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Wave Phase Shift of a Submerged Circular Cylinder

by

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Abstract

Herein the flow past a submerged circular cylinder with a free surface is mapped onto a reference plane, in which the free surface is transformed to a straight line and the cylinder to a certain shape. A global mapping function between two planes is sought in a manner that linear free-surface elevation is generated in the physical plane. Hereby the Froude mumber F_h, based on the submergence depth h', is assumed to be of order $O(1)$ and the ratio a'/h' (a'=cylinder radius) of order $o(1)$.

Waves thus obtained are slightly different in magnitude and phase from usual linear solution. The resulting free wave starts advanced ahead compared to the classical result and its amount depends on Froude number. Based on the present concept wave forces are calculated. In this type of approach the body boundary condition gives more influence on wave resistance than that by the free surface in the speed range $F_h>1$.

1. Introduction

Many attempts have been performed to get an approximate solution for the problem of the flow past a submerged circular cylinder. A general review on this topic is well surveyed in the book of Wehausen and Laitone (1960, p. 574). Usually by introducing a proper smallness parameter, the flow near a free surface and also near the cylinder is perturbed in power series of the parameter.

The main difficulty in this problem is well recognised and it is the non-linearity in the boundary condition on the free surface, which is unknown *a priori*.

Tuck (1965) analysed the flow pattern up to the second order consistently and pointed out that the boundary condition on the free surface is more important to wave resistance than that on the body over all speed ranges.

By employing the method of matched asymptotic expansions, Dagan (1971) obtained the zeroth-order inner and the first-order outer solutions. In this paper the unknown free surface is mapped onto a straight line in a reference plane and a global mapping function between this and the physical plane is sought by following the idea of "displacement potential" in the sense of Nobless (1974).

The displacement potential is obtained as an integral of Havelock source potential with intensity of time-integrated sources on a distribution domain.

The mapping function may be interpreted as gradient of the displacement potential.

2. Description of Problem

A long cylinder with uniform circular cross-section is submerged in a uniform flow U' normal to its axis. The fluid is assumed to be inviscid, incompressible and fluid motion be irrotational. Let the initial

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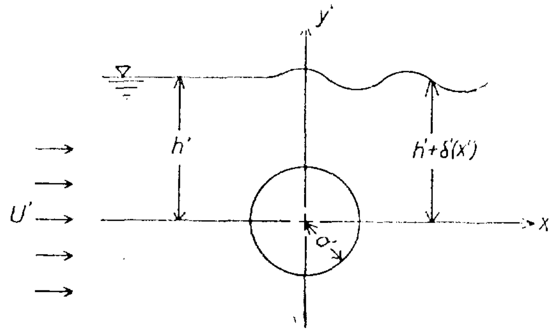

Fig. 1. Physical z'-plane

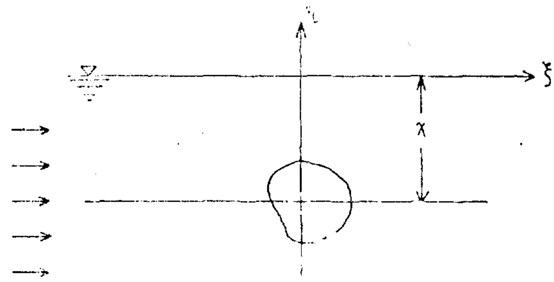

Fig. 2. Reference ζ-plane

transient motion and the surface tension on the free surface be neglected. The fluid is extended laterally to infinity. In rectangular cartesian coordinates the free surface and the cylinder are expressed as $y'=h'+\delta'(x')$ and $|z'|=a'$, respectively, as shown in Fig. 1. Where symbols with appostoph denote quantities with dimensions.

Let $f'=\varphi'+i\Psi'$, $w'=u'-iv'$ and $z'=x'+iy'$ be a complex potential, complex velocity and complex length variable, in this order. The boundary conditions in dimensionless form, dividing through the uniform flow U' and the cylinder radius a' as basic speed and length, are

$$\Psi=h \quad \text{on } y=h+\delta(x) \tag{1}$$

$$\Psi=0 \quad \text{on } |z|=1 \tag{2}$$

$$w=1,\ \delta(x)=0 \quad \text{as } x\to-\infty \tag{3}$$

$$Re\left(\bar{w}^2\frac{dw}{dz}+i\nu w\right)=0 \quad \text{on } y=h+\delta(x) \tag{4}$$

$$\text{with } \nu=\frac{ga'}{U'^2}$$

Radiation condition which states that waves propagate downstreamwards. (5)

Wherd Re and Im are referred to the real and imaginary part of complex quantities, the upper bar stands for their complex conjugate and g is acceleration due to gravity.

This problem is thus characterized as petential boundary-value problem of the Laplace equation, which defies to render an 'extact' solution, even not for numerical approach.

3. Reference Plane

We introduce a reference plane ζ, on which the physical z-plane is conformally mapped in such a manner that the free surface becomes a straight line and the cylinder a certain 'corresponding' one as shown in Fig. 2.

On this plane the kinematic boundary conditions (eqs. (1), (2)) and the upstream condition (eq. (3)) are prescribed on $Im\zeta=\chi$, on the corresponding cylinder and $\zeta\to-\infty$. However, the dynamic boundary condition (eq. (4)) and the radiation condition (eq. (5)) fail to keep any meaning because of the straight free surface $Im\zeta=\chi$.

In this context this reference plane may be considered as the same flow model of the inner problem in the method of matched asymptotic expansions as adopted by Dagan (1971).

Thus the dynamic condition and the radiation condition have to be suspended from being applied until later. Only the kinematic conditions are to be satisfied in the ζ plane.

Accordingly the mapping function between two planes, which is the essence of this problem, should bear some special properties so that the former two conditions could be fulfilled in the physical plane.

Even under the above manipulation, this type of approach is complicated and interwoven non-linearly, more difficult than the matched asymptotic expansion method in some sense.

The merit of this approach, however, may be found in the fact that the free surface conditions could be treated more rationally—at least what the kinematic part and the flow near both ends of the cylinder are concerned. This may be argued by pointing out that the mapping is closely related with the Lagrangian description of fluid particles in the region of the free surface (Wehausen, 1969). Furthermore it has something in common with the streched obserbation on an abrupt change of flow (Van Dyke, 1964, p. 69).

Anyhow in this type of approach, more attention is paid on the free surface and the body boundary condition may not be sufficiently cared.

The body boundary condition is thus replaced by a requirement that corresponding cylinder should give a mapped cylinder as similar as possible to the given one in the physical plane.

In the reference plane the corresponding cylinder may be generated by some singularity distributions.

On this stage of analysis, this singularity distribution will not be clearly figured out until the mapping function is found.

In the light of the above rather loose requirement the singularity distribution, which describes flow exactly in an unbounded fluid for a circular cylinder, is chosen for the first approximation, but with a modification factor α of its intensity.

This modification factor will be determined later as a part of solution in such a manner that the body boundary condition may be improved to some extent.

Thus we have a complex potential in the reference plane as follows:

$$f(\zeta)=\zeta+\alpha^2\left(\frac{1}{\zeta}+\frac{1}{\zeta-2i\chi}\right) \qquad (6)$$

where with $\alpha=1$, this complex potential exactly describes the fluid motion far away from a circular cylinder in an unbounded fluid and χ is associated with the real submergence depth h.

This potential satisfies the kinematic condition on Im $\zeta=\chi$ and the upstream condition as $\zeta\to-\infty$.

The corresponding cylinder is obtained through tracing streamline, beginning with the front stagnation point on the cylinder.

Then this potential satisfies the kinematic boundary condition along the corresponding cylinder. Thus all necessary conditions are satisfied sofar.

4. Mapping Function

Because of the intractable dynamic boundary condition(eq. (4)), it is hardly possible to derive exact mapping function. There are some possible ways to develope an approximate solution. For example, we may define a relation of $Z=z+F(z)$ and expand $F(z)$ into power series of a small parameter(see Noblesse). Where Z is natural coordinate and z undisturbed reference coordinate. Noblesse defined the function $F^{(1)}(z)$ as gradient of the "displacement potential", which is given by an integral of product of usual Havelock source potential and x-integrated sources.

In his analysis an identity exists between the linear wave elevation and the linear vertical displacement

$$\Phi_x=k_0\Psi_y \quad \text{at } y=h$$

where Φ denotes the thin-ship potential, Ψ the displacement potential, the subscripts the partial differentiations and k_0 is fundamental wave number defined by g/U'^2.

Here we utilize this identity and demand that the linear free-surface elevation is to be deduced from the mapping function $F^{(1)}(\zeta)$ along Im $\zeta=\chi$, but possibly with different magnitude and at different location. Thus we at least have the value of the mapping function on the line Im $\zeta=\chi$,

$$\delta(x)\cong-\alpha^2\text{Im}\left\{\frac{1}{\zeta}-\frac{1}{\zeta-2i\chi}-2i\nu\ I(i\nu(\zeta-2i\chi))\right\}$$
$$\text{at } \zeta=\xi+i\chi \qquad (7)$$

where relations between ξ and x, and between χ and h should be determined from the mapping function. $I(\zeta)$ is defined by $e^{-\zeta}E_1(-\zeta)$ (see Salvesen, 1969).

From Abramowitz and Stegun (1964), we can show that $I(\zeta)$ has a series expansion

$$I(\zeta)=e^{-\zeta}\left(-\gamma-\ln\zeta-i\pi-\sum_{n=1}^{\infty}\frac{\zeta^n}{n\cdot n!}\right) \qquad (8)$$

and an asymptotic expansion

$$I(\zeta)\sim-\left(\frac{1}{\zeta}+\frac{1}{\zeta^2}+\frac{2!}{\zeta^3}+\frac{3!}{\zeta^4}\cdots\right)$$
$$-(1+\text{sign }(\text{Im }\zeta))i\pi e^{-\zeta}, \qquad (9)$$

where γ is the Euler constant and the sign is taken in accordance with the radiation condition. By taking the ratio of the cylinder radius to the submergence depth as small parameter, the boundary condition is made linearized in the from,

$$Re\left(\frac{df(z)}{dz}+i\nu f(z)\right)=0 \quad \text{at Im}z=h. \qquad (10)$$

A mapping function $F^{(1)}(\zeta)$, which satisfies this linearized free-surface condition, has a known imaginary part along Im $\zeta=\chi$,

$$\text{Im } F^{(1)}(\xi+i\zeta)=\alpha^2\Big(\frac{2\chi}{\xi^2+\chi^2}+2\nu ReI(i\nu(\xi-i\chi)\Big). \quad (11)$$

This function should be regular in the lower half plane Im $\zeta<\chi$ for harmonic continuation of the mapping. $F^{(1)}(\zeta)$ has a limit value $2\nu\pi\alpha^2(1+\text{sign}(\xi))e^{-\nu\chi-i\nu\xi}$ as $\zeta\rightarrow\pm\infty$, which is bounded by $4\nu\pi\alpha^2e^{-\nu\chi}$ if ν is of order 0(1).

Then the term of $F^{(1)}(\zeta)-F^{(1)}(\pm\infty)$ converges absolutely for sufficiently large $|\zeta|$,

$$|F^{(1)}(\zeta)-F^{(1)}(\pm\infty)|<\frac{M}{|\zeta|}$$

where M is a real constant.

On the other hand the term has a pole at $\zeta=0$ in the lower half plane. We remove this pole by translating it to its image point with respect to the line Im $\zeta=\chi$.

Finally the function $F^{(1)}(\zeta)$ satisfies the Hölder condition and the problem turns to a Riemann-Hilbert problem in the lower half plane Im $\zeta<\chi$. This can be solved by Cauchy integral along Im$\zeta=\chi$ (Muskhelishvili 1953, p. 110).

We again add a term 1/2 (1+sign (Imζ)) $F(\infty)$ to the result in order to meet the radiation condition.

The result of this integral is easily seen as

$$z=\zeta+2\alpha^2\left\{\frac{1}{\zeta-2i\chi}+i\nu I(i\nu(\zeta-2i\chi))\right\}+C \qquad (12)$$

where C is a complex constant.

This complex constant may be considered as one degree of freedom in this concept. This freedom is so utilized that the origin of ζ-plane corresponds to the origin of the physical plane. Accordingly C is given by

$$C=-i\alpha^2\{1/\chi+2\nu I(2\nu\chi)\} \qquad (13)$$

As will be noticed later this constant plays an inportamt role in this problem.

Eq. (12) may be a generalisation of Dagan's result and a modification of the displacement potential.

However, eq. (12) contains no displacement effect of the singularity itself, but only terms from its image singularity. It is due to the special choice of the mapping.

5. Determination of Unknowns α and χ

There are some possible flow models to determine the unknowns (Dagan, 1971).

The underlying idea is that the modification factors α and χ are so determined that the body boundary condition is to be improved to some extent and at the same time the line Im$\zeta=\chi$ should represent the free surface in the physical plane.

Following the line of this thought the abscissa of stagnation point near the leading edge is chosen to be unity with minus sign in the physical plane.

Replacing ζ by $\zeta_{st}=-\chi d+i(1-\hat{e})\chi$ and z by -1 and taking the real part of the right-hand side of eq. (12), we get an expression for α and χ,

$$1=\chi d\left[1+2\Big(\frac{\alpha}{\chi}\Big)^2\left\{\frac{1}{d^2+(1+\hat{e})^2}+\frac{\nu_x}{d}Im\right.\right.$$
$$\left.\left.[I(\nu_x(1+\hat{e})-i\nu_x d)-I(2\nu\chi)]\right\}\right] \qquad (14)$$

where ζ_{st} is a solution of $df/d\zeta=0$ from eq. (6)

$$\text{with } d=\left(\frac{\left\{2\Big(\frac{\alpha}{\chi}\Big)^2+1\right\}^{\frac{1}{2}}+\Big(\frac{\alpha}{\chi}\Big)^2-1}{2}\right)^{\frac{1}{2}}$$

$$\hat{e}=\frac{\left\{4\Big(\frac{\alpha}{\chi}\Big)^2-\Big(\frac{\alpha}{\chi}\Big)^4\right\}^{\frac{1}{2}}}{2d}$$

Furthermore ν_χ is the nondimensional wave number based on χ. From eq. (14) unknowns α and χ are evaluated for given (α/χ) and ν_χ, meanwhile by applying the upstream condition to eq. (12) a relation between h and χ is found,

$$h=\chi-\alpha^2/\chi-2\nu\alpha^2Re(2\nu\chi). \qquad (15)$$

For actual numerical calculation, it is more convenient to prepare a diagram for α and χ/h for given submergence depth h with Froude number $F_h=U'/(gh')^{1/2}$ as parameter.

These are illustrated in Fig. 3 and 4.

From eq. (15) the relation between h and χ can be examined for two special cases: for large values of χ, i.e. for deep submergence, it becomes $h=\chi+\frac{1}{2\nu}\Big(\frac{\alpha}{\chi}\Big)^2+0\Big(\frac{\alpha}{\chi}\Big)^3$.

It implies that h is practically equal to χ not for very high speed. On the other hand for small values of χ, i.e. for shallow submergence, eq. (15) has another form

$$h=\chi-\alpha^2/\chi+2\nu\alpha^2e^{-2\nu\chi}(\gamma+\ln 2\nu\chi).$$

At this degree of approximation, the complex

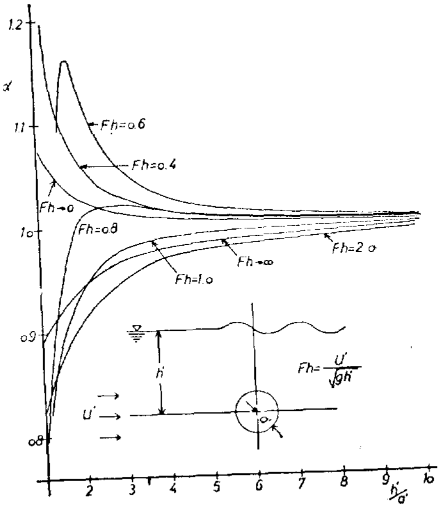

Fig 3. Mofibying factcer α for doublet strength

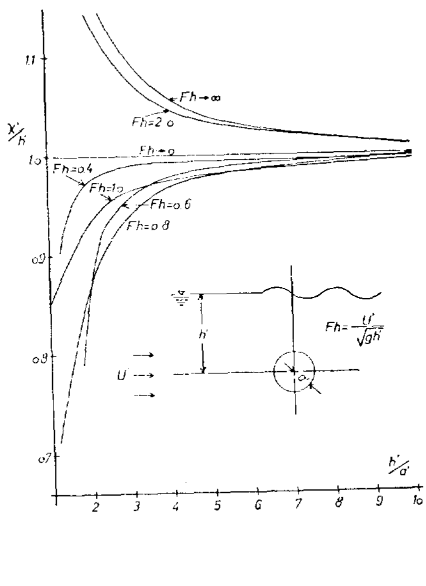

Fig. 4. Modifying factor χ for submergence depth

velocity in the physical plane can only be expressed implicitly in terms of reference variable ζ.

$$w=\frac{df}{dz}=\frac{df d\zeta}{d\zeta dz}$$

$$=\frac{1-\alpha^2\left\{\frac{1}{\zeta^2}+\frac{1}{(\zeta-2i\chi)^2}\right\}}{1-2\alpha^2\left\{\frac{i\nu}{(\zeta-2i\chi)}+\frac{1}{(\zeta-2i\chi)^2}-\nu^2 I(i\nu(\zeta-2i\chi))\right\}} \quad (16)$$

In the lower plane $Im\ z\langle h$, the denominator has neither poles nor zeroes, while the numerator is exactly zero both at the leading and the trailing edges.

6. Waves and Wave Forces

The free surface in the physical plane can be traced from eq. (12) along the line $Im\ \zeta=\chi$. It should be observed that the line $Re\ \zeta=0$ in the range of $0\leq Im\zeta\leq\chi$ is mapped on a curve in the physical plane. Its abscissa is given by

$$x=-2\nu\alpha^2\pi(e^{-\nu\eta}-e^{-2\nu\chi}) \quad (17)$$

In particular eq. (17) becomes $-2\nu\alpha^2\pi e^{-\nu\chi}(1-e^{-\nu\chi})$ on the free surface and thus the free-wave system does not begin with $x=0$, but with an advanced phase-shift of the amount above.

This phase shift increases with decreasing the submergence depth and with being closed to a certain speed ($F_h=0.8$) as shown in Fig. 5.

At the same time the cylinder can be traced in each plane; in the reference plane we can follow the

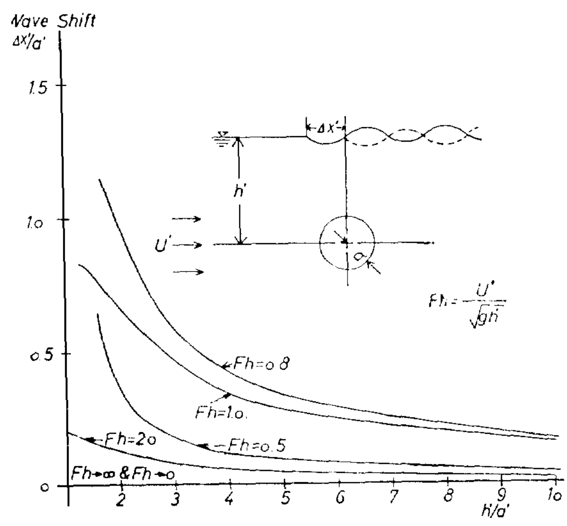

Fig. 5. Wave shift in front of the cylinder

streamline $Imf(\zeta)$ which has the same value as $Im f(\zeta_{st})$ in eq. (6).

It is well known that a doublet in a uniform flow experiences horizontal and vertical forces(Havelock, 1926).

The present result looks very similar to the conventional linear one, but with modification factor α for the doublet strength and modified submergence depth χ for the actual submergence depth h as belows

$$R=4\pi\nu^2\alpha^4e^{-2\nu\chi} \tag{18}$$

and

$$L=-\alpha^4/(2\nu\chi^3)\times(1+2\nu\chi+4\nu^2\chi^2-8\nu^3\chi^3e^{-2\nu\chi}ReI(2\nu\chi)) \tag{19}$$

where all forces are divided by the buoyancy $\rho\pi g$.

For cases of $h=2, 4$ numberical results are demonstrated in Figs. 6~9 in comparison with those of Havelock and Tuck. It is immediately noticed that the present result is most close to the consistent second-order theory of Tuck. If we regard the roles of modification factors α and χ as representing better treatment of the body boundary condition and the free-surface condition, respectively, (in fact it is not easy to clearly separate two effects, especially when the cylinder is not deeply submerged), it leads us to conclude that χ is more important to wave forces in low speed range. In moderate speeds the contributions of α and χ to wave forces are comparable, meanwhile α plays a more important role in high speed range. Indeed this conclusion is restricted to the flow model we take.

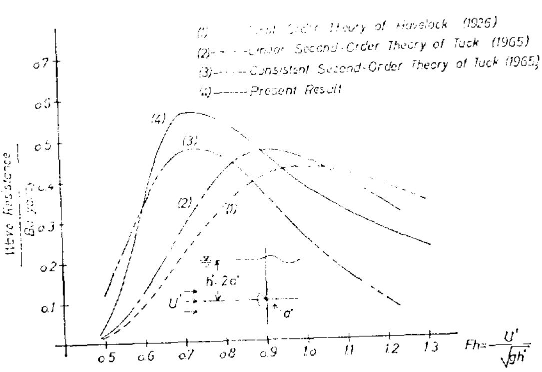

Fig. 6. Wave resistence for h′=2.0a′

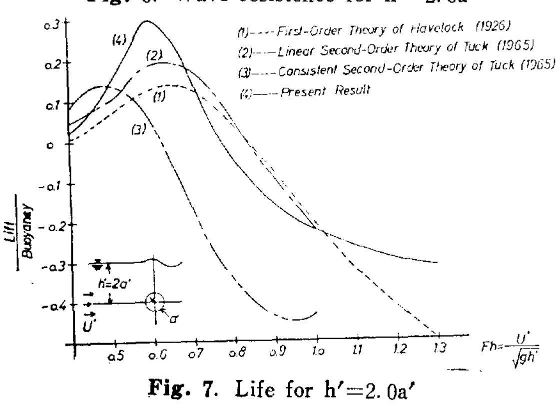

Fig. 7. Life for h′=2.0a′

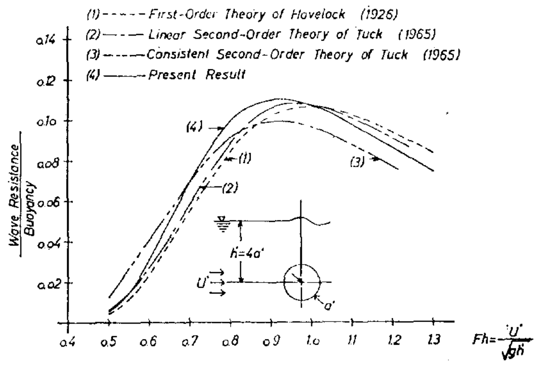

Fig. 8. Wave resistence for h′=4.0a′

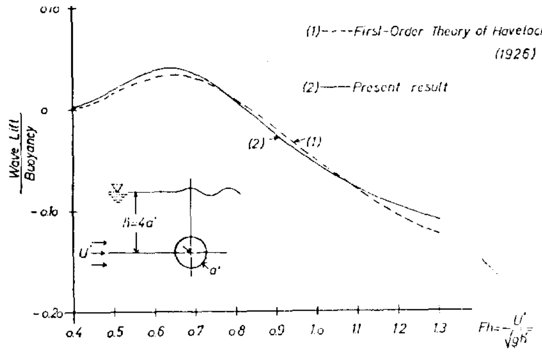

Fig. 9. Life for h′=4.0a′

7. Discussions

For unbounded fluid as a limit case h is set to infinity in eqs. (12) and (16).

From Figs. (3) and (4) it is immediately recognised that the limit values of α and χ/h are both unity for all speeds. As a result the mapping function reduces simply to $z=\zeta$ and the complex velocity to $w=1-1/\zeta^2$, i.e. $w=1-1/z^2$.

This confirms the fact that the fluid motion past a circular cylinder in an unbounded fluid is modulated far from it by a double with unit strength at the center of the cylinder.

Another interesting fact to mention is that the traced dividing body shows very similar configuration to a circle for all submergence depths and speeds.

Finally this result may be regarded as the first approximation with respect to the smallness parameter.

Further approximations may be persued in a similar way as usual perturbation sheme; to add higher-order body singularities and higher-order pressure distribution on the line $Im\zeta=\chi$.

The concept of this method is however inconsistent and the first approximation is already complicated. Thus it is not much recommendable to struggle with the higher-order approximation.

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決 議 文

우리들 科學技術人은 全國民의 生活의 科學化 運動이 祖國近代化와 福祉社會를 이룩하는 捷徑임을 깊이 認識하고 總力을 傾注하여 汎國民運動의 旗手가 될 것을 다짐하면서 다음과 같이 決議한다.

一. 우리는 全國民의 生活의 科學化가 國民 모두에게 擴散되고 汎國民運動으로 結實되도록 最大의 努力을 傾注한다.

一. 우리는 全國民의 生活科學化 運動의 核心的 役軍으로서 科學精神涵養과 科學知識 普及에 積極奉仕한다.

一. 우리는 國民生活의 非科學的 弊習을 打破하고 合理的인 生活科學化 運動을 爲한 支柱的 役割을 擔當한다,

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