〈論文〉

RUNOFF ANALYSIS BY DEAD ZONE LONGITUDINAL DISPERSION ANALOGY

(死帯縦拡散模型에 의한 流出解析)

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Abstract

A prompt subsurface runoff producing mechanism which creates a depletion curve of direct runoff hydrograph is simulated by a dead zone dispersion model technique. Runoff processes are carried out by routing of the outflow resulted from previous linear channel and effective rainfall from its corresponding subwatershed through a series of conceptual linear channels representing subwatersheds of a catchment. Working rules are explained for evaluating the model parameters such as translatory velocity, diffusive factor, and parameters concerning the infiltration and relative magnitude of the prompt subsurface flow region.

要 旨

直接流出 水文曲線의 減水部를 誘発하는 地表下流出 過程이 死帯縦拡散模型技法에 의하여 模擬되었다. 流出過程은 箏時曲線図에 의한 小流域을 대표하는 概急的線型水路를 통하여 上流線型水路의 流出과 該当 小流域의 有効降雨量을 追跡하여 逐行된다. 流出速度, 拡散因子, 浸透와 地表下흐름 領域에 관한 媒介変数의 算出過程이 記述된다.

INTRODUCTION

The determination of a peak flow has been a focus in the research of rainfall-runoff process. But now both the volume and time variation of runoff has become important to water use projects and pollution abatement problems. They require a complete time history of runoff, which calls for knowledge about a depletion curve. The depletion curve is mainly affected by a prompt subsurface runoff drained from the subsurface flow region (top soil layer). The subsurface flow region acts like a capacitor holding an infiltrated rainwater and releasing it slowly. The runoff arrives the outlet later than surface runoff. This phenomenon creates a tailing part of a hydrograph (depletion curve). The prompt subsurface runoff producing mechanism of the subsurface flow region is simulated in this study by a dead zone model technique adopted in a longitudinal dispersion problems in river. Meanwhile in the two parameter convective diffusion model the subsurface flow region is not considered, which has functions of translation and attenuation only.

The runoff producing hydrological system has been also represented by a conceptual model approach employing a linear reservoir and linear channels. In the conceptual models only translation and attenuation function and the subsurface runoff and uneven spatial rainfall distribution in space can be circumvented by placing the rainfall on each subwatershed formed by isochrones as a part of input system to linear channel representing a subwatershed.

DEAD ZONE DISPERSION RUNOFF MODEL

The counterpart of a longitudinal dispersion equation in flow rate

$$\frac{\partial Q}{\partial t} + u\frac{\partial Q}{\partial x} = D\frac{\partial^2 Q}{\partial x^2} \qquad (1)$$

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has been derived from the St Venant equation for a case wihtout lateral inflow and ignoring the inertial terms of the momentum equation (7).

$$\frac{\partial Q}{\partial t} + k\frac{\partial Q^*}{\partial t} + u\frac{\partial Q}{\partial x} = D\frac{\partial^2 Q}{\partial x^2} \qquad (2)$$

Based on the balance of material of a control volume composed of surface flow region and subsurface flow region, a dead zone dispersion runoff model can be formulated as in which Q is surface runoff, Q^* subsurface runoff, u translatory velocity, D diffusive factor, and k the fraction of the subsurface flow region in runoff volume (8).

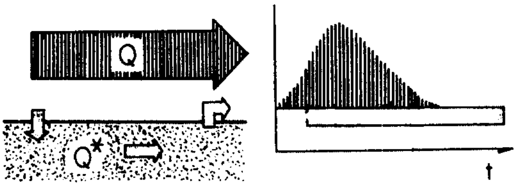

Fig.-1. Mechanics of direct runoff

An initial condition along a linear channel is $Q(t,x) = 0$ for t=0 and all x (3)

The boundary condition at the upstream end of linear channel is given by the effective rainfall converted to flow rate plus the output from the previous linear channel. At downstream end which is pictured as the outlet of a subwatershed, the boundary condition can be defined, if the change in runoff over a short distance is assumed minor, as

$$\frac{dQ(t,x)}{dx} = 0 \quad \text{for all } t \qquad (4)$$

The rate of change in storage in the subsurface flow region may be expressed as

$$\frac{dS}{dt} = fARe - Q^* \qquad (5)$$

in which S is the storage in the subsurface flow region, A the surface area, Re the effective rainfall, and f an infiltration index representing the fraction of flow entering the subsurface flow region. The storage in the subsurface flow region is assumed to be linear with runoff

$$S = KQ^* \qquad (6)$$

Since ARe represents the total runoff Q, Eq. 5 can be written with aid of Eq. 6 as

$$\frac{dQ^*}{dt} = \frac{1}{K}(fQ - Q^*) \qquad (7)$$

With the initial condition of $Q^*=0$ at t=0, the solution of Eq. 7 is given as

$$Q^* = \exp(-\frac{t}{K})\int_0^t \frac{1}{K}\exp(\frac{t'}{K})\, fQdt' \qquad (8)$$

By substitution of Eq.8, Eq.2 takes the form of

$$\frac{\partial Q}{\partial t} + u\frac{\partial Q}{\partial x} = D\frac{\partial^2 Q}{x^2} + \frac{kf}{K^2}\exp(-\frac{t}{K})\int_0^t \exp(\frac{t'}{K})Qdt' - \frac{kf}{K}Q \qquad (9)$$

EVALUATION OF MODEL PARAMETERS

Translatory Velocity and Diffusive Factor. – The two parameters can be estimated from the moments of Q, which is a solution of Eq.1, about the time origin and about the center of area. They are defined as

$$M_r^0(Q) = \int_0^\infty Q(t,x)\, t^r\, dt \qquad (10)$$

$$M_r(Q) = \int_0^\infty Q(t,x)\, (t - M_1^0)^2\, dt \qquad (11)$$

Under assumption of a linear time in variant, the relation between effective rainfall input Re, impulse response function Q, and runoff output Q_0 can be expressed in terms of the moments (6).

$$M_r^0(Q_0) = \sum_{k=0}^{r}\binom{r}{k} M_k^0(Re) M_{r-k}^0(Q) \qquad (12)$$

For r=1 and 2 Eq.12 yeilds

$$M_1^0(Q) = M_1^0(Q_0) - M_1^0(Re) \qquad (13)$$

$$M_2(Q) = M_2(Q_0) - M_2(Re) \qquad (14)$$

Since the two expressions of the first and second moment of Q calculated by Eqs. 10 and 11, the two expressions are solved simultaneously to obtain u and D. For the solution of Eq. 1 given as (7)

$$Q(L,t) = \frac{L}{\sqrt{4\pi D t^3}}\exp(-\frac{(L-ut)^2}{4Dt})\, dt \qquad (15)$$

the two parameters u and D can be evaluated by (2)

$$\frac{X^2}{4D} = \frac{M_1^0(Q)^3}{M_2(Q)}, \quad \frac{u^2}{4D} = \frac{M_1^0(Q)}{M_2(Q)} \qquad (16a, b)$$

Where M_1^0 is the first moment of the impulse reponse Q about the time origin and $M_2(Q)$ is the second moment about the center of area. When the rainfall input Re and the runoff output Q_0 are known from records, the moments $M_1^0(Q)$ and $M_2(Q)$ can be calculated from Eqs.13 and 14. Re and Q_0 in functional form can be approximated by a Lagrangian polynomials or hydrograph equation(3).

In the above approach the subsurface flow region is no taken into consideration. Therefore, the parameters u and D determined from Eqs.13a, b are subject to adjustment. A reliable way of determining the parameters is to find by a trial and error basis for a number of rainfall-runoff data, and they can be related to rainfall characteristics and physiographic factors of the catchment.

Parameters of Subsurface Flow Region. – Direct runoff consists of surface runoff and prompt subsurface runoff. The prompt subsurface runoff is the one which infiltrates the surface soil and enters the stream promptly but later than the surface runoff due to slow movement. Therefore, the infiltration index indicates the extent to what portion of the direct runoff flows through the subsurface flow region. The index now may be defined as the ratio of the prompt subsurface runoff to the direct runoff. The amount of direct runoff and prompt subsurface runoff can be obtained by a recession analysis of a hydrograph on semilogarithmic paper (4).

The fraction of the subsurface flow region is defined as a region which contains and transmits the prompt subsurface runoff. Since the prompt subsurface runoff is an infiltrated runoff, it is entirely dependent upon the infiltrated flow. This implies that the fraction k can be expressed by the infiltration index.

The storage remaining in the basin at depletion stage is mainly drained from the subsurface flow region. The storage at time t is given in terms of direct runoff.

$$S_t = \frac{Q_t}{\ln K_r} \tag{17}$$

in which K_r is a recession constant. Comparing Eqs.6 and 17 the parameter K can be determined by the recession constan ″ . The recession constant is derived by the recession analys of a hydrograph less groundwater.

$$K = \frac{1}{\ln K_r} \tag{18}$$

LINEAR CHANNEL AND RUNOFF PROCESS

The adopted linear channel in this study differs from a conventional linear channel which translates a given discharge without changing the shape of a inflow hydrograph. The adopted linear channel has an additional function of storage action like a capacitor. In both cases the translation time through a given length of channel remains unchanged for a discharge of any magnitude. Such a linear channel is a conceptual representation of subwatershed defined by isochrones. The distribution of subwatersheds has been shown to be obtained analytically from a unit hydrograph of unit duration (5).

Each linear channel has two different inputs, one is a routed runoff through just previous linear channel (upstream subwatershed) termed primary input, and other is an effective rainfall from its corresponding subwatershed named as secondary input. The secondary input is assumed to enter the linear channel at the upper end. The primary input to the first linear channel (upper most subwatershed) does not exist and the secondary input is the effective rainfall converted to flow rate at certain time interval ΔT. The flow rate obtained as such becomes the boundary condition at the upper end of time-space grid of linear channel. The boundary condition at the lower bound is given by Eq.4. With the initial condition of zero discharge the routing is carried out through the first linear channel. The routed outflow becomes the primary input to the second linear channel. The effective rainfall on the second subwatershed is taken as the secondary input to the second linear channel. The total input to the second linear channel is therefore the concurrent sum of the primary input and the secondary input. The total input serves as the boundary condition to the second linear channel. The routing is carried out through the second linear channel and the resulting outflow becomes the primary input to the third linear channel. This process is repeated to the last subwatershed to obtain a complete runoff hydrograph.

CONCLUSIONS

A runoff model was developed for deriving a complete time history of direct runoff. The model requires a prior knowledge of linear channel, translatory velocity of rainwater, diffusive factor, infiltration index, recession constant, and the percentage ratio of flow volume in the subsurface flow region to the direct runoff. Working rules are provided for evaluating the parameters of the model. The study is at the stage of examining both the formulated model and the evaluation of parameters to reproduce a runoff hydrograph from rainfall records for selected catchments, and despite data limitations for the study preliminary check of the model (not reported herein) shows promise for applications in the areas of a complete time variation of runoff for water use and pollution abatement, hydrograph synthesis, and runoff computation for the catchment with considerable variation in rainfall in space and in physiographic factors in the direction of flow. Progress to this point has pointed out a need for continued work for solid working rules for the model parameters with a large number of rainfall events from a large number of catchments. Such subsequent efforts must be considerations relating the parameters to easily obtainable rainfall, physiographic, and geologic factors of the catchment. Continuing work is being undertaken to establish the working rules and runoff process.

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