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論 文
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Feedback Stabilization of Linear Systems with Delay in Control by Receding Horizon

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Abstract

For ordinary systems the receding horizon method has been proved by the author as a very useful and easy tool to find stable feedback controls. In this paper an open-loop optimal control which minimizes the control energy with a suitable upper limit and terminal control and state constraints is derived and then transformed to the closed-loop control. The stable feedback control law is obtained from the closed-loop control. The stable feedback control law is obtained from the closed-loop control by the receding horizon concept. It is shown by the Lyapunov method that the control law derived from the receding horizon concept is asymtotically stable under the complete controllability condition. The stable feedback control which is similar to but more general than the receding horizon control is presented in this paper To the author's knowledge the control laws in this paper are easiest to stabilize systems with delay in control.

I. Introduction

Feedback stabilization has been one of most important design objectives for control systems. There exist several well-known stabilization methods for linear time-invariant ordinary systems The receding horizon method in [1,2,4,5] has been proved a very easy feedback stabilization method, which has a computational advantage and is also applilcabe to time-varying systems. There has been an attempt in [9] to use the result of [4] to stabilize linear systems with delay in state, but it failed to obtain satisfactory results. In this paper the receding horizon concept in [1] is extended to linear systems with delay in control which are represented as follows:

$$\dot{x}(t)=Ax(t)+B_0u(t)+B_1u(t-h) \qquad (1)$$

where $x(t)\ \epsilon R^n, u(t)\epsilon R^m$, and A, B_0, and B_1 are $n\times n, n\times m$, $n\times m$ matrices respectively. The concept of controllability is in order for the analysis of stabilization. In [6,7] there are a few similar controllability concept for the system (1), one of which is as follows: Definition: The system (1) is said to be completely controllable if for every bounded measurable functions $v_i:[-h,0]\to R^m, i=0,1$, and for every x_0, $x_1\epsilon R^n$, there is an admissible control u such that $x(t_1,t_0,x_0,u)=x_1$ where $u_{t_0}=v_0$ and $u_{t_1}=v_1$.

In the above definition u_t is defined by $u_t(s)=u(t+s)$, $-h\leq s\leq 0$. The linear ordinary system $\dot{x}(t)=Ax(t)+Bu(t)$ will be denoted by $[A,B]$. It is shown in [6] that the system (1) is completely controllable for any $[t_0,t_1]$ with $t_1>t_0+h$ if and only if $[A,B_0+e^{-Ah}B_1]$ is completely controlla-

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ble. Historically a general stable constant feedback control has been obtained from the infinite time quadratic cost problem. In [8] it is shown that the steady state optimal control for the quadratic cost problem is given of the form:

$$u(t)=-[(B'_0L_{00}+B'_1L_{01})x(t)+\int_{-h}^{0}[B'_0L_1(\theta)+B'_1L_2(\theta)]B_1u(t+\theta)d\theta] \qquad (2)$$

where $n\times n$ matrices L_{00}, $L_{01}, L_1(\theta)$, and $L_2(\theta)$ are obtained from operator Riccati equations which are represented by partial differential equations and are extremely difficult to compute. The strucure of (2) suggests for a form a stable feedback control law.

Definition: The system (1) is said to be stabilizable if the system (1) is asymtotically stable with a feedback control law of the form

$$u(t)=K_0x(t)+\int_{-h}^{0}K_1(\theta)u(t+\theta)d\theta, \qquad (3)$$

where K_0 and $K_1(\theta)$ are $m\times n$ and $m\times m$ matrices respectively.

A feedback control will be derived from the receding horizon notion and its stability will be proved in Section Ⅱ. By the use of a transformation and some results of Section Ⅱ, a more general stabilizing feedback control law will be obtained in Section Ⅲ.

Ⅱ. Stabilization by a Receding Horison.

Consider the fixed terminal control energy problem: Find the optimal control of the system(1) which minimizes a cost

$$J(u)=\int_{t_0}^{t_1-h}u'(t)u(t),\ u_{t_0}=v_0 \text{ and } u_{t_1}=0 \qquad (4)$$

suject to the terminal constraint

$$x(t_1)=0. \qquad (5)$$

The constraint (5) can be expressed as

$$0=x(t_1)=e^{A(t_1-t_0)}x(t_0)+\int_{t_0}^{t_1}e^{A(t_1-\tau)}[B_0u(\tau)+B_1u(\tau-h)]d\tau, \qquad (6)$$

which can be rearranged to

$$-\left[x(t_0)+\int_{t_0-h}^{t_0}e^{-A(s+h-t_0)}B_1u(s)ds\right]=\int_{t_1-h}^{t_1}e^{-A(\tau-t_0)}B_0u(\tau)\,d\tau+\int_{t_0}^{t_1-h}e^{-A(\tau-t_0)}[B_0+e^{-Ah}B_1]u(\tau)d\tau \quad (7)$$

Let

$$H(t-t_0)\triangleq e^{-A(t-t_0)}B_0+e^{-A(t-t_0)}e^{-Ah}B_1 \qquad (8)$$

The relation (7) can rewritten as

$$-[x(t_0)+\int_{t_0-h}^{t_0}e^{-A(s-t_0)}e^{-Ah}B_1u(s)ds]=\int_{t_0}^{t_1-h}H(\tau-t_0)u(\tau)d\tau \qquad (9)$$

Since the system is completely controllable the optimal solution which minimizes the cost (4) is given by

$$u^*(t)=-H'(t-t_0)W^{-1}(t_1-h-t_0)\ [x(t_0)+\int_{t_0-h}^{t_0}e^{-A(s-t_0)}\ e^{-Ah}B_1u(s)ds \qquad (10)$$

where

$$W(t_1-t_0)\triangleq\int_{t_0}^{t_1}H(\tau-t_0)H'(\tau-t_0)d\tau=\int_0^{t_1-t_0}H(s)H'(s)ds. \qquad (11)$$

The control law (10) is an open-loop control and u_{t_0} is a given initial condition. By replacing t_0 by t from the open-loop control (10) we obtain the closed-loop control

$$u^*(t)=-(B_0+e^{-Ah}B_1)'W^{-1}(t_1-h-t)[x(t)+\int_{-h}^{0}e^{-A\theta}e^{-Ah}B_1u^*(t+\theta)d\theta] \qquad (12)$$

By the receding horizon concept, $t_1=t+T$, we will define a new control law

$$\hat{u}(t)=-(B_0+e^{-Ah}B_1)'W^{-1}(T-h)[x(t)+\int_{-h}^{0}e^{-A\theta}\ e^{-Ah}B_1\hat{u}(t+\theta)d\theta] \qquad (13)$$

It is noted that $H(t-t)=H(0)=B_0+e^{-Ah}B_1$ and that the cost with associated constrainted constraints (4) is chosen so as to give the results in Theorem 2. By the way the control law (13) is obtained it is easy to see that the control law (13) minimizes a moving cost

$$J(u)=\int_t^{t+T-h}u'(t)u(t)dt,\ u_{t+T}=0 \qquad (14)$$

subject to a moving terminal constraint

$$x(t+T)=0 \qquad (15)$$

We summarize the above results.

Theorem 1: The receding horizon control law (13) is the optimal control of the system (1) which minimizes the moving cost (14) subject to the moving constraint (15).

The moving cost like (14) is unusual but has often been used for adaptive control systems. The control law (13) is derived by replacing t_0 and t_1 by t and $t+T$ respectively in order to give a clear understanding of the problem. But it is not difficult to see that by assuming the current time t in (13), (14), and (15) is a fixed value the control law (13) can be derived directly from (10). The stability property of the control law (13) is most important in this paper which is yet to be proved.

Lemma 1: The controllability matrix $W(t)$ in (11) satisfies the following relation:

$$:AW(t)+W(t)A'-[B_0+e^{-Ah}B_1][B_0+e^{-Ah}B_1]'+e^{-At}[B_0+e^{-Ah}B_1][B_0+e^{-Ah}B_1]'e^{-A't}=0 \quad (16)$$

Proof: The controllability matrix $W(t)$ is given by

$$W(t)=\int_0^t e^{-A\tau}[B_0+e^{-Ah}B_1][B_0+e^{-Ah}B_1]'e^{-A'\tau}d\tau \quad (17)$$

From (17) follows that

$$\int_0^t \frac{d}{d\tau}[e^{-A\tau}(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'e^{-A'\tau}]d\tau =-AW(t)-W(t)A'=e^{-A\tau}(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'e^{-A'\tau}\Big|_{\tau=0}^{\tau=t}$$

from which follows the relation (16). This completes in proof.

It is noted that the optimal cost with the open-loop control (10) is given by

$$J(u^*)=[x(t_0)+\int_{t_0-h}^{t_0}e^{-A(s-t_0)}e^{-Ah}B_1u(s)ds]W^{-1}(t_1-h-t_0)\ x[x(t_0)+\int_{t_0-h}^{t_0}e^{-A(s-t_0)}e^{-Ah}B_1(s)ds]' \quad (18)$$

The function (18) will be a Lyapunov function for the system (1) with the control law (13) as shown in Theorem 2.

Theorem 2: If the system (1) is completely controllable, then the system (1) is asymptotically stable with the control law (13) with $T>h$.

Proof: Let $y(t)$ be defined by

$$y(t)\triangleq x(t)+\int_{t-h}^{t}e^{-A(s-t)}\ e^{-Ah}B_1\hat{u}(s)ds \quad (19)$$

Then the receding horizon control law (13) is expressed as

$$\hat{u}(t)=-(B_0+e^{-Ah}B_1)'W^{-1}(T-h)y(t). \quad (20)$$

From (19) and (20) we obtain an important relation:

$$\dot{y}(t)=\dot{x}(t)+e^{-Ah}B_1\hat{u}(t)-e^{Ah}e^{-Ah}B_1\hat{u}(t-h)+A\int_{t-h}^{t}e^{-A(s-t)}e^{-Ah}B_1\hat{u}(s)ds=Ax(t)+B_0\hat{u}(t)+B_1\hat{u}(t-h)+e^{-Ah}B_1\hat{u}(t)-B_1\hat{u}(t-h)+A\int_{t-h}^{t}e^{-A(s-t)}e^{-Ah}B_1\hat{u}(s)ds=Ay(t)+(B_0+e^{-Ah}B_1)\hat{u}(t) \quad (21)$$

$$=[A-(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'W^{-1}(T-h)]y(t) \quad (22)$$

It is noted that the system (21) is an ordinary system. The system (22) is asymptotically stable if and only if the transposed system

$$\dot{\hat{y}}(t)=[A-(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'W^{-1})T-h)]'\hat{y}(t) \quad (23)$$

is asymptotically stable. Take the following Lyapunov function for the system (23):

$$V(\hat{y}(t))=\hat{y}'(t)W(T-h)\hat{y}(t) \quad (24)$$

Taking the derivative of (24) yields

$$\dot{V}(\hat{y}(t))=\dot{\hat{y}}'(t)W(T-h)y(t)+\hat{y}'(t)W(T-h)\dot{\hat{y}}(t) \quad (25)$$

Combining (25) with (22) and (16) yields

$$\dot{V}(\hat{y}(t))=-\hat{y}'(t)[e^{-A(T-h)}(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'e^{-A'(T-h)}+(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)']\hat{y}(t)\leq-\hat{y}'(t)(B_0+e^{-Ah}B_1][B_0+e^{-Ah}B_1]'\hat{y}(t)$$
$$=-\hat{y}'(t_0)e[A-(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'W^{-1}(T-h)](t-t_0)[B_0+e^{-Ah}B_1][B_0+e^{-Ah}B_1]'e[A-(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'W^{-1}(T-h)](t-t_0)\hat{y} \quad (26)$$

Since $[A,\ (B_0+e^{-Ah}B_1)]$ is a completely controllable pair, $[A-(B_0+e^{-Ah}B_1)(B_0+e^{-Ah}B_1)'W^{-1}(T-h)\ (B_0+e^{-Ah}B_1)]$ is a completely controllable pair. Thus the right hand side of (26) is not identically zero and there exists a nonzero constant α such that

$$V(y(t_\delta))-V(y(t_0))\leq-\alpha|y(t_0)|^2 \quad (27)$$

for $t_\delta\geq t_0+\delta$, where $\delta>0$. This indicates that the system (23) and thus the system (22) are symptotically stable. From (19) and (20) follows that

$$|\hat{u}(t)|\leq||(B_0+e^{-Ah}B_1)'W^{-1}(T-h)||\cdot|y(t)| \quad (28)$$

and

$$|x(t)|\leq|y(t)|+h[\max_{-h\leq\theta\leq0}||e^{A\theta}||]\cdot||B_1||\cdot|\hat{u}_t| \leq|y(t)|+h[\max_{-h\leq\theta\leq0}||e^{A\theta}||]\cdot||B_1||\cdot||(B_0+e^{-Ah}B_1)'W^{-1}(T-h)||\cdot|y_t| \quad (29)$$

Since the system (22) is asymtotically stable, $y(t)$ and thus y_t decrease exponentially. Therefore $x(t)$ decreases exponentially from (29). This completes the proof. Theorem 2 canb considered as a generalized result of [4] in the sense that the result in [4] can be obtained by assuming $B_1=0$ from Theorem 2. The stable feedback control law of (13) is shown in Fig. 1.

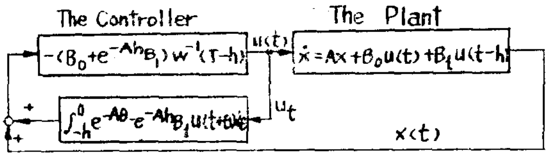

Fig. 1. Feedback Structure of the Receding Horizon Control.

The result of Theorem 2 is applied to a linear system with delay in control:

$$\dot{x}(t)=\begin{pmatrix}2 & 3\\ 0 & 2\end{pmatrix} x(t)+\begin{pmatrix}1\\ 2\end{pmatrix}u(t)+\begin{pmatrix}2\\ 4\end{pmatrix}u(t-h) \qquad (30)$$

The state trajectories and the control value are shown in Fig. 2.

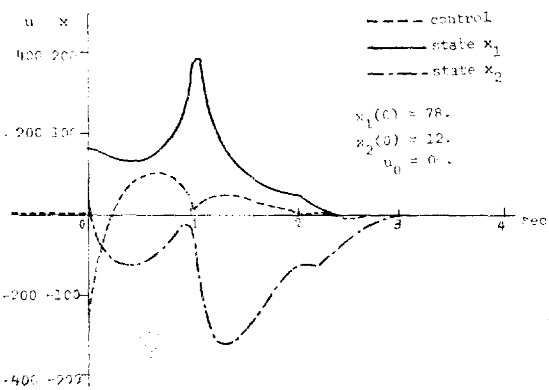

Fig. 2. Responses of the system(30).

By closely investigating the proof of Theorem 2 an unexpected good stabilization method has been found, which will be given in the next section

Ⅲ. A General Stabilization Method.

For the linear system with delay in control (1 let $y(t)$ be defined by

$$y(t)=x(t)+\int_{t-h}^{t} e^{-A(s-t)}e^{-Ah}B_1u(s)ds \qquad (31)$$

for an arbitrary control u_t. For an arbitrary control u_t the system (1) is transformed to

$$\dot{y}(t)=Ay(t)+(B_0+e^{-Ah}B_1)u(t) \qquad (32)$$

If the system (1) is completely controllable, the system (32) is completely controllable. There exist several well known stabilization method [for completely controllable ordinary systems. Let the control

$$u(t)=Fy(t) \qquad (33)$$

be a stabilizing control for the system (32) Then $y(t)\to 0$ and from (31) and (33) we have

$$|x(t)|\le|y(t)|+h[\max_{-h\le\theta\le 0}||e^{A\theta}||]\cdot||B_1||\cdot||F||\cdot|y_t| \qquad (34)$$

From (34) $x(t)\to 0$ as $y(t)\to 0$. The above result can be summarized as follows:

Theorem 3: The completely controllable system (1) is stabilized by a control law

$$u(t)=F[x(t)+\int_{-h}^{0}e^{-A\theta}e^{-Ah}B_1u(t+\theta)d\theta] \qquad (35)$$

where F is a feedback gain which stabilizes the completely controllable ordinary system$[A,\ B_0+e^{-Ah}B_1]$.

The control law (13) can be obtained from the control law (35) with $F=-(B_0+e^{-Ah}B_1)'W^{-1}(T-h)$.

Ⅳ. Conclusion

Though the stabilization of linear systems with delay in control has known to be difficult, easily computable stabilizing feedback control laws (13) and (35) are suggested in this paper under the ocmplete controllability condition. The upper limit t_1-h in the cost and the control constraint $u_{t_1}=0$ in (4) are the author's deliberate choice in order to give the result in Theorem The usual Cost $\int_t^{t_1}u'(t)u(t)d$ will lead to unsatisfactory results. The stabilization of linear systems with delay in state is known to be more difficult [3]. Though the receding horizon concept can be applied to linear systems with delay in state, its stability property has not yet proved.

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