大動脈의 數理 모델을 使用한 새로운 非觀血的 心搏出量 計算方法

論 文
28—12—1

Non-Invasive Cardiac Output Estimation Based Upon A Mathematical Model Of The Aorta; Comparison With Thermo-Dilution Methnd In 13 Patients

閔 丙 九*
(Byung-Ku Min)

Abstract

Cardiac output was calculated for 13 patients using a non-invasive pulse transmission method and then compared with the cardiac output measured simultaneously using a thermo-dilution method. The non-invasive pulse trans-mission method is based upon a parameter optimization technique using an analog model of the aorta[1]. The optimal parameters of the model(taper coefficient, hoop elasticity) are determined from a minimization criterion obtained from the difference between the measured and the model-based transfer function of the fundamental component magnitude of the pressure pulse. This pulse is propagated from proximal(carotid artery) to distal points(femoral or abdominal artery), and the pulses are measured using a piezo-resistive pulse microphone. The aorta diameter is measured using an ultrasonic technique.

In the 16 measurements with 13 patients, the linear regression between cardiac output measured by thermo-dilution, $\overline{Q}_T$, and the calculated cardiac output by the non-invasive pulse transmission method, $\overline{Q}_P$, was related by $\overline{Q}_P=0.78\overline{Q}_T+0.75$ with a correlation coefficient of 0.89.

Keywords: Cardiac output, non-invasive method, mathematical model of the aorta, parameter optimization, thermo-dilution method.

I. Introduction

The detection of changes in the stroke volume of critically ill patients is important in assessing cardiovascular function, and also in evaluating a patient's response to therapy. Determination of the stroke volume by Fick or indicator-dilutlion methods is too cumbersome to perform repetitively, especially in acutely ill patients. For this reason, estimation of stroke volume from instantaneous arterial blood pressure waveforms has been used as an alternative monitoring method. Since the computer-based patient monitoring system is increasingly available in cardiac intensive care unit, clinical application of the pulse contour method is becoming more practical and desirable.

Mcdonald[2] used two pressure recordings 3—5 cm apart in the ascending aorta. Both curves were subjected to Fourier analysis and the apparent phase-velocity was calculated for each of ten harmonics. This phase-velocity and the time-derivative of the proximal pressure curve were then incorporated into the Womersley equation for calculation of aortic flow, and stroke volume

* 正會員 : 서울醫大 助敎授 · 工博
接受日字 : 1979年 8月 9日

as an integral of aortic flow. In animal experiments, it was shown that this pressure-gradient method was less susceptible to sudden changes in peripheral vascular dimensions than the pulse-contour methods based upon a single pressure measurement. One problem that arose in the use of this method was the inaccuracy of the measurement of the pressure gradient along an aorta which has nonuniform geometric and elastic characteristics. To overcome this problem, Mathukrishan and Jaron[3] used a parameter optimization technique to compute aortic input impedance, in a manner similar to Strano's method[4] based upon the aorta model developed by Welkowitz and Fich[1]. Instantaneous aortic flow waveforms were calculated from the input impedance and the proximal aortc pressure. The aortic flow waveform estimated by this analysis closely matched the waveform measured using an electromagnetic flow meter. This method based upon two pressure measurements is designed for invasive stroke volume estimation. It requires a catheterization procedure and the maintenance of continuous flush attachments for the arterial catheter-transducer system.

It would be highly desirable to have a reliable noninvasive method which could be easily applied to detect changes of stroke volume without requiring catheterization procedures. The object of this study is to evaluate a noninvasive pulse transmission method.

Ⅱ. Method

1. Measurement: Using a piezo-resistive pulse transducer (Model PSA, Electronics for Medicine), two pulses (left carotid and left femoral or abdominal), were simultaneously measured. The maximum and minimum values of the carotid pulse were calibrated to the systolic and diastolic pressures as measured with an occluding cuff manometer. The diastolic pressure was also used to calibrate the minimum of the distal pulse based upon the observation that there is only a negligible change in diastolic pressure from proximal to distal locations, while there is a large variation

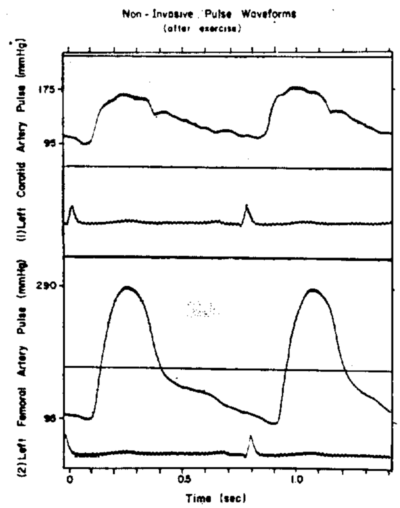

Fig. 1. Non-Invasive pulse waveforms of the left carotid artery (top) and left femoral artery (bottom)

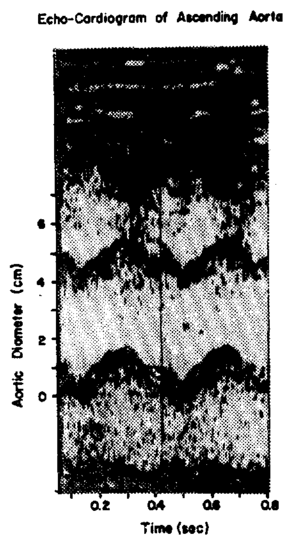

Fig. 2. Echo-Cardiogram of ascending aorta

in systolic pressure. Fig. 1 shows the non-invasive pulse waveforms measured in a patient after moderate exercise. The top figure in Fig. 1 shows the left carotid artery pulse and E.C.G. waveforms, and the bottom figure shows the left femoral artery pulse and E.C.G. waveform. Aorta diameter was measured using a non-invasive ultrasonic technique (Smith-Klein Instrument, #20A-S ultrasonoscope), and the distance between the Proximal and distal pulse locations was also measured. An Echo-cardiogram of the ascending aorta of a patient is shown in Fig. 2. A transducer was positioned at the third or fourth left-intercostal space near the sternum. It was angulated to obtain initially the characteristic echo of the anterior mitral valve leaflet. It was then pointed slightly in a more cethalad direction to obtain the echo of the aortic root. A thermodilution method was used for the control measurement of cardiac output and it was measured after a $10cm_3$ cold saline injection into the right atrium. An Edwards Thermodilution computer was used for this calculation. The recorded pulse waveforms were digitized using the Graf/Pen Sonic digitizer (Model GP-2). The digitized data were used in a Fourier serier analysis program of the PDP-10 computer. At least 60 points were sampled for each pulse waveform analysis.

2. Theory and Computation: Calculations of stroke volume and cardiac output from the transfer characteristics of the pulse are based upon a mathematical model of the aorta and upon a parameter optimization technique. In this model, the aorta is considered as a non-uniform thin walled tube with varying geometric and viscoelastic properties along its length.[1]

The relationships of the pressure and flow transforms of the proximal and distal locations are obtained in the following manner.

In the geometrically and elastically tapered non-uniform aorta model,[1] the proximal pressure transform, $P_1(s)$, and proximal flow transform, $Q_1(s)$, are related to the distal pressure transform, $P_2(s)$, and distal flow transform, $Q_2(s)$, as follows:

$$P_1(s)=A(s)P_2(s)+B(s)Q_2(s) \quad (1)$$

$$Q_1(s)=C(s)P_2(s)+D(s)Q_2(s) \quad (2)$$

where the subscripts, 1 and 2, indicate respectively the proximal and distal quantities, s is the Laplace transform operator, and A,B,C, and D are transmission coefficients which are defined by

$$A=e^{-kl}(\cosh\gamma l+\frac{k}{\gamma}\ \sinh \gamma l) \quad (3)$$

$$B=\frac{R_2}{\gamma}e^{-kl}\ \sinh \gamma l \quad (4)$$

$$C=\frac{(\gamma^2-k^2)}{\gamma R_2}e^{kl}\ \sinh \gamma i \quad (5)$$

$$D=\frac{e^{kl}}{\gamma}(\gamma\ \cosh \gamma i-k\ \sinh \gamma l) \quad (6)$$

where l is the distance between proximal and distal location, k is a taper coefficient, and R_2 is the resistance per unit length at the distal end of the aorta and γ is the propagation constant given by

$$\gamma=\sqrt{k^2+sR_2C_2} \quad (7)$$

C_2 is the hydraulic capacitance per unit length at the distal end.

The inertance of the blood can be taken into account using a lumped inertance parameter, Lo, which is located between the proximal and distal ends. Lo is given by

$$Lo=\frac{\rho l}{A_e} \quad (8)$$

where ρ is the blood density (ρ=1005kg/m³), and A_e is the average aortic area over the test distance.

Then, the overall transmission coefficeints including the inertance effect can be obtained from the matrix product

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\times\begin{pmatrix} 1 & sL_o \\ 0 & 1 \end{pmatrix}\times\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \quad (9)$$

where A_2, B_1, C_1, D_1 are the transmission coefficients of the upper half section and can be obtained from eqs. (3—6) with $l/2$ substituted for l. A_2, B_2, C_2, D_2 are for the lower half section.

Using the overall transmission coefficients of eq. (9), the transfer functin between the proximal and distal pressures T_p and the input impedance Z_1 can be expressed as

$$T_p=\frac{P_2(s)}{P_1(s)}=\frac{1}{A+\frac{B}{Z_2}} \quad (10)$$

$$Z_1=\frac{P_1}{Q_1}=\frac{A+\frac{B}{Z_2}}{C+\frac{D}{Z_2}} \quad (11)$$

Where Z_2 is an arbitrary termination impedance at the distal end of the aorta. From eqs. (3—6) and eq. (9), itcan be observed that the A,B,C,D coefficients are functions of the five parameters,

l, k, heart rate, inside radius of the proximal aorta vessel (R_1), and hoop elasticity at the input to the aorta (ε), where ε is given by

$$\varepsilon=\frac{Eh}{2R_1}=\frac{A_0}{C2} \tag{12}$$

and E is Young's modulus of the vessel wall, while h is the thickness of the vessel wall.

In the present study, the heart rate, input to output distance, and aortic input radius are measured. The optimal values of hoop elasticity (ε) and taper coefficient (k) are determined using eq. (10). Using the above five parameters, A,B,C,D coefficients are computed from eqs. (3—6) and (9). Then the input impedance, Z_1, can be computed from eq. (11), and $Q_1(s)$ can be determined from Z_1 and the measured proximal pressure transform, $P_1(s)$.

Using this analysis, the Fourier harmonic components (1st, 2nd, 3rd) of the distal pulse waveform are computed for various values of the hoop elasticity and taper coefficient using the measured (fixed) values of proximal pulse pressures, aortic root diameter, and the distance between the two pulse measurement locations. The hoop elasticity at the input to the aorta is varied from 1.3×10^4 to 2.4×10^4 Newtons/m^2 in steps of 10^3 Newtons/m^2. Nominal values of blood density and viscosity are used for the computation. The taper coefficient is varied from 2.0 to 4.0 in steps of 0.2.

The optimal values of the hoop elasticity and the taper coefficient are determined based upon the criterion of minimum squared error difference between the magnitude of the fundamental component in the computed distal pulse $|P_{2c}(j\omega_0)|$ and that of the measured distal pulse waveform $|P_{2m}(j\omega_0)|$. A direct search method is used in the parameter optimization technique, where a matrix of the cost functions is formulaued for the various values of taper coefficient and hoop elasticity with a different step size depending upon the sensitivity of the cost function to each parameter. After formulation of the matrix, the parameters providing the minimum value of the cost function are chosen as the optimal parameters of hoop elasticity and taper coefficient.

Using these estimated optimal values, the parameters of the model of the aorta (the distributed capacitance and resistance, and the lumped inertance) are computed and the aorta input impedance is calculated for the first three harmonic frequencies. The phasor components of aortic flow $Q_a(j\omega)$ are computed from the phasors of the proximal pulse pressure and aortic input impedance at each harmonic. Then, the instantaneous aortic flow $q_a(t)$ is computed from $Q_a(j\omega)$ using an inverse Fourier analysis. The integral of $q_a(t)$ starting from the baseline of the mid-diastolic phase is taken as the stroke volume and heart rate is the calculated cardiac output using this non-invasive method.

Ⅲ. Results

Fig. 3 shows a typical aortic flow waveform computed by the inverse Fourier method from the first three harmonics. The baseline is also shown as the mean value of the aortic flow during the diastolic period. Table 1 shows a summary of thireen patient's cardiac output measured by thermo-dilution method and calculated from this non-invasive pulse transmission method.

From the 16 measurement shown in Table 1 we observed the following results:

(a) The linear regression between cardiac output measured by the thermo-dilution method $\overline{Q}_T$, and the calculated cardiac output from the non-invasive pulse transmission method $\overline{Q}_P$ was

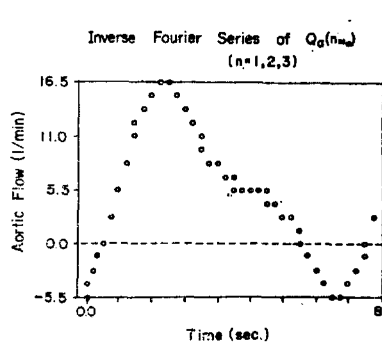

Fig. 3. Instantaneous aortic flow waveform.

TABLE 1. Comparison of Thermo-Dilution and Non-Invasive Cardiac Output
(a) Before Exercise
(b) After Exercise

Patient No.	Thermo-dilution (l/min.)	Non-invasive (l/min.)
1	2.23	2.85
2 (a)	5.16	3.94
(b)	9.12	7.95
3 (a)	8.06	7.39
(b)	9.36	6.85
4 (a)	6.75	4.87
(b)	7.75	7.11
5 (a)	4.73	5.88
(b)	8.96	7.98
6	7.46	7.26
7	6.36	5.69
8	4.91	5.48
9	4.85	3.95
10	2.04	1.72
11	4.80	3.13
12	6.20	6.90
13	6.70	6.37

related by $\bar{Q}_p=0.78\bar{Q}_T+0.75$ with a correlation coefficient of 0.89. The range of $\bar{Q}_T$ was between 2.04 l/min and 9.36 l/min with an average of 6.17±2.25 l/min. $\bar{Q}_p$ changed from 1.72 l/min to 7.98 l/min with an average of 5.56±1.95 l/min.

(b) In the 16 measurements, the error between $\bar{Q}_T$ and $\bar{Q}_p$ was within 15% for 10 measurements, and within 30% for 15 measurements. In 12 measurements, $\bar{Q}_p$ underestimated $\bar{Q}_T$ an average of 16.73±9.68%, and in 4 measurements, $\bar{Q}_p$ overestimated $\bar{Q}_T$ an average of 18.75±8.55%.

(c) In 4 patients, the cardiac output was measured before and after moderate exercise. After exercise, $\bar{Q}_T$ increased in all 4 patients with an average of 49.28±39.38%, and $\bar{Q}_p$ increased in 3 patients with an average of 45.55±42.19%. In patient #3, $\bar{Q}_p$ decreased by 7.3% while $\bar{Q}_T$ increased by 16.1%. This discrepancy may be caused by the time difference of 10 minutes between two measurements for this patient.

(d) A better approximation to $\bar{Q}_T$ resulted from the use of the fundamental component magnitude of the pulse pressure as a criterion for computing the optimal aorta parameters, as compared with the use of either phase angle data or the 2nd or 3rd harmonics of the pulse.

Ⅳ. Discussion

In the non-invasive method of calculating cardiac output, many simplifications are required both in the measurements and the computations. Despite these problems, the present clinical data provides reasonable agreement between the calculated and measured cardiac output. Scme probable causes of error are discussed below.

The presence of soft tissue introduces errors in the measurements of wave amplitude and velocity when these measurements are carried out using sensors placed on the skin. These errors are due to the mechanical compliance of the soft tissue. In the present study, the effects of this error were minimized by using transfer characteristics of the pulse instead of separately analyzing the harmonic components of the proximal and distal waveforms. As shown in Fig. 5, if one can assume that the transfer functions of the pulse from the blood vessel to the skin H1, and H2, in Fig. 5 are the same for both the proximal and distal locations, the effects of these functions will be cancelled out in the computation of the overall transfer function, Ho(nwo). Then the external transfer functions Ho(nwo) equals the internal transfer function, HI(nwo).

Another cause of error in the final results may arise due to the use of thermo-dilution techniques.

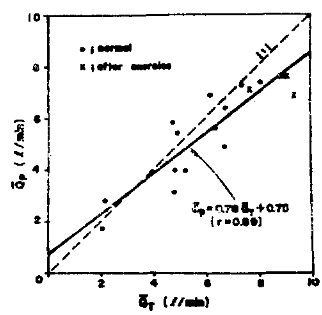

Fig. 4. Comparison between the measured thermo-dilution cardiac output and the computed non-invasive cardiac output

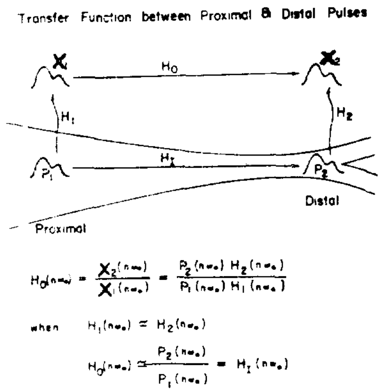

Fig. 5. Transfer functions between proximal and distal pulses

as the control method. While comparisons between thermo-dilution and dye-dilution methods show an excellent correlation[5] recent studies show that there is a 15% over-estimation using the dye-dilution method as compared with the pump flow[6] and electromagnetic flow probe methods[7] in intact animals. In 12 out of the total of 16 present measurements, the non-invasive method underestimated the thermo-dilution cardiac output with an average of 16.7%. Some of this difference may be caused by the over-estimation of the thermo-dilution method as compared with the true blood flow.

A least squared error between the fundamental harmonic magnitudes of the measured and computed distal pressure pulse was used as the criterion for the parameter optimization. This is based upon the observation that the computed cardiac output using the optimal parameters (hoop elasticity and taper coefficient) satisfying this criterion corresponded to the cardiac output value with a minimal error compared to the measured cardiac output. Inclusion of the 2nd and 3rd harmonic amplitudes or the phase angle for the optimization criterion did not improve the overall results. Furthermore, it was found that the computed cardiac output has a well-behaved functional relationship to each of three aorta parameters(taper coefficient, radius, hoop elasticity) when the other parameters are fixed. Since the sensitivity of the cost function to each of these parameters is known, the step size of parameter variation could be chosen so that the most sensitive parameter used a smaller step size than the less sensitive parameter.

In conclusion, the non-invasive pulse transmission method as described can be used to calculate cardiac output with an arror of less than 20% as compared with the thermo-dilution method.

Acknowledgment

This work was supported by the American Heart Association, New Jersey Affiliate under Grant No. 27—4457.

References

1. W. Welkowitz and S. Fich, "A nonuniform hybrid model of the aorta," Trans. New York Academy of Sciences, 29 : 316—331 (1967).
2. D. A. McDonald, Blood Flow in Arteries, 2nd Ed., Williams & Wilkins Co., Baltimore, MD, USA, 436—443 (1974).
3. S. Muthukrishnan, D. Jaron, E. N. Cooper and K. E. Karlson, "Determination of blood flow from pressure measurements: preliminary results," proc. of Third Annual New England Bioengineering Conf., 322—333 (1975).
4. J. J. Strano, W. Welkowitz and S. Fich, "Measurement and utilization of in-vivo blood pressure transfer functions of dog and chicken aortas, IEEE Trans. Biomed. Eng. BME-19 : 261—271 (1972).
5. R.J. Ellis, H. A. Solomon, M. A. Sanmarco and C. W. Lillehei, "Cardiac output determination: Superiority of thermal-dilution," Surg. Forum 20 : 28 (1969).
6. B. G. Min, S. Fich and H. S. Shing, "Accuracy of calibration of electromagnetic flow-probe by dye dilution method," Bull. N. Y. Acad. Med. 53 : 829—840 (1977).
7. D. E. Donald and T. Yipintsoi, "Comparison of measured and indocyanine green blood flows in various orgaus and systems," Mayo Clin. proc. 48 : 492—500 (1973).