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칼만필터의 應用에 關한 硏究

Kalman Filters with Moving Horizons

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Abstract

This paper deals with a modified Kalman filter. An approaching horizon with a suitable initial condition will be considered, which is a little different from the classical Kalman filter. It will be shown in this paper that the new filter with approaching horizons is not only easy to compute but also possesses asymptotic stability properties. Thus this new estimatoris an excellent compromise between the ease of computation and the strict sense of optimality. When this estimator is used for the standard problem, the error covariance bound has been obtained. It is shown that the new estimator can be used as a suboptimal estimator which has a stability property. It is also demonstrated that the steady state Kalman filter can be obtained from the moving horizon estimator by taking the horizon parameter as infinity.

1. Introduction

The Kalman filter is not only a breakthrough in control theories but also has been extensively used in applications. It was developed first by Kalman [1,2] and is still being investigated by many researcher because of its importance [4]. The Kalman filter can be applied to discrete systems [2] and also to continuous systems [1]. In this paper discrete systems will be considered. The linear discrete stochatic systems with white noises are represented by

$$x(i+1)=A(i)x(i)+B(i)w(i),\ x(i_0)=x_0 \quad (1.1)$$

$$y(i)=C(i)x(i)+v(i) \quad (1.2)$$

where $x(i)\ \varepsilon R^n$, measurements $y(i)\ \varepsilon R^p$, and state noise $w(i)$ and measurement noise $v(i)$ are independent Gaussian processes with following conditions:

$$Ex(i_0=\bar{x}(i_0)E(x(i_0)-\bar{x}(i_0))\ (x(i_0)-\bar{x}(i_0))'=F_0$$

$$Ew(i)v'(j)=Ex(i_0)w'(i)=Ex(i_0)v'(i)=0 \quad (1.3)$$

$$Ew(i)=Ev(i)=0,\ Ew(i)w'(j)$$
$$=Q(i)\delta_{ij},\ Ev(i)v'(j)=R(i)\delta_{ij}$$

It is well known that the conditional expectation is the best estimate. When the measurements $\{y(j),\ i_0\leq j\leq i\}$ are known, the best estimate $x(i+1)$ is given by

$$E(x(i+1)\,|\,y(j),i_0\leq j\leq i) \quad (1.4)$$

The best estimate of (1.4) for the systems (1.1)–(1.2) with conditions (1.3) is the so-called Kalman filman filter and is given by

$$\hat{x}(i+1)=A(i)\hat{x}(i)+A(i)P(i_0,i)C'(i)$$
$$[C(i)P(i_0,i)C'(i)+R(i)]^{-1}(y(i)-C(i)\hat{x}(i)),$$
$$\hat{x}(i_0)=\bar{x}(i_0) \quad (1.5)$$

where $P(i_0,\ i)\triangleq$covariance $(\hat{x}(i)-x(i))=E(\hat{x}(i)-x(i))\ (\hat{x}(i)-x(i))'$ is obtained from

$$P(j,i+1)=A(i)P(j,i)A'(i)-A(i)$$
$$P(j,i)C'(i)(C(i)P(j,i)C'(i)+R(i))^{-1}$$
$$C(i)P(j,i)A'(i)+B(i)Q(i)B'(i), \quad (1.6)$$

with $P(j,j)=F_0$. The Kalman filter problem is dual to the deterministic quadratic regulator problem [3]. While the terminal time is fixed in standard regulator problems [3], the receding terminal horizon and its good properties have been studied in recent papers for continuous systems [5,6], discrete systems [7,12,13], and delayed systems [8]. The receding horizon

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concept is applied to a deterministic version of the estimation problem for continuous systems [5]. This paper deals with a filtering problem for linear stochastic systems with approaching horizons. Conditions are derived under which systems are asymptotically stable.

2. Preliminary Results

Let the state transition matrix $\Phi(i,j), i\leq j$, of the system (1.1) be defined by

$$\Phi(i,j)=\begin{cases}A(i-1)A(i-2)\cdots A(j) & \text{for } i\geq j+1\\ I & \text{for } i=j.\end{cases} \quad (2.1)$$

The controllability and observability of systems are defined as follows.

Definition. The pair $\{A(i),B(i)\}$ is said to be uniformly completely controllable, if for some positive integer $l_c\geq 1$ the following conditions hold:

(1) $\alpha_1 I\leq W(i,i+l_c)\leq\alpha_2 I$ for all i (2.2)

(2) $||\Phi(i,j)||\leq\alpha_3(i-j)$ for all i,j (2,3)

where the controllability matrix $W(i,j), i\leq j$, is defined by

$$W(i,j)=\sum_{k=i}^{j-1}\Phi(i,k+1)B(k)B'(k)\Phi'(i,k+1), \quad (2.4)$$

α_1 and α_2 are positive constants, and $\alpha_3(\cdot)$ maps R into R and is bounded on bounded intervals.

Definition: The pair $\{A(i),C(i)\}$ is said to be uniformly completely observable, if for some positive integer $l_0\geq 1$ the following conditions hold:

(1) $\alpha_4 I\leq M(i-l_0,i)\leq\alpha_5 I$ for all i (2.5)

(2) The same as in (2.3) (2.5)

where α_4 and α_8 are positive constants, and the observability matrix $M(i,j), i\leq j$, is defined by

$$M(i,j)=\sum_{k=i}^{j-1}\Phi'(k,i)C'(k)C(k)\Phi(k,i) \quad (2.6)$$

As a consequence of uniform complete controllability and observability there exist positive constants $\alpha_6,\alpha_7,\alpha_8$, and α_9 such that

$$||A(i)||\leq\alpha_6,\quad ||A^{-1}(i)||\leq\alpha_7,$$
$$||B(i)||\leq\alpha_8,\quad ||C(i)||\leq\alpha_9 \quad (2.7)$$

for all i. The invariance of the uniform observability of the system (1.1)−(1.2) is stated in the next theorem. Let $l=\max(l_c,l_0)$

Theorem 2.1 If $\{A(i),C(i)\}$ is uniformly completely observable, then $\{A(i)+F(i)C(i),C(i)\}$ is uniformly completely observable provided $||F(i)||\leq\alpha_{10}$ for some positive constant α_{10}.

The dual result of Theorem 2.1 is well known [9,13] and the proof of Theorem 2.1 can be carried out similarly.

3. Filter with Approaching Horizon

It is a very basic concept that the state can not be measurable for filtering problems. But the initial state's mean and covariance are assumed to be known for the standard Kalman filter as given in (1.3), which is unreasonable. It is more realistic to assume that F_0 is sufficiently large or even infinite, if the initial state is not known. There are many requirements for filters to have good properties. Filter gains in (1.5) must be constant for constant linear systems in order to give easy implementation and the computation of the gain must be easy. In addition filters must possess stability properties and robustness. In order to satisfy some of these properties a steady state filter has been suggested by assuming $t_0=-\infty$, whose gain is constant but its computation of the gain is rather difficult. It is well known concept that the performance index like (1.4) must be chosen in such ways that the resulting filter satisfies as many requirements as possible. It is, of course, not possible to include above requirements explicitly in the performance index. Since solutions of time-invariant systems are invariant on time shift, we use this property to define a moving criterion

$$E(x(i+1)|y(j),\ i-N\leq j\leq i) \quad (3.1)$$

The measurements on $[i-N,\ i]$ are only used for the estimator of $x(i+1)$. The measurements on $[t_0,\ i-N-1]$ are discarded and instead

$$E(x(i-N)-\hat{x}(i-N))(x(i-N)-\hat{x}(i-N))'=F(i-N) \quad (3.2)$$

is assumed. The solution of the above estimation problem is given in the next theorem.

Theorem 3.1 The best estimate $x(i+1)$ of (3.1) with the condition (3.2) is given by

$$\hat{x}(i+1)=A(i)\hat{x}(i)+A(i)P(i-N.i)C'(i)$$

$$[C(i)P(i-N,i)C'(i)+R(i)]^{-1}$$
$$(y(i)-C(i)\hat{x}(i)), \tag{3.3}$$

where $P(i-N,\ i)$ is obtained from (1.6) by replacing j by $i-N$ and $P(i-N,\ i-N)=F(i-N)$.

The compution of matrix $P(i-N,i)$ is easy since it is computed over a finite time interval. Especially for time-invariant systems the estimator is very simple. For time-invariant systems, matrices $A(i),B(i),C(i),Q(i),R(i)$, and $F(i)$ are all assumed to be constant. The filter for time-invariant systems is given as follows:

$$\hat{x}(i+1)=A\hat{x}(i)+AP(N)C'$$
$$[CP(N)C'+R]^{-1}(y(i)-C\hat{x}(i)) \tag{3.5}$$

where $P(N)$ is obtained by

$$P(i+1)=AP(i)A'-AP(i)C'(CP(i)C'+R)^{-1}$$
$$CP(i)A'+BOB',\ P(O)=F. \tag{3.6}$$

It is the constant matrices in (3.5) that characterize the estimator (3.5). Although the estimator (3.3)—(3.5) is easy, its stability is in question, which is most important. We will show in the next theorem that the estimator is asymptotically stable with large values of the matrix F.

Theorem 3.2. (a) Assume

$$0\le Q(i)\le\alpha_{11}I,\ \alpha_{12}I\le R(i)\le\alpha_{13}I \tag{3.7}$$

for all i. If the pair $\{A(i),C(i)\}$ is uniformly completely observable, $B(i)$ is bounded as in (2.7), and $F(i)$ satisfies

$$-F(i+1)+A(i)F(i)A'(i)+A(i)F(i)C'(i)$$
$$[C(i)F(i)C'(i)+R(i)]^{-1}C(i)F(i)A'(i)$$
$$+B(i)Q(i)B'(i)\le 0, \tag{3.8}$$

then for any fixed N with $l_0+1\le N\le\infty$ the estimator (3.3) is uniformly asymptotically stable.

(b) Assume

$$\alpha_{14}I\le Q(i)\le\alpha_{11}I,\ \alpha_{12}I\le R(i)\le\alpha_{13}I \tag{3.9}$$

for all i. If the pairs $\{A(i),B(i)\}$ and $\{A(i),C(i)\}$ are uniformly completely controllable and observable respectively and $F(i)$ satisfies (3.8), then for any fixed N with $l+1\le N\le\infty$ the estimator (3.3) is uniformly asymptotically stable.

The proof of Theorem 3.2 will be given in Appendix II. For time-invariant systems the relation (3.8) is given by

$$-F+AFA'+AFC'[CFC'+R]^{-1}$$
$$CFA'+BOB'\le 0 \tag{3.10}$$

It is noted that the relation (3.8) implies that $F(i)\ge P(i_0,i)$ for initial conditions $F(i_0)=P(i_0,i_0)$ where $P(i_0,i)$ is given in (1.6). For time-invariant systems, F of (3.10) implies $F\ge P$, where P, is the solution of algebraic matrix Riccati equation. When the matrix $F(i)$ in Theorem 3.1 is assumed to be sufficiently large, i.e., $F^{-1}(i)\to 0$, we have more easily computable estimators. Let $K(j,i)=P^{-1}(j,i)$. The following matrix identity is used

$$I-Y'(ZY'+X)^{-1}Z=(I+Y'X^{-1}Z)^{-1} \tag{3.11}$$

whenever the inverses exist. The equation (1.6) can be transformed to

$$K(j,i+1)=P^{-1}(j,i+1)=\{A(i)K^{-1}(j,i)A'(i)$$
$$-A(i)K^{-1}(j,i)C'(i)[C(i)K^{-1}(j,i)C'(i)$$
$$+R(i)]^{-1}C(i)K^{-1}(j,i)A'(i)$$
$$+B(i)Q(i)B'(i)\}^{-1}$$
$$=A'^{-1}(i)\{K^{-1}(j,i)+1)[I-C'(i)(C(i)K^{-1}$$
$$(j,i)C'(i)+R(i))^{-1}C(i)K^{-1}(j,i)]$$
$$+A^{-1}(i)B(i)Q(i)B'(i)A'^{-1}(i)\}^{-1}A^{-1}(i)$$
$$=A'^{-1}(i)\{K^{-1}(j,i)[I+C'(i)R^{-1}(i)K^{-1}(j,i)]^{-1}$$
$$+A^{-1}(i)B(i)Q(i)B'(i)A'^{-1}(i)\}^{-1}A^{-1}(i)$$
$$=A'^{-1}(i)[K(j,i)+C'(i)R^{-1}(i)C(i)]$$
$$\{I+A^{-1}(i)B(i)Q(i)B'(i)A'^{-1}(i)$$
$$[K(j,i)+C'(i)R^{-1}(i)C(i)]\}^{-1}A^{-1}(i) \tag{3.12}$$
$$=A'^{-1}(i)[K(j,i)+C'(i)R^{-1}(i)C(i)]A^{-1}(i)-A'^{-1}$$
$$[K(j,i)+C'(i)R^{-1}(i)C(i)]A^{-1}(i)B(i)H(i)[I$$
$$+H'(i)B'(i)A'^{-1}(i)(K(j,i)+C'(i)R^{-1}(i)$$
$$C(i))A^{-1}(i)B(i)H(i)]^{-1}H'(i)B'(i)A'^{-1}(i)$$
$$[K(j,i)+C'(i)R^{-1}(i)C(i)]A^{-1}(i) \tag{3.13}$$

where $Q(i)=H'(i)H(i)$ for some $H(i)$.

The filter gain of (1.5) can be transformed to

$$A(i)P(j,i)C'(i)[C(i)P(j,i)C'(i)+R(i)]^{-1}$$
$$=A(i)K^{-1}(j,i)C'(i)[C(i)K^{-1}(j,i)C'(i)+R(i)]^{-1}$$
$$=A(i)K^{-1}(j,i)C'(i)[I+R^{-1}(i)C(i)K^{-1}$$
$$(j,i)C'(i)]^{-1}R^{-1}(i)$$
$$=A(i)K^{-1}(j,i)C'(i)[I-(R(i)+C(i)K^{-1}$$
$$(j,i)C'(i))^{-1}C(i)K^{-1}(j,i)C'(i)]R^{-1}(i)$$
$$=A(i)K^{-1}(j,i)[I-C'(i)(R(i)+C(i)K^{-1}$$
$$(j,i)C'(i))^{-1}C(i)K^{-1}(j,i)]C'(i)R^{-1}(i)$$
$$=A(i)K^{-1}(j,i)[I+C'(i)R^{-1}(i)C(i)K^{-1}$$
$$(j,i)]^{-1}C'(i)R^{-1}(i)$$
$$=A(i)[K(j,i)+C'(i)R^{-1}(i)C(i)]^{-1}C'(i)R^{-1}(i). \tag{3.14}$$

From (3.13)—(3.14) the moving horizon estimator and its stability property can be obtained when $F(i)$ becomes infinite.

Theorem 3.3 When $F(i)$ becomes infinite at each time i, the best estimator of (3.3) becomes

$$\hat{x}(i+1)=A(i)\hat{x}(i)+A(i)\hat{K}^{-1}(i-N-1,i) C'(i)R^{-1}(i)(y(i)(y(i)-C(i)\hat{x}(i)) \quad (3.15)$$

where $\hat{K}(i-N-1,i)$ is obtained from

$$\hat{K}(j,i+1)=A'^{-1}(i)\hat{K}(j,i)A^{-1}(i)-A'^{-1}(i) K(f,i)A^{-1}(i)B(i)H(i)[I+H'(i) B'(i)A'^{-1}(i)\hat{K}(j,i)A^{-1}(i)B(i)H(i)]^{-1} H'(i)B'(i)A'^{-1}(i)\hat{K}(j,i)A^{-1}(i)+C'(i+1) R^{-1}(i+1)C(i+1),\ \hat{K}(j,j)=0, j\leq i \quad (3.16)$$

If the matrices $Q(i)$ and $R(i)$ satisfy (3.7), the pair $\{A(i),C(i)\}$ is uniformly completely observable, and $B(i)$ is bounded as in (2.7), then for any fixed N with $l_0+1\leq N\leq\infty$ the estimator (3.15) is uniformly asymptotically stable. Moreover if the matrices $Q(i)$ and $R(i)$ satisfy (3.9), and the pairs $\{A(i), B(i)\}$ and $\{A(i), C(i)\}$ are uniformly completely controllable and observable respectively, then for any fixed N with $l+1\leq N\leq\infty$ the estimator (3.15) is uniformly asymptotically stable.

The equations (3.15) and (3.16) have been obtained from (3.3), (3.13), and (3.14) by replacing $[K(j,i)+C'(i)R^{-1}C(i)]$ by $\hat{K}(j,i)$. The rest of the proof of the theorem will be given in Appendix III. It is noted that the computation of the matrix $K(i-N,i)$ is easy since it is carried over a finite interval. For time invariant systems the estimator (3.15) is given by

$$\hat{x}(i+1)=A\hat{x}(i)+A\hat{K}^{-1}(N+1)C'R^{-1} (y(i)-C\hat{x}(i)) \quad (3.17)$$

where $\hat{K}(N+1)$ is obtained from

$$\hat{K}(i+1)=A'^{-1}\hat{K}(i)A^{-1}-A'^{-1}\hat{K}(i)A^{-1} BH[I+H'B'A'^{-1}\hat{K}(i)A^{-1}BH]^{-1}H'B'A'^{-1} \times\hat{K}(i)A^{-1}+C'R^{-1},\ \hat{K}(0)=0 \quad (3.18)$$

The results in Theorems 3.2 and 3.3 can be obtained from the following basic properties. The proof of Lemma 3.1 will be given in Appendix I.

Lemma 3.1. (a) If the matrix $F(i)$ satisfies (3.8) then the solution of (1.6) with $P(j,j)=F(j)$ satisfies the following order relation:

$$P(j_1,i)\leq P(j_2,i) \text{ for } j_1\leq j_2\leq i \quad (3.19)$$

(b) The solution matrix of (3.16) satisfies the following order relation

$$\hat{K}(j_1,i)\geq\hat{K}(j_2,i) \text{ for } j_1\leq j_2\leq i \quad (3.20)$$

(c) If $Q(i)$ and $R(i)$ satisfy (3.7), $\{A(i),C(i)\}$ is uniformly completely observable, $B(i)$ is bounded as in (2.7), and $F(i)$ satisfies the relation (3.8), then for any fixed N with $l_0+1\leq N<\infty$ there exist positive constants α_{15} and α_{16} such that

$$\alpha_{15}I\leq P(i-N,i)(\text{or } K(i-N-1,i))\leq\alpha_{16}I \quad (3.21)$$

(d) If $Q(i)$ and $R(i)$ satisfy (3.9), $\{A(i),B(i)\}$ and $\{A(i),C(i)\}$ are uniformly completely controllable and observable respectively, and $F(i)$ satisfies (3.8), then for any fixed N with $l+1\leq N\leq\infty$ there exist positive constants α_{17} and α_{18} such that

$$\alpha_{17}I\leq P(i-N,i)(\text{or } K(i-N-1,i))\leq\alpha_{18}I \quad (3.22)$$

The estimator (3.3) is the best estimator for (3.1), (3.2) and (3.8) and the estimator (3.15) is optimal for the (3.1), (3.2) and $F(i)=\infty$. Even though they are optimal in their own rights, it will be very interesting to compare with the standard Kalman filter problem. The covariance of the error $e(t)=x(i)-\hat{x}(i)$ will be stated as follows.

Theorem 3.4. If the filter (3.3) (the filter (3.15) respectively) is used, instead of the Kalman filter (1.5) for the standard problem(1.1), (1.2), and (1.3) then the error covariance has the following bounds.

$$\tilde{P}(i_0,i_1)\leq E[e(i_1)-\bar{e}(i_1))(e(i_1)-\bar{e}(i_1))'] \leq\begin{cases} P(i_1-N,i_1)+\Phi_P(i_1,i_0) (F_0-P(i_0-N,i_0))\Phi_P'(i_1i_0) \\ (K^{-1}(i_1-N-1,i_1)+\Phi_K(i_1,i_0) (F_0-K^{-1}(i_0-N-1,i_1))(\Phi_K'(i_1,i_0)\text{rep.}) \end{cases}$$

where $\tilde{P}(i_0,i_1)$ is obtained from (1.6) with the initial condition $\tilde{P}(i_0,i_0)=F_0$, $P(i-N,i)$ from(1.6) with (3.8), $K(i-N-1,i)$ from (3.16), $\Phi_P(i,i_0)$ is the state transition matrix of $\{A(i)-A(i) P(i-N,i)C'(i)[C(i)P(i-N,I)C'(i)+R(i)]^{-1}C(i)\}$, and $\Phi_K(i,i_0)$ is the state transition matrix of $\{A(i)-A(i)\hat{K}^{-1}(i-N-1,i)C'(i)R^{-1}(i)C(i)\}$

The proof of the Theorem 3.4 is given in Appendix IV. Since $\Phi_P(i_1,i_0)$ and $\Phi_K(i_1,i_0)$go to zero as i_1 goes to infinity from Theorems 3.2 and 3.3, the steady state error covariance difference from the steady state Kalman filter is no more than $P(i_1-N,i_1)-\tilde{P}(i_0,i_1)$ as i_1 goes to infinity. Since this difference becomes zero for $N=\infty$, the estimators (3.3) and (3.15) can be considered suboptimal estimators for the standard Kalman filter. For time invariant systems the steady state error covariance difference is $P(N)$ (or$\hat{K}^{-1}(N+1)) - P_s$ where P_s is the steady state solution of (1.6). $P(N)$ (or$\hat{K}^{-1}(N+1)$) goes to P_s as N goes to infinity.

4. Conclusion

The Kalman filter has been applied to so many fields by so many researchers. Because of its importance, properties and modifications of the filter need to be investigated thoroughly. A modified criterion (3.1) and (3.2) is introduced and the optimal estimator for this problem is given in (3.3). For control systems designs, several properties including stability, easy computation, and ease of implementation need to be considered along with the optimality. It is not possible to define a criterion like (1.4) which renders required necessary properties. Therefore it is a usual practice to decide a form of criterion first and then check whether the resulting system has required properties. The optimal estimators (3.3) and (3.15) are shown to have stability properties. The computation of the filter gain is easy since the computation is carried over the finite-time intervals. For time-invariant systems these filters are easily implemented since its gains are constants. Thus the optimal crit erion (3.1) and (3.2) with certain constraints on F, which renders the estimators (3.3) and (3.15), is an excellent compromise to add better properties in the filters but to sacrifice a little for the strict sense of optimality. For those problems which require the standard approaches, (1.1) through (1.4), the estimators (3.3) and (3.15) can be used as suboptimal filters.

Appendix

1. Proof of Lemma 3.1.

(a) Consider two matrix difference equations

$P_1(i+1)=A(i)P_1(i)A'(i)-A(i)P_1(i)C'(i)(C(i)P_1(i)C'(i)+R(i))^{-1}C(i)P_1(i)A'(i)+B(i)Q_1(i)B'(i)$

where $P(i_0)=F_1(i_0)$

$P_2(i+1)=A(i)P_2(i)A'(i)-A(i)P_2(i)C'(i)(C(i)P_2(i)C'(i)+R(i))^{-1}C(i)P_2(i)A'(i)+B(i)Q_2(i)B'(i)$

where $P_2(i_0)=F_2(i_0)$. It is known that $P_1(i)\geq P_2(i)$, if $F_1(i_0)\geq F_2(i_0)$ and $Q_1(i)=Q_2(i)$, and $P_1(i)\geq P_2(i)$ if $Q_1(i)\geq Q_2(i)$ and $F_1(i)=F_2(i)$. From above it follows that $P(j_1,i)\leq F(i)$ and thus $P(j_1,j_2)\leq F(j_2)=P(j_2,j_2)$, which in return implies $P(j,i)\leq P(j_2,i)$.

(b) The equation (3.16) can be obtained from (1.6) by replacing $A(i)$,$C(i)$,$B(i)$,$Q(i)$, and $P(j,i)$ with $A'^{-1}(i)$, $R^{1/2}(i)H'(i)B'(i)A'^{-1}(i)$, $C'(i+1)$, $R^{-1}(i+1)$, and $\hat{K}(j,i)$ respectively. It is true that $\hat{K}(j_1,i)\geq 0$ and thus $\hat{K}(j_1,j_2)\geq 0=K(j_2,j_2)$. The relation (3.20) follows from this and the properties of matrix Riccati equations mentioned in (a).

(c) The upper bounds of $P(i-N-i)$ and $\hat{K}(i-N-1,i)$ are obvious from (2.3), (2.7), and summation over a finite interval. The lower bounds can also be obtained in such similar methods as dual problems of regulators [7,13,11].

(d) The case of a finite time N is almost·same as in (c). For the case of $N=\infty$ $P(i-N,i)$ is given in [10]. Since $\hat{K}(j,i)$ is derived from a cerived from a certain estimation problem as shown in (b), the property of $\hat{K}(i-N-1,i)$ follows also from [10].

2. Proof of Theorem 3.2.

We only sketch the proof. The error vector of the estimator (3.3) is given by

$$e(i+1)=[A(i)-A(i)P(i-N,i)C'(i)(C(i)P(i-N,i)C'(i)+R(i))^{-1}C(i)]e(i).$$
$$=A(i)[I-\tilde{P}^{-1}(i-N,i)C'(i)R^{-1}(i)C(i)]e(i) \quad \text{(A-1)}$$

where $\tilde{P}(i-N,i)=C'(i)R^{-1}(i)C(i)+P^{-1}(i-N,i)$.

Consider a Lyapunov function for (A-1),

$$V(e(i),i)=e'(i)A'^{-1}(i-1)\tilde{P}(i-N-1,\ i-1)A^{-1}(i-1)e(i) \quad \text{(A-2)}$$

which is a ture Lyapunov function from Lemma 3.1 (c,d) and (2.7). From Lemma it can be shown that

$$V(e(i+1),i+1)-V(e(i_0),i_0)\leq -\alpha_{19}|e(i_0)|^2 \quad \text{(A-3)}$$

for $i\geq l+1$, from which follows the asymptotic stability of the system (A-1).

3. Proof of Theorem 3.3.

We only sketch the proof. The error vector of the stimator (3.15) is given by

$$e(i+1)=A(i)\,[I-K^{-1}(i-N-1,i)\,C'(i)R^{-1}(i)C(i)]e(i) \qquad (A\text{-}4)$$

Consider a Lyapunov function

$$V(e(i),i)=e'(i)A'^{-1}(i-1)\,K(i-N-2,i-1)A^{-1}(i-1)e(i). \qquad (A\text{-}5)$$

Then from Lemma 3.1 (b,c,d) it can be shown that the relation like (A-3) is satisfied.

4. Proof of Theorem 3.4.

The lower bound is obvious. Let $L(i)=A(i)P(i-N,i)C'(i)(C(i)P(i-N,I)C'(i)+R(i))^{-1}$. Then the error covariance of the estimator for arbitrary gain $L(i)$ is given by

$$S(i_0,i+1)=(A(i)-L(i)C(i))S(i_0,i)(A(i)-L(i)C(i))'+B(i)Q(i)B'(i)+L(i)R(i)L'(i) \qquad (A\text{-}6)$$

where $S(i_0,i_0)=F_0$. It can easily be shown that $P(i-N,i)$ satisfies

$$P(i-N,i+1)=(A(i)-L(i)C(i))P(i-N,i)(A(i)-L(i)C(i))'+B(i)Q(i)B'(i)+L(i)R(i)L'(i) \qquad (A\text{-}7)$$

Let $T(i)=S(i_0,i)-P(i-N-1,i)$. Then we have

$$T(i+1)=(A(i)-L(i)C(i))(T(i)-T(i))(A(i)-L(i)C(i))' \qquad (A\text{-}8)$$

where $T(i)=P(i-N,i)-P(i-N-1,i)\geq 0$ and $T(i_0)=F_0-P(i_0-N,i_0)$, from which follows Theorem 3.4 for the case of the case of the estimator (3.3). For the estimator (3.15) let $L(t)=A(i)\hat{K}(i-N-1,i)C'(i)R^{-1}(i)$. Then the relation(A-6) holds and $K^{-1}(i-N,i)$ satisfies the relation(A-7) wtih $P(i-N,i)$ replaced by $K^{-1}(i-N,i)$. Let $T(i)=S(i_0,i)-K^{-1}(i-N-1,\ i)$. Then the relation (A-8) holds with $\tilde{T}(i)=K^{-1}(i-N,i)-K^{-1}(i-N-1,i)$. Since it can be shown that $T(i)\geq 0$ it follows that Theorem 3.4 holds for the estimator (3.15). This completes the proofs of Appendix.

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