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The Optimal Design of Digital Controller for Deadbeat Performance.

論 文
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Abstract

This paper presents in some detail a discussion of the design of the digital control system without nonlinear elements to exhibit deadbeat performance in response to step inputs. The transfer function of digital controller is obtained by calculating the sequence of inputs and outputs of the digital controller for deadbeat performance. Digital controllers are designed for unity feedback multivariable systems by this method, which is well adapted for obtaining a solution by a digital computer.

1. Introduction

In the previous decade, much effort has been made toward the analytical design of the digital control system which uses a digital computer as controller[1,2,3,4,5]. In particular, much attention has been focused on the design of system with deadbeat performance, by which is meant that a system responds to a step input in the quickest manner without overshoot.

Z-transform method is used in the design of the digital controller of digital control systems with deadbeat performance[2,4]. This classical method yields good solution for simple systems. However, it presents severe limitations and difficulties when applied to the design of high-order and multivariable systems.

The state-transmission method is applied to the design of digital controller[3]. This state-transmission method becomes more advantageous than the classical Z-transform method in the system design when nonlinearities are not disregarded. But it is not adequate to the design of multivariable systems.

In this paper, much effort is devoted to the analytical design of digital controller of multivariable systems without nonlinear elements which can respond with deadbeat performance for step input[6]. In order to analyze the response of the digital control systems which are taken for design examples, the digital computer simulation is set up.

2. Design of the digital controller for deadbeat performance

We design the digital controller that allows digital control systems to respond in a deadbeat manner to a vector step input. Consider the digital control system shown in the block diagram of Fig. 1

This system is described by the linear vector differential equations

$$\dot{X}=FX+GU \tag{1}$$

$$Y=CX \tag{2}$$

where X is the $n\times 1$ state vector

U is the $m\times 1$ control vector

Y is the $p\times 1$ output vector

The objective of this problem is to design $D(Z)$ to cause $Y(t)$ to respond in a deadbeat ma-

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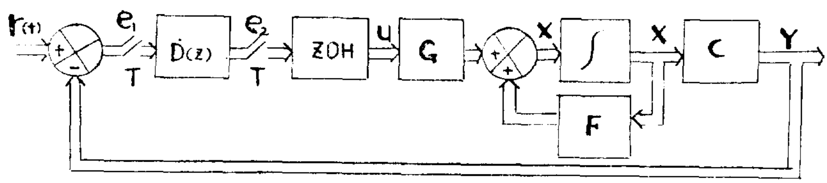

Fig. 1. Block diagram of digital control system

nner to a vector input $r(t)$, in the case of zero initial conditions.

The computation of $D(Z)$ proceeds in two parts. First, the sequence of the output of digital controller, $e_2(k)$ is computed to obtain the deadbeat response of $Y(t)$ in a minimum number of sampling periods. From the $Y(t)$ response, the sequence of the input of digital controller, $e_1(k)$ may be computed. Then, $D(Z)$ is to be computed such that $e_2(k)$ is generated from $e_1(k)$.

Because of the zero-order hold, the discrete model for a given continuous system is given by

$$X[(k+1)T]=A(T)X(kT)+B(T)e_2(kT) \tag{3}$$

$$Y(kT)=CX(kT) \tag{4}$$

where

$$A(\mathrm{T})=exp(FT)$$

$$B(T)=\int_0^T exp\ (FT)Gdt$$

and T is a sampling period. For simplicity, T is omitted. Since $X(o)=0$, $X(k)$ and $Y(k)$ may be computed as follows;

$$X\ (k)=\sum_{l=0}^{k-1}A^{k-l-1}Be_2(l) \tag{5}$$

$$Y\ (k)=\sum_{l=0}^{k-1}CA^{k-l-1}Be_2(l) \tag{6}$$

Let N be the number of sampling intervals until deadbeat response is achieved. For a vector step input of arbitrary amplitude, denoted r_0, we want e_1 to go to zero in the minimum number of sampling periods. Thus,

$$r_0=Y(N)=\sum_{l=0}^{N-1}CA^{N-l-1}Be_2(l) \tag{7}$$

or, in matrix form,

$$[CA^{N-1}B\ \ CA^{N-2}B\cdots CB]\begin{pmatrix} e_2(0)\\ e_2(1)\\ \vdots\\ e_2(N-1)\end{pmatrix}=r_0 \tag{8}$$

In order to guarantee a deadbeat response, $X(N)$ must be zero. Since $U(t)$ is constant in the interval $NT\leq t<(N+1)T$ and $\dot{X}(NT)=0$, then $X(t)$ cannot change from NT to $(N+1)T$. Thus, $U(N)$ will be the control required to effect the step change in the output response.

$$U(t)=U(N)=e_2(N)\ \text{for}\ NT\leq t \tag{9}$$

From the state equation

$$\dot{X}(N)=FX(N)+Ge_2(N) \tag{10}$$

Equating $\dot{X}(N)=O$ and substituting Eq. (5) into Eq. (10), we obtain

$$F\left[\sum_{l=0}^{N-1}A^{N-l-1}Be_2(l)\right]+Ge_2(N)=O \tag{11}$$

From Eq. (11)

$$\sum_{l=0}^{N-1}FA^{N-l-1}Be_2(l)+Ge_2(N)=O \tag{12}$$

and in matrix form, Eq. (12) may be written as

$$[FA^{N-1}B\ \ FA^{N-2}B\cdots FBG]\begin{pmatrix} e_2(0)\\ e_2(1)\\ \vdots\\ e_2(N)\end{pmatrix}=O \tag{13}$$

Eqs. (8) and (13) may be combined as Eq. (14) to form the system equations that must be solved to determine the sequence of e_2

$$\begin{bmatrix} CA^{N-1}B & CA^{N-2}B\cdots CB & O\\ FA^{N-1}B & FA^{N-2}B\cdots FB & G\end{bmatrix}\begin{pmatrix} e_2(0)\\ e_2(1)\\ \vdots\\ e_2(N)\end{pmatrix}=\begin{bmatrix} r_0\\ O\end{bmatrix} \tag{14}$$

Let

$$\begin{pmatrix} CA^{N-1}B & CA^{N-2}B\cdots CB & O\\ FA^{N-1}B & FA^{N-2}B\cdots FB & G\end{pmatrix}=S \tag{15}$$

then, from Eq. (14),

$$\begin{pmatrix} e_2(0) \\ e_2(1) \\ \vdots \\ e_2(N) \end{pmatrix} = S^{-1} \begin{pmatrix} r_0 \\ O \end{pmatrix} \tag{16}$$

The sequence of e_2 is obtained from the solution of Eq. (16). Let us denote that solution in the following way;

$$e_2(l) = P(l) r_0, \quad l = 0, 1, \cdots N \tag{17}$$

Matrice $P(l)\,(l=0,1,\cdots N)$ can be obtained in the course of computation as shown in the following section.

Then, let us consider computation of the e_1 sequence. From the error equation, we have that

$$e_1(k) = r_0 - Y(k) \tag{18}$$

From Eq. (6),

$$e_1(k) = r_0 - \sum_{l=0}^{k-1} C A^{k-l-1} B e_2(l) \tag{19}$$

Substituting Eq. (17) into Eq. (19) and manipulating it, we obtain

$$e_1(k) = \left[1 - \sum_{l=0}^{k-1} C A^{k-l-1} B P(l)\right] r_0 \tag{20}$$

Finally, $D(Z)$ may be computed as follows; Taking the Z-transform of the $e_1(k)$ sequence, we get

$$E_1(Z) = \sum_{k=0}^{\infty} e_1(k) Z^{-k} \tag{21}$$

The infinite series due to the Z-transform of $e_1(k)$ is terminated at $N-1$, since the coefficients $e_1(k)$ for $k > N-1$ are identically zero. Thus,

$$E_1(Z) = \left\{\sum_{k=0}^{N-1} Z^{-k}\left[1 - \sum_{l=0}^{k-1} C A^{k-l-1} B P(l)\right]\right\} r_0 \tag{22}$$

The Z-transform of $e_2(k)$ sequence is

$$E_2(Z) = \sum_{k=0}^{\infty} e_2(k) Z^{-k} \tag{23}$$

Since the input to the system is constant after $N-1$ sampling periods, we have

$$e_2(k) = P(N) r_0 \text{ for } k \geq N \tag{24}$$

Thus,

$$E_2(Z) = \left[\sum_{k=0}^{N-1} P(k) Z^{-k} + P(N) \sum_{k=N}^{\infty} Z^{-k}\right] r_0 \tag{25}$$

or,

$$E_2(Z) = \left[\sum_{k=0}^{N-1} P(k) Z^{-k} + P(N) \frac{Z^{-N}}{1-Z^{-1}}\right] r_0 \tag{26}$$

Since $E_2(Z) = D(Z)\ E_1(Z)$, from Eqs. (22) and (26)

$$\left[\sum_{K=0}^{N-1} Z^{-k} P(k) + P(N) \frac{Z^{-N}}{1-Z^{-1}}\right] r_0 = D(Z)$$

$$\left[\sum_{k=0}^{N-1} Z^{-k} \left(1 - \sum_{k=0}^{k-1} \left\{C A^{k-l-1} B P(l)\right\}\right)\right] r_0 \tag{27}$$

Since Eq. (27) must hold for arbitrary r_0, we obtain the result as follows;

$$D(Z) = \left[\sum_{k=0}^{N-1} Z^{-k} P(k) + P(N) \frac{Z^{-N}}{1-Z^{-1}}\right]$$

$$\left[\sum_{k=0}^{N-1} Z^{-k} \left(1 - \sum_{l=0}^{k-1} \left\{C A^{k-l-1} B P(l)\right\}\right)\right]^{-1} \tag{28}$$

3. Design examples

Based upon the technique developed above, the design of the digital controller of linear digital control systems can be carried out systematically in four major steps, which are summarized as follows;

(i) Compute the transition matrix A and input transition matrix B.

(ii) Determine the minimum number of sampling periods, N, by the inspection of Eq. (14)

(iii) Compute the matrix S of Eq. (15) and determine $P(l)\ (l=0,1,\cdots N)$ from the matrix S^{-1}.

(iv) Determine the transfer function of the desired digital controller by use of Eq. (28)

Two steps among them, i.g., (i) and (iii) are

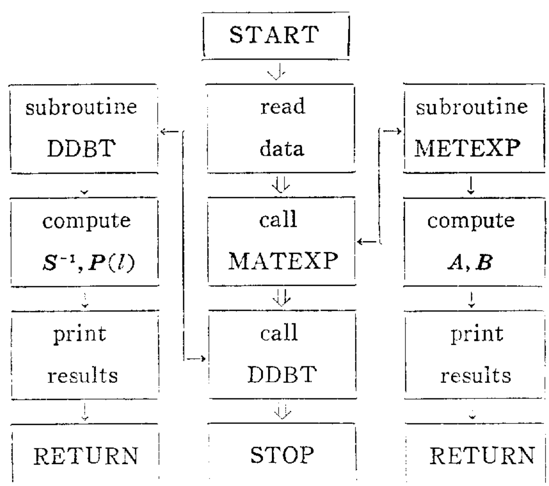

Fig. 2. The simplified flow chart for design of deadbeat controller

carried out by use of digital computer. The simplified flow chart is shown in Fig. 2. The design procedure as outlined above is best illustrated by the following numerical examples.

3.1 Single input-single output system secondorder system: n=2, m=p=1

Consider a system shown in Fig. 3. The sampling period is assumed to be 1 sec. The transfer function of the control process is

$$G(S)=\frac{1}{S(S+1)}$$

The performance specifications are that (1) the system response to step input is to have no ripple and zero error in the steady state, and(2) the transient component must decay to zero in the shortest possible time. Design a digital controller to meet these requirements.

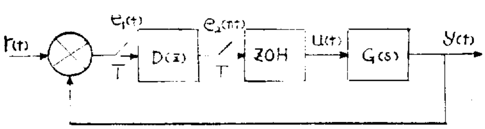

Fig. 3. Digital Control System

The state equation of the process can be expressed as follows;

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}=\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}+\begin{pmatrix} 1 \\ 0 \end{pmatrix}u \tag{29}$$

The output equation is given by

$$y=[0\ \ 1]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{30}$$

Thus, matrices $\boldsymbol{F}, \boldsymbol{G}$, and $\boldsymbol{C}$ are given as

$$\boldsymbol{F}=\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} \quad \boldsymbol{G}=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \boldsymbol{C}=[0\ \ 1]$$

An inspection of Eq. (14) reveals that $N=2$.

The state transition matrix $\boldsymbol{A}$ and the input transition matrix $\boldsymbol{B}$ are calculated for the case $T=1$ sec by use of digital computer.

$$\boldsymbol{A}=\begin{pmatrix} 0.36788 & 0 \\ 0.63212 & 1 \end{pmatrix} \quad \boldsymbol{B}=\begin{pmatrix} 0.63212 \\ 0.36788 \end{pmatrix}$$

Other matrices needed are also calculated. Thus, Eq. (16) becomes

$$\begin{pmatrix} e_2(0) \\ e_2(1) \\ e_2(2) \end{pmatrix}=\begin{pmatrix} 1.582 & -1.243 & -1.582 \\ -0.582 & 0.243 & 0.582 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} r_0 \\ 0 \\ 0 \end{pmatrix} \tag{31}$$

From Eqs. (17) and (31), we obtain

$\boldsymbol{P}(0)=1.582$, $\boldsymbol{P}(1)=-0.582$, $\boldsymbol{P}(2)=0.0$

Therefore, the digital transfer function is obtained from Eq. (28) as follows;

$$D(Z)=\frac{1.582-0.582Z^{-1}}{1+0.418Z^{-1}} \tag{32}$$

3.2 Multiple input-multiple output system fourth-order system; n=4, m=p=2

Consider a multiple input-multiple output process characterized by the transfer matrix

$$\boldsymbol{G}(S)=\begin{pmatrix} \frac{1}{S+2} & \frac{1}{S+3} \\ \frac{1}{S+1} & \frac{1}{S} \end{pmatrix}$$

Design a digital controller so that the process may respond with deadbeat performance.

The state equation of the process can be expressed as follows;

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix}=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}+\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{33}$$

The output equation is

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{34}$$

Thus, matrices $\boldsymbol{F}, \boldsymbol{G}$, and $\boldsymbol{C}$ are obtained as

$$\boldsymbol{F}=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{G}=\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \quad \boldsymbol{C}=\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are obtained for the sampling period $T=1$ sec by use digital computer.

$$\boldsymbol{A}=\begin{pmatrix} 0.13533 & 0 & 0 & 0 \\ 0 & 0.36788 & 0 & 0 \\ 0 & 0 & 0.04978 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{B}=\begin{pmatrix} 0.43233 & 0 \\ 0.93212 & 0 \\ 0 & 0.31674 \\ 0 & 1 \end{pmatrix}$$

Because the system has two inputs and two outputs, it follows that $N=4$ from Eq. (14). Hence, Eq. (14) takes on the form

$$\begin{pmatrix} 0.05851 & 0.01577 & 0.43233 & 0.31674 & 0.0 & 0.0 \\ 0.23254 & 1.0 & 0.63212 & 1.0 & 0.0 & 0.0 \\ -0.11702 & 0.0 & -0.86467 & 0.0 & 1.0 & 0.0 \\ -0.23254 & 0.0 & -0.63212 & 0.0 & 1.0 & 0.0 \\ 0.0 & -0.0473 & 0.0 & -0.95021 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

$$\begin{pmatrix} e_{21}(0) \\ e_{22}(0) \\ e_{21}(1) \\ e_{22}(1) \\ e_{21}(2) \\ e_{22}(2) \end{pmatrix} = \begin{pmatrix} r_{01} \\ r_{02} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (35)$$

The matrices $P(l)$ may be obtained upon inverting the coefficient matrix of Eq. (35). Thus,

$$\begin{pmatrix} e_{21}(0) \\ e_{22}(0) \\ e_{21}(1) \\ e_{22}(1) \\ e_{21}(2) \\ e_{22}(2) \end{pmatrix} = \begin{pmatrix} 3.6592 & 0.0 & 6.8029 & -6.8029 & 1.2197 & -1.2197 \\ -2.1048 & 1.0524 & -1.0524 & 1.0524 & 0.4059 & -0.4059 \\ 1.8178 & 0.0 & -0.9206 & 0.9206 & 0.6059 & -0.6059 \\ 0.1048 & -0.0524 & 0.0524 & 0.0524 & -1.0726 & 1.0726 \\ 2.0 & 0.0 & 1.0 & 0.0 & 0.6667 & -0.6667 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} \begin{pmatrix} r_{01} \\ r_{02} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (36)$$

Now,

$$P(0) = \begin{pmatrix} 3.6592 & 0.0 \\ -2.1048 & 1.0524 \end{pmatrix} \quad P(1) = \begin{pmatrix} 1.8178 & 0.0 \\ 0.1048 & -0.0524 \end{pmatrix}$$

$$P(2) = \begin{pmatrix} 2.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix}$$

Finally, the digital transfer function can be obtained by substitution of the numerical values of the corresponding matrices into Eq. (28) ; that is

$$D(Z) = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \qquad (37)$$

where

$$d_{11} = \frac{3.6592 - 2.0331Z^{-1} + 0.2787Z^{-2}}{1 - 0.9677Z^{-1} - 0.0323Z^{-2}}$$

$$d_{12} = \frac{1.2196Z^{-1} - 0.6137Z^{-2}}{1 - 0.9677Z^{-1} - 0.0323Z^{-2}}$$

$$d_{21} = \frac{-2.1048 + 0.4342Z^{-1} - 0.0164Z^{-2}}{1 + 0.0323Z^{-1} - 0.0738Z^{-2}}$$

$$d_{22} = \frac{1.0524 - 0.6646Z^{-1} + 0.0305Z^{-2}}{1 + 0.0323Z^{-1} - 0.0738Z^{-2}}$$

4. Digital computer simulation of digital control systems

In order to analyze the deadbeat response of the digital control systems described in the last section for the step input, it is desired to set up the digital computer simulation. The computation of the response of continuous-time part between sampling periods can be carried out by the utilization of numerical integration techniques. The integration routine used in the digital computer simulation is the variable stepsize Runge-Kutta-Merson algorithm[7]. The linear recursion equation of the digital controller is simulated by use of the transfer function obtained in the previous section. A general flow chart for the simulation is shown in Fig. 4.

The output response to a unit-step input of the simple system taken for design example in the previous section is plotted in Fig. (5), which shows that the system response reaches steady state and settles to zero error in two sampling periods without overshoot. The process input, $u(t)$ becomes zero in two sampling period, because the given control process contains a free integrator.

Fig. (6) also shows the output response of the multivariable system to two unit-step inputs. One of two process inputs, $u_2(t)$ reaches to zero in two sampling periods because of a free integrator involved in the control process.

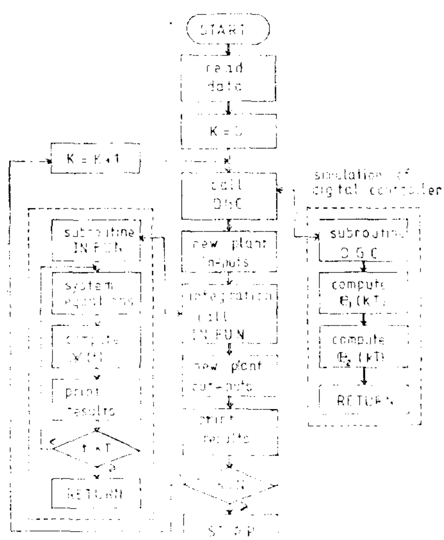

Fig. 4. A general flow chart for computer simulation

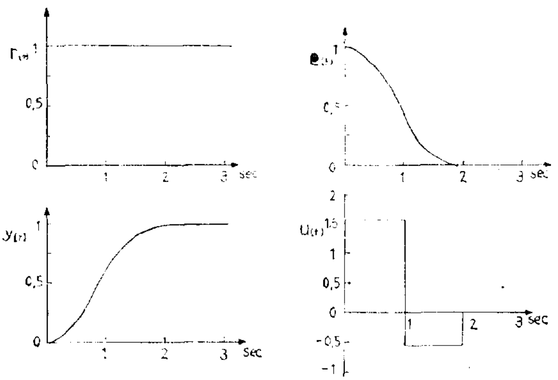

Fig. 5. Step-function response of the simple system of section 3.1

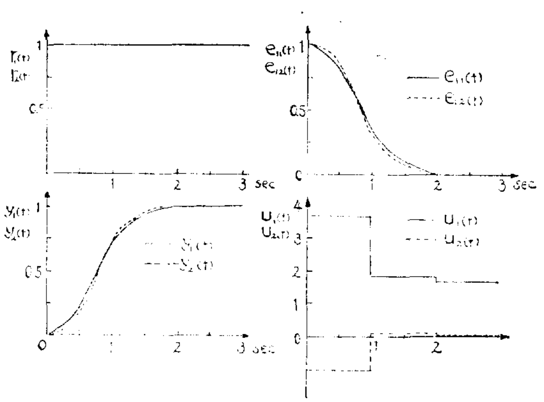

Fig. 6. Step-function response of the multivariable system of section 3.2

5. Conclusions

This paper presents in some detail a discussion of the design of the digital control system to exhibit deadbeat performance in response to step inputs. The transfer function of digital controller is obtained by calculating the sequence of inputs and outputs of the digital controller for deadbeat performance. This design procedure is well adapted for obtaining a solution by a digital computer.

In this paper, digital controllers are designed for unity feedback system. However, this method may be used for nonunity feedback system. This design technique gives good results, in particular, in multivariable systems without nonlinear elements. This method requires further studies for system with nonlinear elements.

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