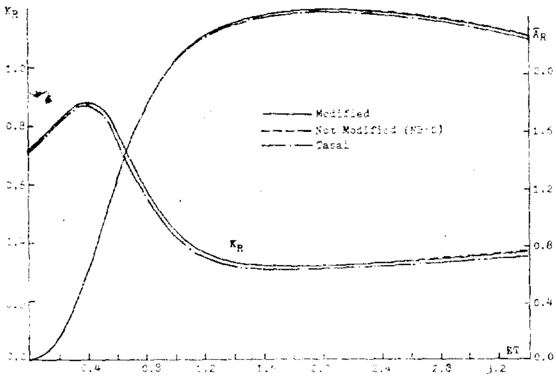

Fig. 12. Roll added mass and damping coefficient for a Lewis form B/2T=0.2, $\sigma^*=0.6$

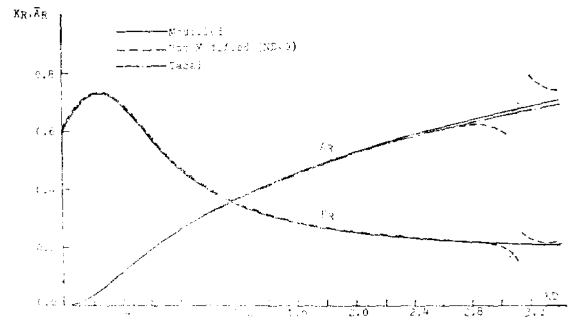

Fig. 13. Roll added mass and damping coeffi6cient for a Lewis form B/2T=1.2, $\sigma^*=0.6$

(24), (25) and (26) for ψ into equation (18) and by substituting equations (29) into equation (19).

The source distribution is approximated by some step functions, and the boundary condition is imposed at the midpoint of each segment.

To examine the convergence of the computer program, we devide the source distribution into 16 segments on h and calculate the hydrodynamic forces with increasing the number of line segments on f_i.

The test results are provided for a circular cylin der at $KT=1.0$(T means the draft of the cylinder).

Fig. 2 shows the results for heaving motion, and Fig. 3 for swaying motion.

In Fig.2 and Fig. 3 NB means the number of line segments on the half of f_i, and dotted line corresponds to the case of NB=0. When NB=10, the relative error between the present and Ursell's results becomes within 3.5% for heaving motion.

When NB=6, the relative error between the present and Ohmatsu becomes within 7.3% for swaying motion.

Next we perform the calculations for various cylinders, such as circular and Lewis form cylinders, with 16 line segments on h and with $NB=6$ on f_i.

Fig. 4 and Fig. 5 show the heave added mass coefficient and the heave damping coefficient for a circular cylinder and Fig. 6 and Fig. 7 show the sway added mass coefficient and the sway damping coefficient for a circular cylinder.

The hydrodynamic coefficients of two Lewis form cylinders, which have half-beam to draft rations of 0.2 and 1.2 and the same sectional area coefficient of 0.6, are presented in Figs. 8-13 for heave, sway and roll motions.

From the these results we conclude that the irregular frequency phenomena can be removed by the present method.

VI. Conclusion

Many ideas and methods were developed to remove the irregular frequency phenomena of singularity distribution method. The results of the present calculation were compared with other's such as Ursell's, Tasai's or Ohmatsu's with good agreements.

From the numerical calculation results, we conclude that the irregular frequencies are removed without regard to the body section shape, but the calculated values of added masses and damping coefficients are little higher than those of unmodified singularity distributions. This result is considered as the influence of the kinetic energy of the inner fluid flow.

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