디지탈여유 시스템에서의 신뢰도를 위한 MTIF 및 패라메타 해석

論 文
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MTIF and Parameter Analysis of Reliability for the Redundant Digital System

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Abstract

In this paper we deal with a hardware redundency, using the replications of an original module to enhance the reliability of the system in view of static and dynamic redundancy. As results of the study the following facts have been proved that (1) if the mission time is small ($T \leq 0.5$), the effect of spare modules cannot be expected significantly, (2) for the small mission time ($0.1 \leq T \leq 0.3$) the SPR system is more reliable than the other redundant systems. In addition to above facts, it is also proved that for the large mission time ($T \simeq 1$), the dynamic redundant system is more reliable than the static redundant system, and that the SR system is more reliable than the HR system in the dynamic redundant system.

I. Introduction

The increase of need for the ultra-reliable digital system led to the development of various redundancy techniques, which are especially indispensible to the fault-tolerant computer design [10]. This paper is focused on the hardware redundancy, using the replications of an orginal module to enhance the reliability of the system. They are classified with such two groups as static and dynamic redundancy [3].

A. Static redundancy :
   - Self-purging redundancy
   - N-tuple modular redundancy

B. Dynamic redundancy:
   - Stand by redundancy
   - Hybrid redundancy

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The static redundancy can mask the failures and needs no procedure for the elimination of a fault. The intermittent failure cannot influence on the static redundant system. However, the dynamic redundancy requires two-step procedure to eliminate a fault: first, fault detection, location: second, replacement of failed module. The permanent and intermittent failures cannot be distinguished in standby redundancy. The advantages of the dynamic over the static redundancy is described by Avizienis, etc.[10].

They have voting mechanism except standby redundancy. But in this paper the voters are assumed to be perfect.

A comparative analysis of redundant schemes was given by D.S. Taylor [8], whose paper is concerned with only dynamic redundant systems. But in this paper both static and dynamic redundancy are treated and compared on the basis of the two parameters-MTIF and cost factor, which is approximately equal to the number of modules, to assess analytically the performance of the system [7].

II. Theory

(1) Self-purging Redundancy

Self-purging redundant system [1] shown in Fig. 1 consists of modules, disagreement detectors, flipflops, control AND gates, and a threshold voter. The voter output is 1 if and only if the sum of its inputs is equal to or greater than its threshold. This system purges the failed modules by the control AND gates. When a module fails, the failure is detected by the comparison of the module output to the voter output Then the output of disagreement detector is 1, which reset the flipflop. So the flipflop output is 0. Therefore the failed module output is not transmitted to the voter. And retry procedure should be used to distinguish between permanent and intermittent failures, because many failures in digital components are intermittent failures.

The reliability with perfect switch is

$$R(N,M,O)[T]=\sum_{i=M}^{N}\binom{N}{i}R_0^{\,i}(1-R_0)^{N-i}\cdots\cdots\cdots(1)$$

Exact reliability for the SPR system is derived in consideration of the effect of switch unreliability by J. Losq [1], but it is too complicated, and needs cumbersome program for the digital computation. So the author derives a simple expression, assuming that the reliabilities of disagreement detectors and flipflops are constant and the control AND gates are perfect.

These assumptions are reasonable, for the failure rates of the disagreement detectors or flipflops are much less than those of the modules which consist of approximately 1,000 gates in general. In this case, the reliability for the SPR system is easily derived, assuming that $P_x=1$

$$R(N,M,O)[T]=\sum_{i=1}^{N}B_iR_0^{\,i}\ \left(M\leq\frac{N}{2}\right)\cdots\cdots\cdots(2)$$

$$B_i\triangleq(-1)^i\binom{N}{i}\sum_{k=1}^{i}(-1)^k\binom{i}{k}V_k\ \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

$$V_k\triangleq\begin{cases}0 & \text{for } 1\leq K\leq M-1\\ \sum_{i=0}^{M-1}\binom{N-k}{i}C_s^{\,N-k-i}(1-C_s)^i & \text{for } M\leq K\leq N-M\\ 1 & \text{for } N-M+1\leq K\leq N\end{cases}\cdots\cdots(4)$$

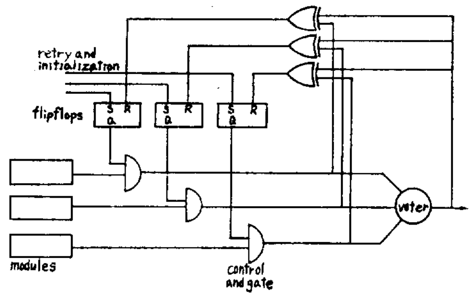

Fig. 1. SPR system with N modules and a voter threshold of M.

(2) N-tuple Modular Redundancy

N-tuple modular redundant system shown in Fig. 2 is formed by a set of odd identical replications of the orginal module and a majority voter [2]. The output of the majority voter is a value corresponding to the majority of its input values. But even if the majority of the module fail, the system might still survive. Such cases are called compensating failures. If one is stuck-at-0 and the other is stuck-at-1 in NMR system, then the system becomes a (N-2)MR system. So the reliability for NMR system is subjected to the values of P_0, P_1 and P_x. Since the only types of failure considered are stuck-at-0, stuck-at-1, and stuck-at-X in general,

$$P_0+P_1+P_x=1$$

If compensating failures are not considered($P_x=1$), the reliability is

$$R(N,M,O)[T]=\sum_{i=M}^{N}\binom{N}{i}R_0^{\,i}(1-R_0)^{N-i}\ \cdots\cdots\cdots(5)$$

$$N=2n+1,\ \ M=n+1$$

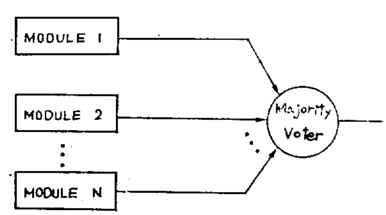

Fig. 2. NMR system with N modules.

The reliability equation for NMR system with compensating failures shown in [2], [3] is

$$R(N,M,O)[T]=\sum_{i=1}^{N}B_iR_0^{\,i} \cdots\cdots (6)$$

$$B_i \triangleq (-1)^i \sum_{k=1}^{i}(-1)^k \binom{i}{k} V_k \cdots\cdots (7)$$

$$V_k \triangleq \begin{cases} 1 & \text{for} \quad n+1 \leqq d \leqq 2n+1 \\ \displaystyle\sum_{i=n+1-k}^{n} \binom{N-k}{i} P_1^{\,i} \sum_{j=n+1-k}^{N-k-1} \binom{N-k-i}{j} P_0^{\,j} P_x^{\,N-k-i-j} & \text{for } 1 \leqq k \leqq n \end{cases} \cdots\cdots (8)$$

for $P_x \neq 0,\ P_0 \neq 0,\ P_1 \neq 0$

In the case $P_x=0$, the compensating functions of NMR system is modified as follows:

$$V_k \triangleq \begin{cases} 1 & \text{for } n+1 \leqq k \leqq 2n+1 \\ \displaystyle\sum_{i=n+1-k}^{n} \binom{N-k}{i} P_1^{\,i} P_0^{\,N-k-i} & \text{for } 1 \leqq k \leqq n \end{cases} \cdots (9)$$

(3) Standby Redundancy

Standby redundant system consists of a working module, several standby modules, a monitor, and switches. This redundancy scheme has been studied by several parpers [4], [8], [9].

The monitor decides whether working module fails or not. If it fails, is switched off and replaced by a standby module. This scheme is shown in shown in Fig. 3. Standby modules are kept in power-off state until they are required to replace the failed module. Therefore, the system reliability can be increased for the failure rate of modules with power-off state is lower than that of modules with power-on state.

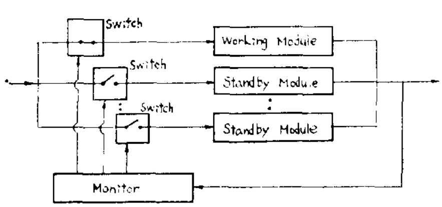

Fig. 3. Standby redundant system configuration.

When the dormancy factor is not 0, the reliability equation for SPR system with S spares shown in [4], [9] is

$$R(1,1,S)[T]=R_0 \sum_{i=0}^{S} \binom{i-1+\frac{1}{\rho}}{i} C^i (1-R_S)^i \cdots (10)$$

In the case $\rho=0$, the reliability equation shown in [4] is

$$R(1,1S)[T]=R_0 \sum_{i=0}^{S} \frac{(\lambda T)^i}{i!} \cdots\cdots (11)$$

(4) Hybrid Redundancy

Hybrid redundant system shown in Fig. 4 consists of standby modules, a voter, and odd number of identical replication of original module, forming a NMR core. This redundancy scheme is combined with standby redundancy and NMR redundancy.

If the disagreement detector discovers a discrepancy, the switch remove the disagreeing module for the NMR core, and replace it by a spare until spare modules are exhausted. The detailed description and design of HR system is presented in [5].

When the dormancy factor is not 0, the reliability equation of HR system shown in [3] is

$$R(N,M,S)[T]=\sum_{j=1}^{N}B_j^{\,S}R_0^{\,j}+\sum_{i=1}^{S}B_{N+i}^{\,S}R_0^{\,N}R_S^{\,i} \quad (12)$$

$$B_j^{\,S} \equiv \left[\binom{NC/\rho+S}{S} \Big/ \binom{S+(N-j)/\rho}{S}\right] B_j,\ j=1,2,\cdots N \cdots (13)$$

$$B^S_{N+i} \equiv \binom{NC/\rho+S}{S}\left[\sum_{m=0}^{i-1}\binom{NC/\rho}{m}(-1)^m-\sum_{j=0}^{N}\right.$$
$$\left.\left[\sum_{j=1}^{i}(-1)^{i-k}\binom{NC/\rho+i}{i-k}\binom{NC/\rho+k}{k}\Big/\binom{k+(N-j)/\rho}{k}B_j\right]\right]$$
$$i=1,2,\cdots,S \cdots\cdots (14)$$

then B_j is shown in equation (7),

$M=n+1,\ N=2n+1$

In the case $\rho=0$, the reliability for HR system shown in [3] is

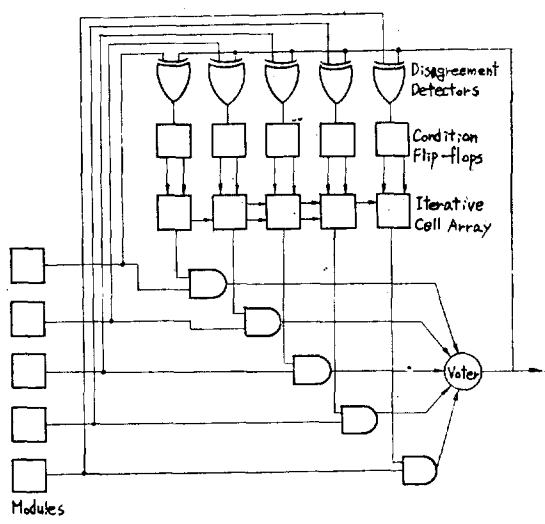

Fig 4. HR system with a TMR core and 2 standby spares.

$$R(N,M,S)[T]=\sum_{j=0}^{N} B_j^S R_0^j \cdots\cdots(15)$$

where

$$B_j^S \triangleq (CN/(N-j))^S B_j \quad j=1,2,\cdots,N-1 \cdots\cdots(16)$$

$$B_N^S \triangleq \sum_{i=0}^{S-1}\frac{(CN\lambda T)^i}{i!}+\frac{(CN\lambda T)^S}{S!}B_N -(CN)^S\sum_{j=0}^{N-1}\left[\sum_{k=0}^{S-1}\frac{(\lambda T)^k}{(N-j)^{S-k}k!}\right]B_j \cdots\cdots(17)$$

then B_j is shown in equation (7),

$M=n+1,\ N=2n+1$

Computational Results

The following results are drawn out from the output of Fortran program for the equations (1)—(17). They are the results of the prameter analysis of reliability and MTIF-cost plots for each system.

(1) Threshold

In SPR system the threshold value is a very critical parameter. The figures (5)—(6) show the variation of the system reliability as the threshold value is varied.

(i) When the coverage is perfect, the system reliability is high if the threshold value is low.

(ii) When the coverage is not perfect and the mission time is small (T≦0.1), the system reliability is high if the threshold value is high. But when the coverage is notperfect and the mission time is large (T≧1), the system reliability is high if the threshold value is low.

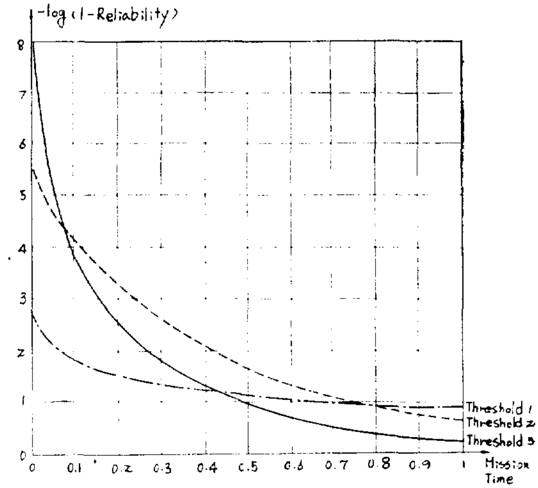

Fig 6. SPR system with 5 modules ($C_s=0.98$)

(2) Stuck-at fault probability distribution

The figures (7)—(8) show how the system reliability varies as the stuck-at fault probability distribution is changed in NMR system.

i) As P_x converges to zero and P_0 to P_1, the system reliability converges to its maximum value.

ii) P_x is decreased or P_1 converges to P_0, the time when the crossover point occurs is larger, and finally the crossover point doesn't occur.

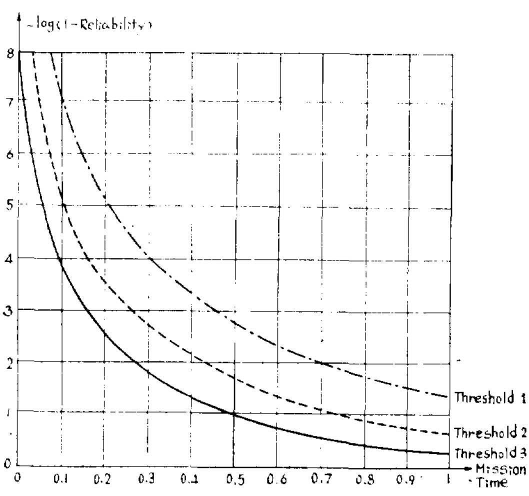

Fig 5. SPR system with 5 modules ($C_s=1$)

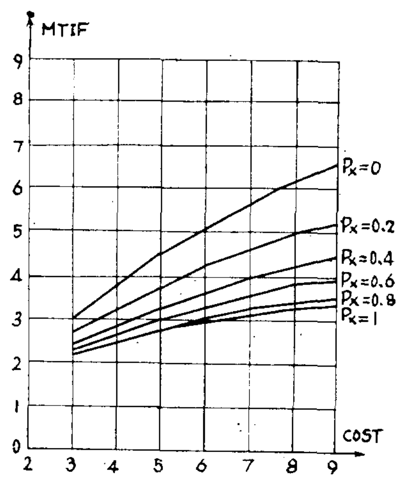

Fig 7. NMR system ($R_{ref}=0.9$, $P_0=P_1$)

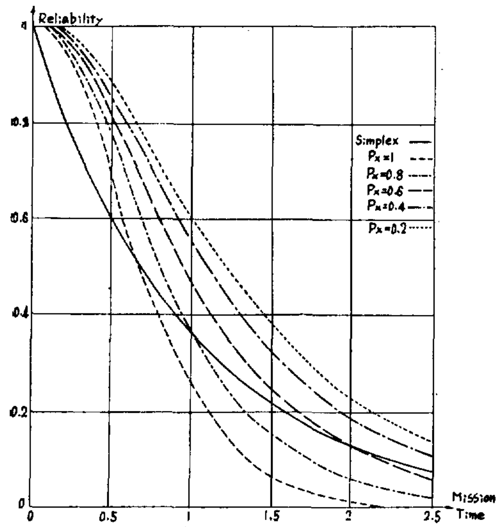

Fig 8. NMR system with 5 modules ($P_0=P_1$)

(3) **Coverage factor**

The figure(9) shows how the system reliability varies as the coverage factor is changed.

The effect of the coverage factor is more sensitive as the threshold value is lower in SPR system. And when the threshold is one, the system reliability is improved as the coverage become smaller in SPR system.

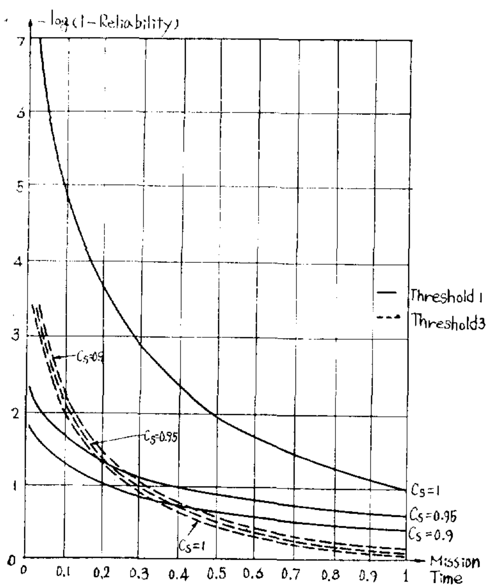

Fig 9. SPR system with 5 modules

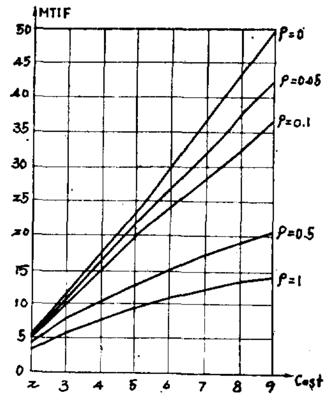

Fig 10. SR system ($R_{ref}=0.9$, $C=1$)

(4) **Dormancy factor**

The figure (10) shows the variation of the system reliability as the dormancy factor is varied.

(5) **Number of module**

The figures (11)—(13) show the change of the system reliability as the number of module is increased.

i) The standby redundant systems are more reliable than the static redundant system with the same number of modules when the coverage is perfect.

ii) The SPR system is more reliable than NMR system with the same number of modules if the mission time is not very short (T≧0.1),

iii) When the coverage is not perfect in HR system, the more spares degrade the system

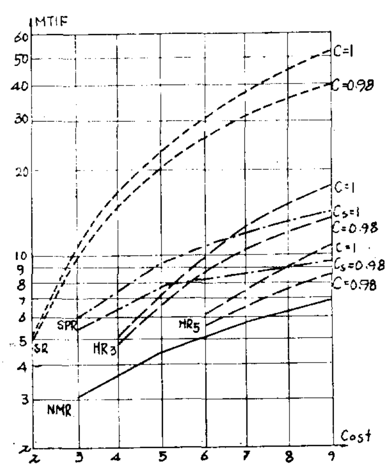

Fig 11. A comparison of MTIF ($R_{ref}=0.9$, $\rho=0$)

reliability in some case.

iv) The maximum gain of reliability is obtained when the first spare module is added in the dynamic redundancy.

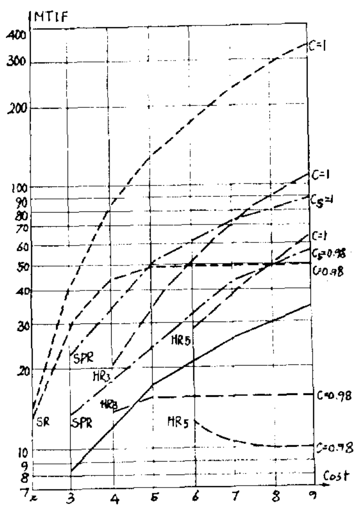

Fig 12. A comparison of MTIF(R_{ref}=0.99, ρ=0)

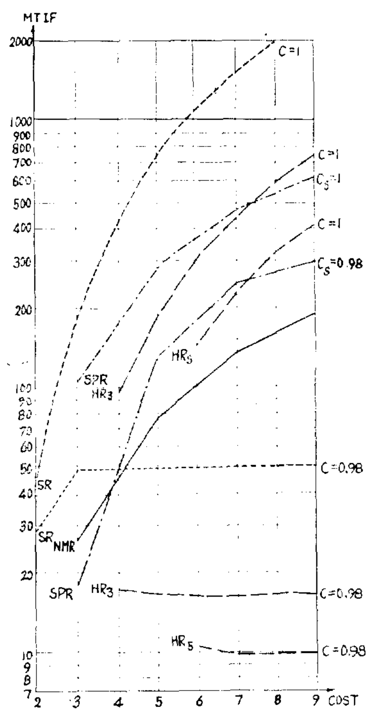

Fig. 13. A comparison of MTIF (R_{ref}=0.999, ρ=0)

Conclusions

For every application, we should compare all kind of redundancy with their performance, cost, simplicity. And their model should be examined for the confidence that should be given to the results. This paper does not deal with real data, but the following conclusions, which are drawn out from the computational results, may be useful to the designer.

(1) If the mission time is small ($T \leq 0.5$) or the coverage is not perfect in the dynamic redundant system, the effect of spare modules cannot be expected significantly. So the optimal number of spare modules is one or two in this case.

(2) There exists an optimal number of spare modules and the more spares degrade the system if the coverage is not perfect.

(3) The threshold value in SPR system should be determinal in consideration of the coverage factor and the mission time.

(4) For the very short mission time ($T \leq 0.05$) the NMR system, which has no switchs, is more reliable than the other redundant system with switches whose coverage is not perfect.

(5) For the small mission time ($0.1 \leq T \leq 0.3$) the SPR system is more reliable than the other redundant systems. In this case the dynamic redundant system cannot take advantage of the dormancy factor.

(6) For the large mission time ($T \simeq 1$), the dynamic redundant system is more reliable than the static redundant system, and the SR system is more reliable than the HR system in the dynamic redundant system.

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Defintions and Nomenclatures

λ: constant failure rate of a core module

λ_s: constant failure rate of a standby module

ρ: dormancy factor $\rho=\lambda_s/\lambda$

R_o: EXP $(-\lambda T)$

R_s: EXP$(-\lambda_s T)$

T: mission time

N: number of core modules

M: minimum threshold number of core modules that must be fault-free to assure a proper operation of the system when compensating failures are not considered.

S: number of spare modules at the beginning of the time interval

R(N,M,S)[T]: reliability of the system with N core modules, S spares, threshold of M. at the mission time T.

Simplex: a nonredundant system

SPR: the Self-Purging Redundancy

NMR: the N-tuple Modular Redundancy

SR: the Stand-by Redundancy

HR: the Hybrid Redundancy

Coverage: Pr[fault detection, location, and appropriate switching are properly performed/fault occurs]

C: coverage in standby or hybrid redundant system. Probability that a failed core module is replaced by a spare one.

C_s: coverage in self-purging redundant system, Pr [the output of a failed module is 0 or forced to send 0/A module fails]

TMAX(RI): maximum mission time at a specified minimum reliability RI, i.e.the time it takes for the reliability to drop to the reference reliability RI.

SIMTMAX: a maximum mission time in a simplex system

MTIF: the mission time improvement factor which is defined as TMAX(RI)/SIMTMAX (RI).

V_i: a compensating function Pr [output of the system is still giving the correct result/(N-i) of core modules have failed]

stuck-at-0: a state of failed module, whose output is stuck at a constant logic 0.

stuck-at-1: a state of failed module, whose output is stuck at a constant logic 1.

stuck-at-X: a state of failed module, whose output is undetermined, viz., not stuck at a constant logic value.

P_i: Pr[stuck-at-i/a module fails] $i=0, 1, X$

P_r: stands for Probability