<Research paper>

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A Dynamic Programming Model for the Project-Sequencing Problem

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Abstract

For many capacity expansion problems, distinct capacity types must be specified to identify capacity at different locations or capacities with different costs and operating characteristics.

In this study, a project-sequencing model is developed that allows operating costs to influence the timing decisions for project establishment. Under certain conditions, the power expansion formulation is derived that may be solved through the dynamic programming approach, and its first application to planning in electric power systems is selected to investigate an optimal policy and to show the impact of requiring system to service more than one type of demand.

Several sample testing results indicate that in some systems the efficiency of the large nuclear plants is higher than that of small ones that it may overcome the effects of the drop in reliability.

1. Introduction

For many capacity planning problems, distinct capacity types must be specified to identify capacity at different locations[1] or capacity with different costs and operating characteristics[2]. In the simplest planning model with differentiated types of capacity, the sequencing of a finite number of capacity expansion projects is selected to meet the projected demand at minimum discounted cost[6].

In this study, a project-sequencing model is developed where each project is defined by its fixed capacity and investment cost, and a new project is added when increases in demand exhaust capacity already established. The overall system costs are associated with a system operating costs, and one-time cost of installing a plant of certain type at each planning time.

The system operating costs consist of two kinds of costs associated with the forced outage and the annual energy supply at time t with the system.

Each system operating cost is derived from a continuous sub-optimization of one or more planning models. Solution procedure has the following three important assumptions:

(a) the optimal timing of one installation in an optimal sequence is independent of the other optimal installation times.

(b) Operating costs at a particular time are assumed to depend only on the set of projects in operation at that time and on exogenous quantities, such as demands, that are directly related to time.

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Given these assumptions, the project-sequencing model is formulated that may be solved through the dynamic programming model technique, and its application to the existing sample system from the Mexican electric power system[7]*, is selected to investigate an optimal policy and to show the impact of requiring the system to service more than one type of demand (i.e. Peak power and annual energy). The optimal policy is to select the sequencing of a finite number of capacity expansion projects, and timing decision for each project which is independent of those for other projects. A simple criteria which minimizes the total discounted system operating and investment costs, is derived for determining timing decisions.

The mathematical formulation for the total cost of the capacity expansion program is based on the author's paper[4] with revised subprograms and modified constraints. The solution technique is also implemented with respect to the forward dynamic programming given by the eqs.[14] and [15], and the linear programming method for the determination of the cost of the annual energy supply instead of a simple constructive method as in the paper[3, 4].

Furthermore, to reduce the computational effort, a network construction program is used at each plant installation stage.

2. Development of the Mathematical Model.

2.1. General Structure of the Model.

The project-sequencing model for the power capacity expansion problem has two major interconnected components:

(1) A dynamic programming model of choosing the types of plants to install in an existing system and best time to bring each into production.

(2) An annual system operating cost submodel for both fuel and expected forced outage. Parameters of this submodel are the demand for peak power, the demand for annual energy and the types of plants in the system.

The general structure of this project-sequencing model consists of the main body program, three supporting subroutines; FIVCM, CONVL, BCONVL, and two functional subprograms; DEL 2 and DALS1 (see chapter 4 for detailed discussion).

2.2. The Dynamic Programming of the Project Sequencing Model(PSM)

2.2.1. The capacity expansion Model.

The main objective of the capacity expansion problem is to select the types of plants to be installed, and the timing decisions of each installation.

In this study, the continuous time capacity expansion model is developed for finding the sequence of a finite set of capacity expansion projects that minimizes the total discounted system operating and investment costs.

The structure of this model consists of two major cost components such as the system operating costs associated with the forced outage and annual supplying energy costs in the existing system and the one-time cost of installing a plant of certain types of plants at each planning time.

To develop the model, let $k(i)$ be the i^{th} installed plant of type k, and τ_i be the time at which the i^{th} plant of type k is installed. Let $S=\{k(1),\ k(2),\ \cdots\cdots\}$ be a sequence of unit installations

(*) For the Korean system, Data are not completely available to run the program. However, the results of the program with some Korean data & other foreign data are available.

of plant of type k, and $\tau=\{\tau_1, \tau_2, \cdots\cdots\}$ be a sequence of installation times. Then the capacity expansion program is given as the pair (S, τ).

When the sequence S of installation of the plant of type k is finite, the total cost of capacity expansion program with discount rate r is calculated as the present value of the sum of the system costs as the eq. (1)

$$J(S,\tau)=\sum_{n=1}^{N}\left\{\int_{\tau_{n-1}}^{\tau_n} C(Z^{n-1},t)\exp(-rt)dt+v_{k(n)}\exp(-r\tau_n)\right\} + \int_{\tau_N}^{\infty} C(Z^N,t)\exp(-rt)dt \quad (1)$$

Where

$J(S,\tau)$=the total cost of the program which is related to the present value of the fixed costs and the system operating costs.

r=the discount rate

N=the total number of installation with the choice of plant type $k=k(n)\in S$, $n=1,2,\cdots,N$

K=the total number of each plant of type k

$v_k=v_{k(n)}$* : the one-time cost of the n^{th} installed plant of type k. This cost represents the capital costs of construction plus the present value (at installation time) of the fixed operating costs over an infinite horizon.

$C(Z^n,t)$=the system operating cost with the state vector Z^n of the system after n new generating plants have been installed, in year t.

Thus

$$Z^n=Z^0+\sum_{i=1}^{n}\omega_{k(i)} \quad (2)$$

Where

Z^0=the initial state vector at the start of the planning period.

$\omega_{k(i)}$=a K-dimensional unit vector with 1 if the k-type of plant is selected at the n^{th} installations, and zero elsewhere. This represents a new installing K×1 plant vector.

To make it possible to compute numerical values, the demand is assumed to be constant after some planning period T, and so can show there is a finite optimal program. Since the given system is increasingly costly to operate as time goes on, the ststem operating cost $C(Z,t)$ is bounded on $[0, T]$ and non-decreasing in t for fixed Z.

It is permissible that $C(Z,t)$ is bounded on $[0,\infty]$ for large T.

2.2.2. Development of the dynamic programming for PSM.

As mentioned earlier, the project-sequencing model is to find the optimal sequence of new plant types and its corresponding installation times which minimizes the total cost of the program $J(S,\tau)$.

So the objective function of the project sequencing model is given by the following type of dynamic programming model with respect to $\{k(n)\}$ and $\{\tau_n\}$:

$$J(S^*,\tau^*)=\underset{\{k(n)\}\in S}{\mathrm{Min}}\ \underset{\{\tau_n\}\in\tau}{\mathrm{Min}}\left[J(S,\tau)=\int_0^{\infty}C(Z^N,t)\exp(-rt)dt+G(Z^{n-1},k(n),\tau_n)\right] \quad (3)$$

(*) $v_k=v_{k(n)}$ is defined such that

$$v_k=\begin{cases}v_k(t) & \text{; convex decreasing function in } t\leq T\\ v_k(T) & \text{; constant for } t>T\end{cases}$$

Where T is the length of the planning period.

where

$$G(Z^{n-1}, k(n), \tau_n)=\sum_{n=1}^{N}\left\{\int_{\tau_{n-1}}^{\tau_n} C(Z^{n-1}, t)\exp(-rt)dt+v_{k(n)}\exp(-r\tau_n)\right\} \tag{4}$$

and $G(Z^{n-1}, k(n), \tau_n)$ is interpreted as the presented value of the total costs associated with the installation and operation of plant $k(n)$ in the system at time τ_n.

Since

$$v_{k(n)}\exp(-r\tau_n)=v_{k(n)}-\int_0^{\tau_n} rv_{k(n)}\exp(-rt)dt \tag{5}$$

Substituting the eq. (5) into the eq. (4) gives

$$G(Z^{n-1}, k(n), \tau_n)=\sum_{n=1}^{N}\left\{\int_{\tau_{n-1}}^{\tau_n} [C(Z^{n-1}, t)-r\cdot v_{k(n)}]\exp(-rt)dt+v_{k(n)}\right\} \tag{6}$$

Since the first part of the right-hand-side of the eq. (3) is independent of τ_n, the minimum total cost of the program $J(S^*, \tau^*)$ given by eq. (3) becomes as

$$J(S^*)\equiv J(S^*, \tau^*)=\underset{\{k(n)\}\in S}{\mathrm{Min}}\left[\int_0^{\infty} C(Z^N, t)\exp(-rt)dt+\sum_{n=1}^{N} G(Z^{n-1}, k(n))\right] \tag{7}$$

Where $G(Z^{n-1}, k(n))=\underset{\tau_n}{\mathrm{Min}}[G(Z^{n-1}, k(n), \tau_n)]$ (8)

Let

$$G_T(Z^N)=\int_0^{\infty} C(Z^N, t)\exp(-rt)dt$$

$$=\int_0^{T} C(Z^N, t)\exp(-rt)dt+\frac{1}{r}C(Z^N, T)\exp(-rT) \tag{9}$$

Then, the eq. (7) can be rewritten as

$$J(S^*)=\underset{\{k(n)\}\in S}{\mathrm{Min}}\left[\sum_{n=1}^{N} G(Z^{n-1}, k(n))+G_T(Z^N)\right] \tag{10}$$

Define the set of all permutations of project indices in I;

$$\Omega_I=\{I_0, I_1, \cdots, I_N | I_{i+1}=I_i\cup k(i+1) \text{ for } i=0, 1, \cdots, N-1\} \tag{11}$$

Where I_i corresponds to the system state vector Z^i of the i^{th} installation of the plant type k such that

$$Z^i=Z^{i-1}+\omega_{k(i)}=Z^{i-1}+\omega_{Si} \text{ where } S=\{k(1), k(2), \cdots, k(N)\}\equiv\{S_1, S_2, \cdots, S_N\}$$

Let $\mu(I)$ be the number of elements in I, and

$$f(I)=\underset{\{k(n)\}\in \Omega_I}{\mathrm{Min}}\left\{\sum_{n=1}^{\mu(I)} G(I_{n-1}, k(n))\right\} \tag{12}$$

Then, the eq. (10) can be written as

$$J(I^*)=\underset{\{k(n)\}\in \Omega_I}{\mathrm{Min}}\left[\sum_{n=1}^{\mu(I)} G(I_{n-1}, k(n))+G_T(Z^N)\right] \tag{13}$$

Where S^* is equivalent to $I^*\in\Omega_I$.

Thus, finding $f(I^*)$ is equivalent to finding $J(I^*)$ since

$$J(I^*)=f(I^*)+G_T(Z^N) \tag{14}$$

Note in eq. (13) that each successive term $G(I_n, k(n+1))$ in a minimum cost sequence depends on the preceding project set I_n but not on the sequence in which the projects in I_n are undertaken.

If I is the set of the first $\mu(I)$ projects in a minimum-cost sequence, it is easily shown that the sequencing of the projects in I must be minimum-cost as defined in eq. (12). It is possible to have several sequences of projects in some subset of I which may provide equal minimum-cost values as the alternative optimum solutions. If this happens, one can adopt the tie-breaking rule of selecting the minimum-cost sequence that has the latest establishment time $\tau^*=\tau^*(I,k)$ for the last project k added to complete the set I.

Consequently, the latest minimum-cost solution for the formulation given by eq. (7) may determine through the following forward dynamic programming relationship:

$$f(I)=\underset{k\in I\subset \Omega_I}{\text{lmin}}\{f(I-k)+G(I,k)\} \tag{15}$$

$$f(\phi)=0$$

whe lmin stands for the latest minimization, and the solution procedure terminates with the solution $f(I^*)$, which may be converted to the solution $J(I^*)$ through eq. (14).

2.3. System Operating Cost Sub-Model.

In the previous section, the costs related to the power capacity expansion project were the system operating costs associated with the forced outage and annual energy supply in the existing system, and one-time cost of installing a plant of certain types at each planning time.

Particularly, the system operating costs $C(Z,t)$ consist of two kinds of costs associated with; (i) the cost of forced outage at time t with system Z, called $C_1(Z,t)$, and (ii) the cost of supplying the annual energy at time t with system Z, called $C_2(Z,t)$. Thus $C(Z,t)=C_1(Z,t)+C_2(Z,t)$.

In order to calculate the costs of $C_1(Z,t)$, and $C_2(Z,t)$, the following two fundamental assumptions are undertaken:

(1) the cost of forced outage, $C_1(Z,t)$ is proportional to the expected unsupplied power at the yearly peak, and

(2) the cost of the annual supplying energy requirement is obtained by using the annual load duration curve, $L(y,t)$.

On the basis of the above assumption, each cost can be computed by using the following two sub-models.:

$$C_1(Z,t)=8760\,h\cdot\int_{Q-D(t)}^{Q}[X-(Q-D(t))]\,dF(X;Z) \tag{16}$$

$$C_2(Z,t)=8760\cdot\int_0^1 R(L(y,t),Z)\,dy \tag{17}$$

Where Q=total power capacity to produce peak power computed as $Q=\sum_{k=1}^{K}\theta_k Z_k$ where θ_k is the name plate rating capacity of a plant of type k and Z_k is the number of plants of type k in the system.

X=the amount of failed (or unavailable) power capacity which represents a Bernoulli random variable.

$D(t)$=Annual peak power demand in year t.

h=Penalty cost per unit forced outage (\$/MWH).

$F(X;Z)$=Cummulative distribution of failed capacity on forced outage with given system Z such that

$F(X;Z)$=Prob[amount of failed capacity on forced outage$\leq X$, given system Z]

$R(L,Z)$=Minimum cost of supplying L units of power with system Z.

$L(y,t)$=Load duration curve in year t, and $0\leq y\leq 1$. This represents that L units of power must be supplied for $0\leq y\leq 1$ of year t.

3. Determination of Some System Sub-Models of the PSM.

3.1. Determination $C_1(Z, t)$

Let the cost of failed power capacity per year, given X, be proportional to the amount of unsatisfied demand. Then this cost, $\Delta C_1(Z, t)$ can be written as

$$\Delta C_1(Z, t)=\begin{cases}8760\,h[X-(Q-D(t))], & \text{if } X>Q-D(t)\\ 0, & \text{if } X\leq Q-D(t)\end{cases} \tag{18}$$

Where h is the proportionality constant defined as the penalty cost per unit shortage (\$/MWH) and $D(t)=D(T)$ for $t\geq T$.

Thus the cost of forced outage at time t with the system Z is determined by the expected value of $\Delta C_1(Z, t)$ with its failed capacity distribution function $F(X\,;Z)$:

$$C_1(Z, t)=8760\,h\,E[X-(Q-D(t))]$$
$$=8760\,h\int_{Q-D(t)}^{Q}[X-(Q-D(t)]dF(X\,;Z)$$

Where the eq. (19) is the same as eq. (16).

The eq. (19) can be also rewriten as

$$C_1(Z, t)=8760\,h\int_{Q-D(t)}^{Q}\overline{F}(X\,;Z)dX$$

Where $\overline{F}(X\,;Z)=1-F(X\,;Z)$ [11].

For the detailed computation $C_1(Z^n, t)$; the cost of forced outage with the system Z^n after n installations, it requires the value of the distribution of failed capacity after n installation, $F(X\,;Z^n)$, and the cost per unit outage during the peak period; h.

3.1.1. Determination $F(X\,;Z^n)$

Assuming that each plant of type k in the system has the probability P_k of being down at time t, and also assuming that each plant failure is statistically independent of the other plants in the system, then for the given initial value of $F(X\,;Z^0)$, the distribution of failed capacity after m installations $F(X\,;Z^m)$ is given by

$$F(X\,;Z^m)=P_{k(m)}F(X-\theta_{k(m)}\,;Z^{m-1})+(1-P_{k(m)})F(X\,;Z^{m-1}) \tag{21}$$
$$m=1,2,\cdots,n$$

Thus the $F(X\,;Z^n)$ can be calculated as the convolution of $F(X\,;Z^0)$ such that

$$F(X\,;Z^n)=\begin{cases}1, & X>Q^n\\ \text{linear combination of} & \\ F(X\,;Z^0),\cdots,F(X-\sum_{i=1}^{n}\theta_{k(i)}\,;Z^0), & 0<X\leq Q^n\\ 0, & X\leq 0\end{cases} \tag{22}$$

Where

$$Z^n=Z^0+\sum_{i=1}^{K}\omega_{k(i)}$$

$$Q^n=Q^0+\sum_{i=1}^{n}\theta_{k(i)} \quad \text{for } Q^0=\sum_{k=1}^{K}\theta_k Z_k^0 \tag{23}$$

and

$$F_0(X)=F(X\,;Z^0)=\begin{cases}1, & X>Q^0\\ \text{continuous increasing in } X, & 0<X\leq Q^0\\ 0, & X\leq 0\end{cases} \tag{24}$$

3. 1. 2. Determination h

The cost per unit outage during the peak period, h ($/MWH) is usually very difficult to estimate but it affects mainly the timing of installation rather than the choice of the plant types. In this study, it is chosen as the value of 50 times the average price of energy in the sample data.

3. 1. 3. Determination $C_1(Z^n, t)$

With the determined $F(X ; Z^n)$ and h from the above sections 3. 1. 1 and 3. 1. 2, the cost of forced outage, $C_1(Z^n, t)$ can be calculated as

$$C_1(Z^n, t)=8760\, h\int_{Q_n-D(t)}^{Q_n} \bar{F}(X ; Z^n)dX \tag{25}$$

Where

$$Q^n=Q^0+\sum_{i=1}^{n} \theta_{k(i)} \text{ for } Q^0=\sum_{k=1}^{K} \theta_k Z_k^0$$

For simplicity, it has taken a discrete distribution for $F(X ; Z^n)$ and 50 MW Metric power. Consequently, $C_1(Z^n, t)$ can be calculated as

$$C_1(Z^n, t)=8760\, h \sum_{i=1}^{I\max} \bar{F}(X_i ; Z^n)\times 50 \tag{26}$$

$$\text{for } Q^n-D(t)\leq X_i \leq Q^n,\ \forall_i$$

3. 2. Determination $C_2(Z ; t)$

The cost of supplying the annual energy, $C_2(Z, t)$, arises from the need to meet the energy demands per unit time of the customers. These demands are represented by the load duration curve. The cost $C_2(Z, t)$ can be derived from two major components associated with a load duration curve for fixed t; $L(y, t)$ for $0\leq y\leq 1$, and a cost per unit time of energy supplied by each plant type k; α_k ($/MWH).

Mathematically, $L(y, t)$ is a decreasing function for fixed t and $0\leq y\leq 1$. Practically, this $L(y, t)$ is defined as the minimum power required during the fraction y of the year t. Hence the annual energy demand in year t is given by

$$E(t)=\int_0^1 L(y, t)dy \tag{27}$$

Where $\partial L(y, t)/\partial t>0$ for all $y\in[0, 1]$ and all t.

For a given cost per unit time of energy supplied by each plant of type k, α_k is the minimum cost of supplying L units of power with system Z^n after n installations; $R(L, Z^n)$ can be determined from the following optimization formulation;

$$R(L, Z^n)=\underset{X_k}{\text{Min}} \sum_{k=1}^{K} \alpha_k X_k \tag{28}$$

subject to

$$\sum_{k=1}^{K} X_k \geq L$$

$$0\leq X_k \leq u_k Z_k^n,\quad k=1, 2, \cdots, K$$

$$n=0, 1, \cdots, N$$

Where $u_k=\theta_k\xi_k$; θ_k is a name plate rating capacity of plant of type k, and ξ_k is an annual utilization factor of plant type k, and α_k is a cost of energy supplying a plant of type k, ($/MWH), X_k; the total power outputs of all plants of type k, and Z_k^n is the number of plants of type k in the system with n installations.

On the other hand, $R(L, Z^n)$ can be obtained by the following nonlinear programming method, i.e.

$$R(L, Z^n) = \underset{X_k, \lambda_1, \lambda_2}{\mathrm{Min}} \left[\sum_{k=1}^{K} \alpha_k X_k + \lambda_1 \left(-\sum_{k=1}^{K} X_k + L \right) + \lambda_2 (X_i - u_i Z_i^n) \right] \tag{29}$$

$$i = 1, 2, \cdots, K, \quad n = 0, 1, \cdots, N$$

Subject to

$X_k \geq 0$ for all k

Where the λ_1 and λ_2 are Largrange Multipliers.

Furthermore, for computational simplicity of $R(L, Z^n)$, one can also adapt a constructive solution approach in the papers [3, 4]. This approach characterizes the system at time zero by grouping the initial and decision sets of plants according to the increasing variable costs of energy (i.e. merit order). So $R(L, Z^n)$ can be computed as the linear combination of the initial function $R_0(L) = R(L, Z^0)$.

In this study, the linear programming method is used to determine $R(L, Z^n)$ under the following assumptions: The load duration curve, $L(y, t)$ for fixed t is assumed to have the following form;

$$L(y, t) = L(y) e^{gt}, \quad y \in [0, 1] \tag{30}$$

Where $L(y) = L(y, 0)$ for $t = 0$, and g is an increasing rate of demand for each segment of $y \in [0, 1]$. Next, since each plant can only produce ξ% (utilization factor) of its peak power because of the down-time due to maintenance and some possiblities of plant failures etc., the utilization factor for the nuclear power plant is assumed to be $\xi_k = 0.85$, and for the fossil plant, $\xi_k = 0.90$, respectively.

3.3. Network Configuration

The network is constructed in order to find the cost sequence of installations of plants starting at the existing system Z^0 and ending at Z^N. This can be considered in the dynamic programming phase in the network. That is, for changing N, the program constructs a dummy final node Z^F to which each Z^N is connected at cost $G_T(Z^N)$. This is to find the shortest route from Z^0 to Z^F for an optimal unit installation sequence for a given planning time period T.

This network is built from starting system Z^0 by constructing arcs due to each installation of k-type of plant.

The cost on each arc is related to $G(Z^0, k)$, and its ending node represents the new state of the system $Z^0 + \omega_k$ for $k = 1, 2, \cdots K$. This network building is continued until the optimal installation time is at least the end of the planning period T. Thus the number of installations in each stage depends on the size of the plant types. Actually the total number of installation of k types of plants is given by K^n, $n = 1, 2, 3, \cdots, N < \infty$, thus this number is very large one when K and n are large.

So, some node elimination procedure can be taken in order to reduce the number of the same states generated from each previous states. Under the node elimination of the network, the total number of distinct states created by all combinations of n installations, $\Pi(n)$, from the existing system Z^0, can be given as

$$\Pi(n) = \sum_{i=0}^{K-1} \binom{n+i-1}{i}, \quad \text{for all } n \geq 1 \tag{31}$$

Table 1 shows some results of the eq. (31).

Table 1. Numbers of Distinct Nodes at Each Installation with K Different Types of plants.

K \ n	$\Pi(n)$							
	1	2	3	4	5	·	·	·
1	1	1	1	1	1	·	·	·
2	2	3	4	5	6	·	·	·
3	3	6	10	15	21	·	·	·
4	4	10	26	35	56	·	·	·
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

According to such an elimination approach, a considerable computational times connected with $K^n - \Pi(n)$ unnecessary nodes from all possible generating nodes K^n can be saved. Further elimination of nodes with the high cost states can also be expected through the computing process.

3.4. Optimal Installation Time

For a given any sequence of plant of the type k after n installations, $\{k(n)\}$, the selection of the optimal timing decisions in eq. (4–6) seperates into N independent subproblems $G(Z^{n-1}, k(n))$ associated with the eq. (10) where $G(Z^{n-1}, k(n), \tau_n)$ is given as the eq. (4) or (6).

First of all, the minimum-cost timing solution for eq. (10) can be obtained by

$$\frac{\partial G(Z^{n-1}, k(n), \tau_n)}{\partial \tau_n} = 0$$

where τ_n is the time of the n^{th} installation, and $\tau_n \in [\tau_{n-1}, \tau_{n+1}]$ for all n.

This result gives us that the optimal installation time to install the n^{th} plant in an optimal sequence is when the system operating cost savings become equal to the equivalent annual fixed cost.

That is,

$$C(Z^{n-1}, \tau_n) - C(Z^n, \tau_n) = r \cdot v_{k(n)} \qquad (32)$$

As in Marglin's analysis [10], the eq. (32) defines the optimal timing decision $\tau^* = \tau_n(Z^n, k(n))$ at the time when the marginal benefit rate, given by the savings on operating costs provided by adding the plant type k to the set $Z^{n-1} = Z^n - k(n)$ equals the marginal cost rate for capital charges on the investment. This type of timing condition can be generalized to several cases where the investment cost v_k varies over time [5].

4. Computer Simulation Model

In order to test the project-sequencing model system under some reasonable conditions, simulations, simulation method should be chosen because of lack of understanding of the various subsystems of the project-sequencing systems, including their relative importance and interdependence upon one another. In this study, the simulation modelling is considered under the following main objectives:

(1) To generate all different installation times within the planning time period.

(2) To allocate the hydro to load duration curve within peak constraints, and to determine the number of ordered distinct state created by all possible combinations of N installations from the original system Z^0.

(3) To calculate node total capacity and capacity failure probability $\overline{F}(x:Z)$.

(4) To identify the system performance statistics of the project-sequencing system with different types of plants, installation times, and costs etc.

On the other hand, the proposed dynamic programming associated with the eqs. (14) and (15) is used to determine the optimal solution of the PSM.

Figure 1 shows the main structure of the project-sequencing model. Through the dynamic programming solution procedure, three supporting subroutines; FIVCM, CONVL, BCONVL, and two functional subprograms; DEL 2 and DALS 1 are considered internally in the main program. The

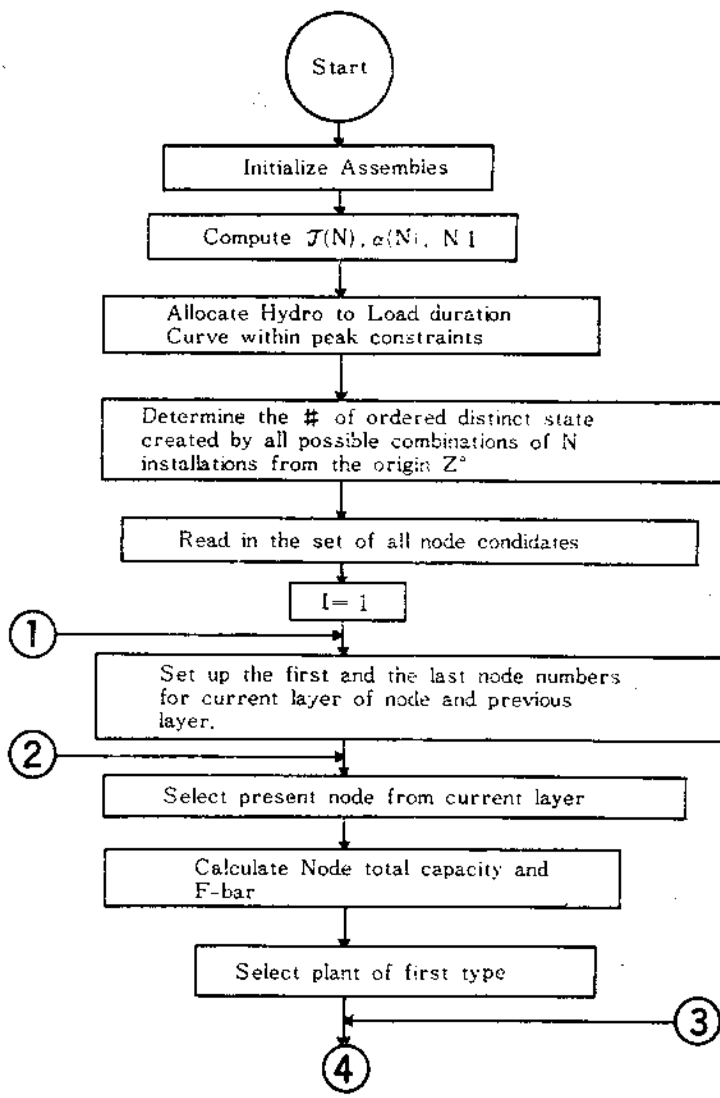
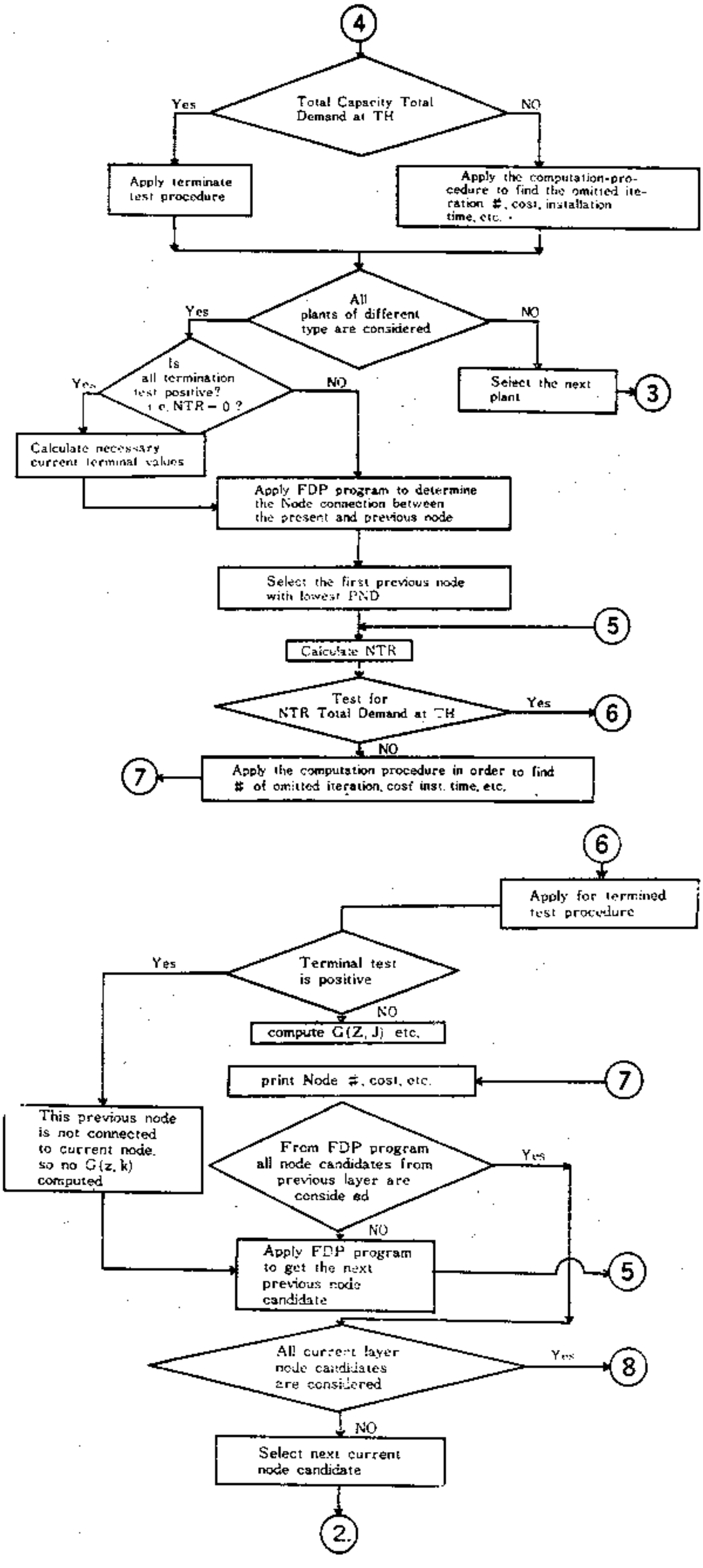


Figure 1. Main Flowchart of Project-sequencing Model


4
Total Capacity Total Demand at TH
Yes
NO
Apply terminate test procedure
Apply the computation-procedure to find the omitted iteration #, cost, installation time, etc.
All plants of different type are considered
Yes
NO
Select the next plant
3
Is all termination test positive? i.e. NTR = 0 ?
Yes
NO
Calculate necessary current terminal values
Apply FDP program to determine the Node connection between the present and previous node
Select the first previous node with lowest PND
5
Calculate NTR
Test for NTR Total Demand at TH
Yes
6
NO
Apply the computation procedure in order to find # of omitted iteration, cost inst. time, etc.
7
6
Apply for termined test procedure
Terminal test is positive
Yes
NO
compute G(Z, J) etc.
print Node #, cost, etc.
7
This previous node is not connected to current node, so no G(z, k) computed
From FDP program all node candidates from previous layer are conside ed
Yes
NO
Apply FDP program to get the next previous node candidate
5
All current layer node candidates are considered
Yes
8
NO
Select next current node candidate
2.

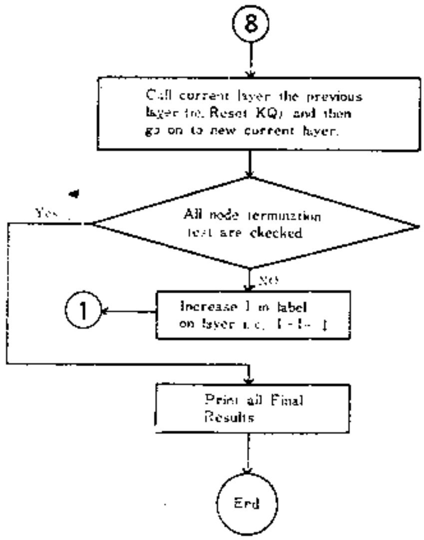


Figure 1. Continue

brief descriptions are as follows:

FIVCM…To calculate the probability distribution of failure capacity for each power plant.

CONVL…The CONVL accepts the vector X of the length N and vector Y of length M. It convolves them and returns X of length $(N+M-1)$.

BCONVL…The BCONVL accepts a real vector X of length NX and a Bernoulli variable B with $Pr(0)=BP$ and $Pr(NB-1)=1-BP$. It assumes $NB \leq NX$. It also assumes X with B and yields the result as X of length $(NX+NB-1)$.

DEL 2…(i) To calculate demand levels for load duration curve and to allocate Hydro energy to load duration curve within peak constraint.

(ii) To calculate $\Delta F(X\,;Z)=F(X\,;Z-K)-F(X\,;Z)$.

DALS1…To calculate $\Delta C(Z,\tau)=C(Z-K,t)-C(Z,t)$

4.2. Sample Study

The mathematical model and some computational method have been discussed in the previous sections to investigate the optimal type of new generating installations in different electric power system.

In this section, sample data from the Mexican Electric Power System [7] are used for the analysis of the system performance. The initial input data are provided as follows:

Length of study period …… 3 or 5 year
existing installed capacity in period 0 …… 6,250 MW
peak demand in period 0 …… 5,000 MW
peak power supply …… 3,800 MW
Annual hydro supply of total energy …… 15,410 GW/ann.
Growth rate of demand …… 9%/ann.
Discount rate for present worth …… 8%/ann.
Cost of forced outage …… $800/MWH

utilization rate;

Nuclear power plant 0.85

fossil power plant 0.90

The alternative plant's characteristics of type k being installed are described in Table 2.

In particular, the capacity of each plant of type k gives information to generate a number of 50 MW unit for each plant, and thus this gives KC=[20, 10, 20] for Z_k, $k=1, 2, 3$.

Table 3 is for the existing plant capacity for failure capacity probability and KC vector calculations. Table 4 is for the existing plant capacity for energy and KC calculations, and also gives information for variable cost of energy. Table 5 is for energy demand for the load duration curve.

The results of the dynamic programming for the project sequencing problem are shown in Table 6. As shown in Table 6, two alternative planning periods are taken for the system analysis, and the resulting optimal policies are turnd out as; (i) for the three year planning period, the optimal, installed plants are mixed with type 1, type 1, type 2 and type 1 though the optimal installation sequential times, 0.23, 1.62, 2.77, and 2.95 year. For the 5 year planning period, it turned out, S^*=(type 1, type 1, type 1, type 2) and τ^*=(1.3, 2.1, 3.4, 4.5) year respectively. Furthermore, for each case, the optimal total system operating costs are given as \$838.10^6 and 855.10^6, respectively.

Table 2. Alternative Plants of Type k.

Plant type Z_k	capacity of Z_k	capital cost $\$ \times 10^6$ (CG)	fixed op. cost $\$ \times 10^6$/year (FOC)	Prob. of failure Cap. (P_k)	Variable op. cost \$/WH (VC)
Z_1	1000 MW Nuclear	173.2	2.0	0.053	1.26
Z_2	500 MW Nuclear	101.6	1.6	0.040	1.35
Z_3	1000 MW Fossil	86.0	1.4	0.053	2.96

Table 3. Existing Plant Capacity for Probability Calculations

plant size MW	No. of Units	Prob. of failure cap.	Total Capacity MW	Remarks for (KC)
50	36	0.0108	1800	36
100	5	0.0108	500	10
150	9	0.0233	1350	27
200	9	0.01	1800	36
300	1	0.03	300	6
500	1	0.04	500	10
Total			6250	125

Table 4. Existing Plant Capacity for Energy Calculations (Fossil Plants)

Item \ Number	1	2	3	4	5	6	7	Total
Plant capacity (MW)	500	300	1000	200	100	50	100	2250
Energy cost ($\$ \times 10^6$)	3.11	3.13	3.37	3.67	4.13	4.72	5.29	21.42
KC	10	6	20	4	2	1	2	

Note. KC is the number of 50 MW unit for each plant capacity.

Table 5. Energy Demands for Load Duration Curve at Time Zero.

	Energy Demand (GW)	Hours used
1	4.45	1000 hrs
2	3.07	5000 hrs
3	2.70	2760 hrs
Total	27.25 TWH	8,760 hrs = 1 year

Note (i) 27.25 TWH = 4.45 GW × 1000 H + 3.07 GW × 5000H + 2.70 GW × 2760 H = 27,252 GWH

(ii) 1 MW = 10^5 watts
1 GW = 10^3 MW
1 TW = 10^3 GW

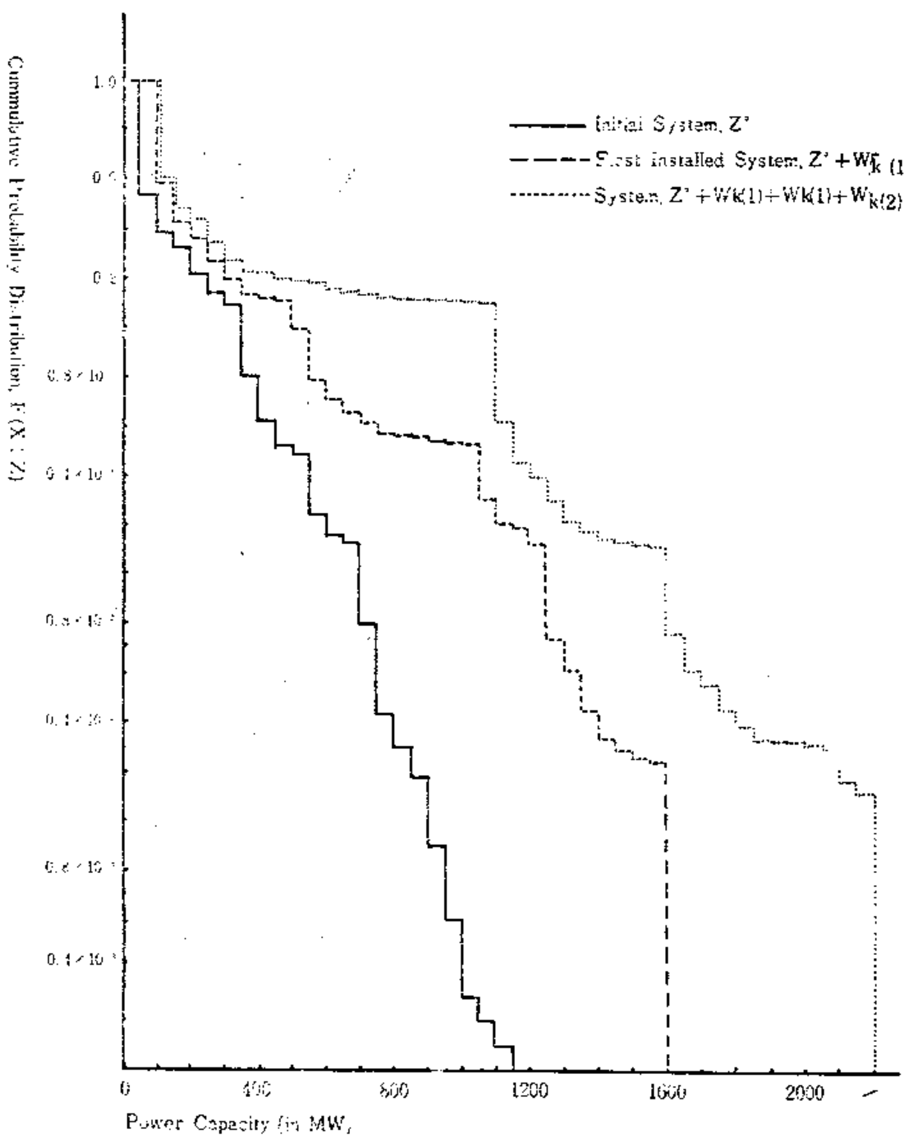


Figure 2. The Cummulative Probability Distribution of Failed Capacity, $\overline{F}(X ; Z) = 1 - F(X ; Z)$

Table 6. Results of the Computer Simulation.

Planning Period (year)	Extreme policies: CASE A — Policy Install. Time Total cost ($\$ \times 10^6$)	CASE B — Policy Install. Time Total cost ($\$ \times 10^6$)	CASE C — Policy Install. Time Total cost ($\$ \times 10^6$)	Max. Diff.	% Max. Diff.	Optimal policy — S^* ; policy, τ^* ; Opt. Install. Time (year), TC* ; Opt. cost ($\$ \times 10^6$)
3	(1, 2, 1, 1)	(2, 2, 2, 1)	(3, 3, 2, 1)	—	—	$S^*=(1, 1, 2, 1)$
	(0. 23, 1. 82, 2. 50, 2. 95)	(1. 11, 1. 82, 2. 50, 2. 95)	(1. 32, 2. 12, 2. 77, 2. 95)	—	—	$\tau^*=(0.23, 1.62, 2.77, 2.95)$
	839	880	959	120	14. 3	TC*=833
5	(1, 1, 1, 1)	(2, 2, 2, 2, 2, 2)	(3, 3, 3, 3)	—	—	$S^*=(1, 1, 1, 2)$
	(1. 3, 2. 1, 3. 4, 4. 6)	(1. 3, 2. 1, 2. 8, 3. 5, 4. 2, 4. 9)	(1. 4, 2. 2, 3. 3, 4. 5)	—	—	$\tau^*=(1.3, 2.1, 3.4, 4.5)$
	882	881	903	22	2. 5	TC*=855

Note that, A policy $[S=(1, 2, 3)]$ means a sequence of installation of three different types of plants, $(Z_k\ ;\ k=1, 2, 3)$, i.e. $Z_1=1000$ MW nuclear plant, $Z_2=500$ MW nuclear plant, $Z_3=1000$ MW fossil plant.

The figure 2 shows the probabilistic effect for the 3 year planning illustration. As shown in figure 2, one case is the plot of the failure capacity probability $\overline{F}(x : Z)$ of adding a first 1000 MW nuclear plant to the given initial system with Z^0, and the second is the case of the failure probability $\overline{F}(x : Z)$ of installing three power plants such as a 1000 MW nuclear, 1000 MW nuclear, and 500 MW nuclear power plants, sequentially to the given system Z^0. Each point on the curve represents the probability that the amount of unavailable capacity at any time t is greater than the value of the abscissa.

On the other hand, further nodes are eliminated through each installation stage because of the energy and other cost restrictions. Particularly, for the three year planning period, only 7 nodes are considered instead of 10 distinct nodes at the third installation stage.

5. Concluding Remarks

A dynamic programming model for the project-sequencing problem is developed in this study, and the following points of view are critisized from the system performance analysis:

i) The optimization determines not only the number of each plant of different type to install but also the optimal installation time in year. It is also able to calculate the trade off between the change of capital costs due to the selection of particular plant unit size and the resulting change in operating costs, provided always that the reliability criterion is satisfied.

ii) There is a good indication of a great difference between the total present worth costs of the three alternative extreme policies considered, based upon the results shown in Table 6.

iii) To compute the analysis, the procedure can be utilized to indicate the sensitivity of the optimal plans to changes in discount rate, cost of forced outage, growth rate of demand, utilization rate, etc. thus providing further information to guide the final decision.

iv) The optimization technique is used for selecting an optimum installation time of power plant operating cost. However, this technique can be revised for further extension with a great efficiency.

v) The constructive approach Shows that a change in planning horizon is reflected in changing the optimal timing of an installation, but if not too large, it does affect slightly the choice of plant types.

vi) Several sample testing results indicate that in some systems the efficency of the large nuclear plants is higher than that of small ones, so that it may overcome the effects of the drop in reliability.

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